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THE REAL RATE OF INTEREST: A THEORETICAL ANALYSIS

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A theoretical view of the real rate of interest, such as is provided by models of economic growth, is presented. That question is of compelling interest, even though the issues are so long-run as to be of little practical importance. Models reviewed include the Solow model, and its disaggregated extension by Stiglitz; endogenous growth models; the Ramsey model; and the Diamond capital model. All these models are less than fully adequate to answer key questions. Solow-type models are good at demonstrating the influence of grand changes, such as alterations in saving rates, or demographic changes. However key variables—particularly the saving rate—are treated as constants. The Ramsey model, on the other hand, assumes in effect that a major influence on the real rate is a given impatience parameter. The Diamond model is ideal for economies dominated by pension fund saving, but does not describe any actual economy.

I. WHY BOTHER?

Given that our world is faced with many pressing policy problems, which range from the economic transformation of ex-socialist economies, to international trade and the environment, why should we worry about the real rate of interest? The question assumes that the issue is a long-term one, not an issue of macroeconomic management in the relatively short-run. Perhaps the true answer is that the problem is so intriguing that, pressing or not, it is nearly irresistible to the economic mind.

In the final chapter of the *General Theory*, entitled 'Concluding Notes', Keynes (1936, p. 376) wrote:

> I see, therefore, the rentier aspect of capitalism as a transitional phase which will disappear when it has done its work. And with the disappearance of its rentier aspect much else in it besides will suffer a sea-change. It will be, moreover, a great advantage of the order of events which I am advocating, that the euthanasia of the rentier, of the functionless investor, will be nothing sudden, merely a gradual but prolonged continuance of what we have seen recently in Great Britain, and will need no revolution.

Just over 60 years after this eloquent passage was penned, the rentiers are doing rather well, and radical economic revolution, far from being advocated with rising urgency, is reduced to the mantras of half-crazy fringe groups. If a view of where the long-run real rate of interest is headed can be so drastically wrong, it is worth asking why, and whether, economic theory can provide a better prognosis.

II. LEADING QUESTIONS

This article takes a long-run view of the real rate of interest. That involves thinking of the real rate as a single stable value, which applies world-wide in all markets open to the free movement of international capital. The equilibrium real rate represents an invariable value, changing so slowly over time that little error is involved in treating it as a constant. Naturally, shocks to the world economy, such as the 1973 oil shock, may create disequilibrium in markets that are less than perfectly flexible; which is almost all markets in practice, including the market for capital. However, unless the shock concerned changes the world equilibrium permanently, as the first oil shock seems not to have done, the real rate of interest should gravitate back to its long-run equilibrium value.

It will be obvious that the very concept of an equilibrium long-run real rate of interest is an abstraction which sets aside many particularities of time, place, and historical episode. To model it we have to pretend that the world is simpler and more stable than it is in fact. The late Joan Robinson called the world created by such an exercise a golden age, meaning that, as she put it (Robinson, 1958, p. 99): 'We may describe these conditions as a *golden age* (thus indicating that it represents a mythical state of affairs not likely to obtain in any actual economy).'

The advantage of operating at such an extreme level of abstraction is that it allows us to consider some questions which it is difficult to pose, leave alone resolve, in a realistic model. Two examples of such questions are the following.

(i) Is the level of the real rate of interest uniquely determined by market equilibrium conditions, or could there be multiple equilibrium values?

(ii) Even if the market equilibrium value of the real rate of interest is unique, could government intervention move the rate to a different level, and would that change be desirable?

The answer to the first question is more problematic than may be evident, because the real rate of interest—'the price of long-term capital'—is not a price determined simply by supply and demand, as one might conceive the price of milk to be.[1] I mean here particularly that, unlike the milk market, the market for aggregate capital is too large to be considered in isolation, using partial equilibrium analysis. That said, it will be seen below that thinking in terms of an equality between the aggregate demand and the aggregate supply of capital can be a useful way to model the global forces which make their influences felt on the real rate of interest.

III. NEOCLASSICAL GROWTH MODELS: THE SOLOW MODEL

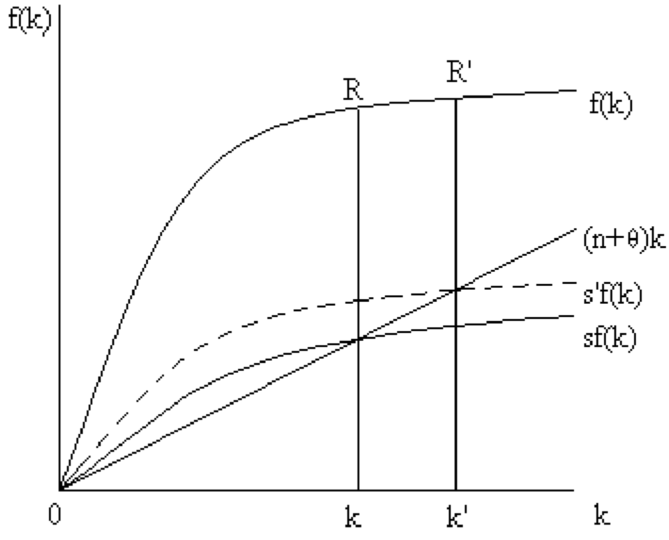
The Solow model (Solow, 1956) is a beautifully simple aggregate model of capital accumulation. Output is generated by the cooperation of two inputs: capital and labour. This is described by an aggregate, constant-returns-to-scale, production function. The growth of the working population is not explained by the model: it is exogenous. Similarly, technical change, assumed to be of the labour-augmenting variety, is exogenous. The accumulation of capital is explained by the model, but only from a simple direct assumption. A constant proportion s of output is saved and invested. These assumptions lead to a simple dynamic equation for the change in per-capita capital, k:

$$\frac{dk}{dt} = s\,f\,[k] - (n+\theta)\,k \tag{1}$$

where s is the saving rate, n is the rate of growth of the employed labour force, and θ is the rate of

[1] In Europe, and elsewhere, the price of milk is highly regulated, and is not determined solely by private supply and demand. So the reference to the market for milk here should be read as applying to an unregulated milk market, such as is found at the present time in New Zealand.

Figure 1
The Effect of Higher Saving in the Solow Model



labour-augmenting technical progress. All these last values—s, n, and θ—are parameters which measure grand influences on the equilibrium real rate of interest.

In steady state, capital grows at the same rate as effective labour—that is raw labour augmented by the benefits of technical progress so k does not change:

$$s\,f\,[k^*]-(n+\theta)\,k^*=0 \qquad (2)$$

where k^* is the steady-state level of capital relative to effective labour. The long-run real rate of interest is the real marginal product of aggregate capital:

$$f_1\,[k^*] \qquad (3)$$

where the subscript denotes partial differentiation.

By manipulating equation (2), we can see how varying the model parameters affects the long-run real rate of interest. For example, differentiating (2) totally with respect to s gives:

$$f\,[k^*]+\{s\,f_1[k^*]-(n+\theta)\}\frac{dk^*}{ds}=0. \qquad (4)$$

For the Solow model to be stable, the term between curly brackets must be negative. Therefore

$$\frac{dk^*}{ds}$$

must be positive. This leads at once to:

Proposition 1: A rise in the saving rate lowers the long-run real rate of interest.

Similar manipulations lead to:

Proposition 2: A rise in either the rate of population growth, or the rate of technical progress, raises the long-run real rate of interest.

Figure 1 illustrates the above conclusions. The real rate of interest is the slope of the production function where the curve $sf(k)$ intersects the line through the origin of slope n + θ. When that last slope is altered, the conclusions stated follow simply. The figure illustrates the effect of a change in s. The dotted function is for a higher value of s. With the low value of s, equilibrium capital per head is at k, and the real rate of interest is the slope of the production function at R. With the high value of s (s')

equilibrium capital per head is at k', and the real rate of interest is the slope of the production function at R'.

IV. LONG-TERM MOVEMENTS IN THE REAL RATE OF INTEREST

When a model is as simple as the Solow model, it should be applied to reality only with extreme care. However, in the knowledge that the short-run dynamics of the model do reflect Propositions 1 and 2 of the previous section, and in the hope that the model may capture at least the effects of the great forces under consideration, the following discussion of long-run movements in the real rate of interest is offered with a handle-with-care warning.

It is the case that real rates of interest have risen in the major industrial countries over the last 25 years. What this means for our present purposes is not so easy to discern. Macroeconomic developments have probably been the major influence. Inflation and the real rate of interest have been negatively correlated; and the last quarter century has been one of declining inflation.

Leaving aside the macroeconomic argument, many of the longer-term underlying influences on the real rate of interest which have operated in recent decades are such as would be expected to lower the rate. Since the world economy is dominated by the rich OECD countries, we can reasonably confine attention to the changes which have been experienced in these countries in the last two or three decades. These could be characterized crudely as follows.

Changes in aggregate net saving
These have been less marked than might be supposed. There have been cut-backs in government dis-saving (borrowing), but those have not been dramatic. The slow ageing of populations has had marked demographic effects on the age composition of populations. Some economic theories say that lower government dis-saving should be offset by lower private saving. Something like that did happen in OECD countries, but according to cross-section regression analysis reported in Edwards (1995), only 50 per cent of differences in public saving is offset by changes in private saving. Simple life-cycle models predict that ageing of the population should decrease national saving. However, empirical studies show that the old save more than the young. In fact, private saving rates have not changed greatly.

A fall in the rate of growth of the working population
This is specially marked in Europe.

Possibly a diminution of the rate of technical progress
This is most uncertain and controversial. Those who advocate this position point to the slow growth of the US economy, particularly when it is measured in productivity terms. Others argue that US growth is underestimated.

On feeding these effects into the Solow model, clear conclusions do not emerge. In any case, the argument points to very slow changes in the real rate of interest, which may need many decades to work themselves out. While dreaming of the long run, we have to live out our brief lives in a sequence of short runs. This is a conclusion disappointing to our theoretical enquiry. Not just the short run, conceived as a brief transitory episode, but extended historical periods, amounting to much of a lifetime, may be dominated by 'temporary' macroeconomic disequilibrium conditions.

V. EXTENSIONS OF THE SOLOW MODEL

(i) Stiglitz's Model: Disaggregation and Economic Convergence

In a superb paper, Stiglitz (1969) asked and answered a pointed question concerning the Solow model. This model is highly aggregated and in more than one respect. At the time there was great concern that capital is treated as a simple aggregate in the model. Stiglitz addressed the issue that economic agents are treated as if they were one huge representative agent, and he investigated what would happen if agents were to be differentiated according to what shares they enjoy in the ownership of aggregate capital. Assuming that agents all supply the same quantity of labour, earn the same rate of return on whatever capital they own, and all save the same share of total income, Stiglitz established the following result.

Proposition 3 (Stiglitz): Regardless of the initial distribution of wealth, the per-capita wealth holdings of all agents will converge to k^*, where k^* is the long-run Solow solution for capital per head.

This is a convergence result, which should be cited in every paper on economic convergence, and never is. The force driving the result is easy to understand. For a poor (rich) agent the share of capital income in total income is low (high). Therefore the rate of growth of wealth, which is proportional to income owing to the constant saving rate out of income, is high (low) for poor (rich) agents.

The Stiglitz theorem tells us not to worry about aggregation, at least not where the uneven distribution of wealth is concerned. Such inequalities will disappear with time, the argument says; although the time required may be long. In any case, the model behaves in aggregate exactly as if there is no inequality.

The theorem of course is as solid as its algebra. It will be clear, however, that it depends, as does any theorem, on its assumptions. And the correctness of these is far from evident. In particular:

- the model assumes that all agents supply labour of equal value, regardless of wealth;
- the model supposes a perfectly integrated global capital market, so that all wealth holders earn the same return on their savings;
- the model supposes that all agents save the same share of total income, regardless of its level and source.

Similar assumptions underlie all convergence models. There is no need to rehearse the likely failure of these assumptions in detail, as their doubtful nature will be plain. The main reason why labour income varies greatly from agent to agent is that human capital varies likewise. We might then include human capital in wealth. That would make the assumption of an integrated capital market less tenable. Barro *et al.* (1995) explore the implications of the international immobility of human capital. In any case, even for physical capital, there is less than perfect mobility, either for capital as such, or for technology. These same immobilities imply that savers everywhere do not earn the same return on their capital.[2]

As a matter of fact, neither are saving rates similar for different individuals, nor are they much the same for different nations. Edwards (1995) describes the spread of observed saving rates in a panel of 32 countries, and details how econometric analysis can explain the differences which exist. These particular findings may be noted.

- National saving rates vary greatly, both across different types of country (e.g. Latin America versus Asia), and within such groups.
- Private saving rates can be partly accounted for by differences in numerous variables. These include government saving and social security arrangements. But in most cases less than 50 per cent of the variance in saving rates is accounted for by the variables used.

It seems that to think of the world as a massive Solow model, even with unequal distribution of wealth, is to mis-describe it. Even so, some of the driving force of Stiglitz's theorem still applies. The current real rate of interest chiefly depends on the wealth owned by the rich: their wealth counts for more, because they have more of it. The growth of wealth, in the long run, however, is strongly influenced by the saving of the poor, because their wealth may grow faster. That last tendency can be offset by low saving rates among the poor, or low wages.

[2] Much research on convergence has focused on so-called 'conditional' convergence. This means that such influences as an economically benign political system, or a good standard of education, need to be right before a nation can participate in the process of globalized converging economic growth. Others have emphasized the importance of infrastructure quality, although this may simply be an expression of similar causes.

(ii) Endogenous Growth

Starting with Romer (1986), a radically different view of economic growth from that proposed by the Solow model has been developed. These ideas build on earlier models of growth, including the von Neumann model and Arrow's learning-by-doing model (see Arrow, 1962; Solow, 1994). A large literature has resulted, to which justice cannot be done here. A good review is to be found in Barro and Sala-i-Martin (1995), particularly in chapters 4 and 5. Our chief concern here will be the question of what endogenous growth implies for the long-run rate of interest.

An example illustrates the kind of implications which emerge. Consider first a model in which there are no diminishing returns to capital taken alone (the so-called AK type of model), so that labour is not a constraint on production. In this kind of model the long-run rate of interest is a technologically determined parameter, being the constant return to capital given by the production technology. This means the discount rate of consumers has to adapt. Now, in contrast to the Solow model, saving affects growth, but has no effect on the long-run rate of interest.

In the Ramsey model described below, the real rate of interest adjusts to equal the steady-state net discount rate; the adjustment mechanism being capital accumulation, which affects the variable real rate of return. If the real rate of return is a constant, and we assume discounting again, it follows that the steady-state net discount rate must bear the burden of adjustment. The discount rate can fulfil that role when it depends on the rate of growth of consumption per head owing to diminishing marginal utility. Then the rate of growth becomes the crucial adjustment mechanism which makes possible a steady long-run equilibrium state.

Concepts of endogenous growth provide a challenging way of looking at economic growth. These models were mainly designed to explain the long-run rate of growth, for which reason they make that value endogenous. By so doing they sometimes make the long-run real rate of interest an endogenous value also. And qualitative conclusions can be radically affected. For instance, the Solow model says that a higher rate of saving will lower the long-run real rate of interest, but will not influence the long-run growth rate. The endogenous growth model family includes models in which a higher rate of saving will influence the long-run growth rate, and may or may not affect the long-run real rate of interest. Sadly, these models take us even further from the simple and definite answers which we might desire.

VI. THE RAMSEY MODEL

Solow's production function was not his invention. Even for the economic dynamics which he developed considerably, it had been employed nearly 20 years earlier by Frank Ramsey in his classic (1928) paper. In short, the critical difference between Ramsey's approach and Solow's is that, while in the latter the saving rate is a parameter, determined by forces not explicitly examined within the model, in the Ramsey model saving is determined by the model itself, the chief purpose of which is to provide a model of optimal saving.

The Ramsey model had little impact when it was first published in 1928. It was before its time, appearing when only a tiny fraction of economists had even a rudimentary mathematical training. When it attracted great interest in the 1960s, as part of a huge flowering of mathematical economics at that time, it was seen almost exclusively as a model of optimal planned development. Recently it has been promoted as a descriptive model, particularly by Robert Barro, who has made it the lynch-pin of his theory of economic convergence (see Barro, 1991).

The merits of the two models are not all on one side. While the Solow model treats the saving rate as a parameter, it is able, as we have seen, to determine the long-run real rate of interest, and to show how it is affected by model parameters. In the Ramsey model, on the other hand, the long-run real rate of interest is in effect a parameter of the model. In a simple basic version of the model, this is easily seen, because unless the real interest rate is equal to the rate at which savers discount utility, it is always optimal to accumulate more capital, or to decumulate capital. Therefore in a steady state, in which capital per head and consumption per head are both constant, the real rate of interest is equal to the time discount parameter.

In such a case, it follows that the Ramsey model is not much use for understanding the determination of the long-run real rate of interest. It says that it is determined by how impatient savers are. And if there are changes in the long-run real rate, these must be explained by alterations in the impatience of savers. And what would explain such alterations? Ignoring these root questions, it is interesting to note that in the Ramsey model the relationship between model parameters and the long-run rate of interest is different from that shown by the Solow model.

This may be seen simply as follows. Suppose that the production function is:

$$F[K, L, t] \tag{5}$$

where K and L are inputs of capital and labour respectively, and t is time.

The labour force grows at rate n; thus the labour force at time t is $L_0 e^{nt}$. Let the instantaneous national utility function be:

$$L_0 e^{nt} U\left\{\frac{C(t)}{L_0 e^{nt}}\right\} e^{-\delta t} \tag{6}$$

where $C(t)$ is total consumption at time t. Aggregate utility depends upon consumption per head, multiplied by the number enjoying that utility. In addition there is discounting of aggregate utility at rate δ. Each nation is taken as having the same rates of growth of labour and technical progress.

Let labour-augmenting technical progress take place at exponential rate θ. For a Ramsey solution to be consistent with steady-state growth, the utility function should take the form:

$$U\left\{\frac{C(t)}{L_0 e^{nt}}\right\} = \left\{\frac{C(t)}{L_0 e^{nt}}\right\}^{\alpha}. \tag{7}$$

Substituting (7) into (6) gives an aggregate utility function:

$$L_0^{1-\alpha} e^{[n(1-\alpha)-\delta]t} C(t)^{\alpha}. \tag{8}$$

What (8) amounts to is interesting. It says that considering just the objective function, the model behaves as if population were stationary and of size $L_0^{1-\alpha}$, and utility is discounted at rate:

$$\delta - (1-\alpha)n. \tag{9}$$

If there is no technical progress, the long-run rate of interest will be given by (9). It follows that the Ramsey model differs from the Solow model in one critical respect. *Population growth does affect the long-run rate of interest, but in the opposite direction from that which rules in the Solow model.*

The reason for this conclusion is that population growth in the Ramsey model has the effect that the future counts for more (i.e. is discounted less), because more people will be weighted in the future for a given consumption per capita.

With technical progress, the position is different. It will cause a faster per-capita growth of consumption, even in steady state.

Now allow for a positive value of θ. In steady state per-capita consumption will grow at rate θ. The marginal utility of individual consumption is:

$$\alpha c^{\alpha-1} \tag{10}$$

and its rate of decrease is:

$$(1-\alpha) c^{-1} \frac{dc}{dt}. \tag{11}$$

In steady state, (11) takes the value $(1-\alpha)\theta$, i.e. the rate of growth of consumption per head equals the rate of labour-augmenting technical progress. This will equal the rate of interest less net discounting. That is:

$$r = (1-\alpha)\theta + \delta - (1-\alpha)n. \tag{12}$$

Notice the contrast to the Solow model shown by (12). In the Solow model, population growth and labour-augmenting technical progress have identical effects. In the Ramsey model they have opposite effects. This is because the growth of human numbers causes the future to be weighted more heavily. Technical change, on the other hand, means that people will be richer in the future, and for that reason their consumption at the margin counts for less.

The Ramsey model always was, and remains, fascinating. However, when our chief concern is a positive explanation of the real rate of interest, it is of limited interest. When we turn to normative questions, such as whether policy interventions should try to lower (or raise) the real rate of interest, the Ramsey model might have more application. Unfortunately, the highly aggregated, representative agent versions which mostly appear in the literature, are of limited help in this connection.

VII. DIAMOND'S CAPITAL MODEL

Diamond built a powerful model by marrying Samuelson's pure-consumption loan framework to a neoclassical capital model, with a production function and investment. Consider a simple version of the model, perfectly adequate for our purposes, in which the consumer lives for two periods. In the first period of her life she supplies 1 unit of labour inelastically and earns the wage rate corresponding to the marginal product of the capital which the previous generation saved for its retirement. She may also save part of her wage and this becomes the capital saved until the next period. Population grows at a positive rate α, so that each generation is $(1 + \alpha)$ the size of the previous generation.

Now the production function gives gross output as a function of gross capital. It is as if capital is corn, and young workers lend part of their corn wages to farmers, who plant it in the ground, and pay the additional yield of corn resulting next period—when it provides a pension.

Details of how this model works can be found in Blanchard and Fisher (1989). I take advantage of that fact to offer here the informal and intuitive discussion which is needed to show how this approach paints the determination of the real rate of interest in quite a different light from that seen in the Solow or Ramsey models. A more formal treatment is provided in the Appendix. We see, in particular, that:

- in the Diamond model the long-run equilibrium real rate of interest may not be unique;
- whether or not it is unique, an equilibrium solution is unlikely to be optimal—sometimes, for example, compulsory saving schemes could make everyone better off;
- there may be a role for national debt, positive or negative, to take the economy towards its optimum.

The budget constraint of a young consumer is:

$$\text{consumption} + \text{saving} \leq \text{wage} \tag{13}$$

where all the variables in (13) refer to the first period of the life concerned. Imagine that saving goes to buy units in a private or public pension fund. The older generation which co-exists (overlaps) with this young generation will sell all the units which it owns. These will be sold without dividend: i.e. not including the earnings on the units. The dividend will be part of the older generation's expenditure. In steady state the fund will grow at rate α. Let the fund held by an older generation at some time be of size $K/(1+\alpha)$. The older generation will sell their $K/(1+\alpha)$ units. The younger generation will buy those units, but will require an extra $\alpha K/(1+\alpha)$, because of its larger size. The pension fund will issue those units.

The pension fund lends its capital to firms. These can be represented by a single competitive profit-maximizing firm with production function:

$$F[K_t, L_t] \tag{14}$$

where K_t and L_t are aggregate factor inputs at time t. In the period in which the working population has size 1, and saves K units, (14) will read:

$$F\left[\frac{K}{1+\alpha}, 1\right]. \tag{15}$$

The young generation works with less capital than it will save, on account of population growth. Denoting partial differentiation by subscripts, the wage earnings of the generation of size 1 will be:

$$F_2\left[\frac{K}{1+\alpha},1\right]. \tag{16}$$

When they are retired pensioners, these individuals will consume the with-dividend value of their units, which will be:

$$K\cdot(1+F_1[K,1+\alpha]). \tag{17}$$

Notice that (17) incorporates the fact that when the capital of the young generation of size 1 comes to be applied to production, the labour force with which it is joined will be of size $1 + \alpha$. Note also that (17) shows that the pension bought by an investment of size K is K multiplied by the with-dividend gross rate of return, $1 + F_1[K,1 + \alpha]$. This last value is treated as a constant by workers who decide how much to save, although collectively they influence it, as the higher the value of K, the lower is its marginal product, or return, $F_1[K,1 + \alpha]$.

Now assume K is chosen to maximize a lifetime utility function:

$$U[w-K,K\cdot(1+r)] \tag{18}$$

where w must be equal to (16), with the K in that case being the value chosen by the previous generation. And r must be equal to $F_1[K,1 + \alpha]$ with K being the value chosen by the current generation. In steady state, all three values of K—that chosen by the previous generation, that chosen by the current generation, and that defining the value of r—will be equal.

One can usefully view the optimizing decision of a single generation as a mechanism which maps from the K value chosen by one generation into a value of K chosen by the following generation. One can see this in the maximization of (18), where the following effects are evident:

- the higher are the savings of the previous generation, the higher will be the wage rate of the present generation;
- the higher are the savings of the present generation, the lower will be the gross rate of return which that generation will enjoy.

These two effects, together with the assumption of regular preferences, imply the following result:

Proposition 4: Assuming future consumption to be a strictly normal good,[3] if the previous generation saves more, then so will the present generation.

The result is true because a higher wage implies a higher demand for future consumption (regular income effect). Therefore that higher demand must manifest itself in higher saving, unless a price effect resists that movement. That price effect would be a fall in the gross rate of return to saving. But that could only happen if there were more saving. So there must be more saving in any case.

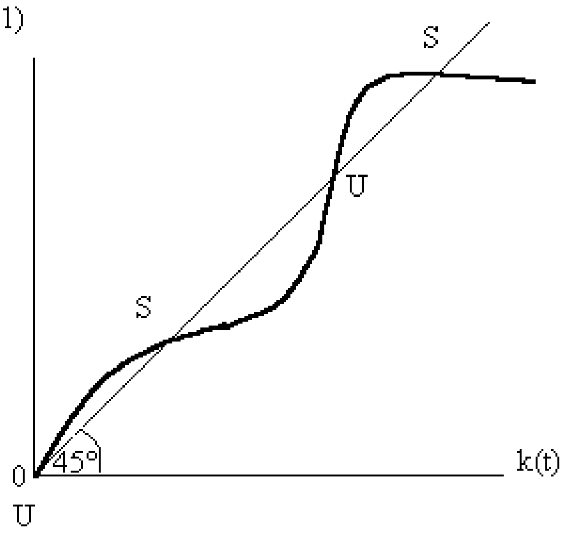
While the slope of the relationship between past and current saving must be positive, it may be greater or less than 1, and this permits multiple steady states, as shown in Figure 2. The states are labelled S or U to indicate whether they are stable or unstable.

If we compare a high-capital stable steady state with a low-capital stable steady state, we reach a conclusion which many people would regard as quite natural, yet which economists are trained to find surprising. *The rich economy is rich because it is rich. The poor economy is poor because it is poor.* It follows that the basic structure and parameters of the model do not determine uniquely the long-run real rate of interest. The Appendix shows that the multiple solution case does not require absurd assumptions, although what is required might be regarded as implausible.

When we turn to optimality, we again encounter surprising results. Private decentralized saving decisions are almost bound to be non-optimal. Why?

[3] Assuming future consumption to be a strictly normal good rules out the case in which the consumer has a fixed target for the pension.

Figure 2
Stable and Unstable States in the Diamond Model



There are two externalities in the Diamond model. They go in the opposite directions. Yet only an extraordinary chance could lead to them cancelling each other exactly.

Consider a substantial[4] group of identical young individuals deciding how much to save. When they have chosen the optimal level of saving, they are by definition indifferent between a slightly smaller and a slightly higher level of saving, these two values bridging the precise optimum level. The following generation would strictly prefer the higher of the savings levels. The higher the rate, the better the wage rate which they will enjoy. The existing generation, regarded as a collective, would prefer a lower rate of saving from this group, because that would raise the gross rate of return on their own savings. In this comparison the divergent interests concern generations alive (or at any rate young) at different times. In steady state, however, the interests of the different generations are in effect summed to make up the lifetime interest of a representative individual. With two offsetting effects the question of whether a different rate of saving could give a higher lifetime utility to the representative agent is uncertain. That granted, intuition suggests that it could only be by virtue of a fluke that the two externalities would just balance each other, and make the private maximizing decision the same as the social optimum.

Notice that a steady state which can be dominated by another steady state is not necessarily Pareto-inefficient by virtue of that fact. This is because one has to find a transition path which is Pareto-superior. Even so, for the Diamond model, if a given steady state is dominated by one with a lower saving rate, it is trivial to find such a transition path. On the other hand, if steady-state saving is below the optimum which would be chosen by a 'social planner', a transition path is likely to involve intergenerational conflicts.

VIII. CONCLUSION: SOLOW OR DIAMOND?

If we imagine the world economy as a huge, globally integrated system grinding out a world real rate of interest, it is plain that it is a hugely complicated system. Therefore no highly simplified, highly aggregated model can ever do it justice. While that is true, it would be comforting to have a simple abstract model of an absurdly simplified world which could explain the determination of the real rate of interest at least for that make-believe place. Sadly, we lack even that basic starting point. We have simple aggregated models which determine the real rate in a mathematical sense. However, on examination, all those which have been discussed above suffer from serious defects.

[4] In this context, substantial means 'non-atomistic', so it can be quite small.

The Solow model and its offspring make the solution for the real rate of interest depend upon model parameters, particularly the saving rate, which should be economically determined values. The Ramsey model virtually decides the real rate by its choice of an impatience parameter. Diamond's capital model determines the real rate of interest in the most subtle and satisfying manner. Saving is fully endogenous, and no single parameter decides the real rate of interest by itself. Here the problem is that the model is highly stylized, and depicts an economy in which pension funds are the sole vehicle for intermediating between savers and firms.

This is not realistic. It is true that pension funds are big in Anglo-Saxon capital markets, though much less so in other countries, and that their role has tended to increase over time. Yet the fact remains that in all capitalist countries there are huge concentrated wealth holdings that pass down the generations through bequests, and which are not remotely liquidated during the old age of their owners. For such wealth dynasties, something more like the Solow model seems appropriate.

It is, of course, the case that the simple models can be improved upon considerably. For instance one could take the view that, while the Diamond model is an excellent representation of pension capital accumulation, the Solow model is better for non-pension wealth. Then the two models could be combined to give equilibrium in a capital market shared by the two types of savers. This could be done, though I have never seen such a model developed.

If the saving rate is to be made endogenous in a Solow-style model, some version of a Ramsey optimal saving model seems to be inescapable. Then the simple Ramsey model could be developed, so as to make the impatience rate endogenous. Considerable work has been done in that direction. The theory becomes technically challenging, and it cannot be exposited here. The interested reader is referred to Becker and Boyd (1997).

The lengthy argument developed above leads to the conclusion that we do not have a good theory of the long-run real rate of interest.[5] Simple models seem to be inadequate, although they may throw light on some important issues. We can glimpse more convincing models of the supply of capital via saving. They are quite complicated, and have sometimes not been developed explicitly. It may be that the simple Solow model, for all that it will never satisfy a theoretical purist, delivers more effective insight than more complicated models can ever do. To be really insightful, it is imperative that a model should be disaggregated, although no usable model can be as disaggregated as realism would dictate. Where pension saving is important, and as it becomes more important as state non-funded schemes are placed by funded schemes, Diamond-style influences have to be included. However various influences are weighted, it will be the case that saving rates, demographics, and technical progress will be the grand forces driving the real interest rate. There is no reason to suppose that the same would not be true for better and more complicated models, if these could be further developed, and if we could get our minds on top of them.

[5] Those who remember the capital theory controversies of the 1960s, and my own role in them, may be tempted to see this statement as a belated conversion on my part. Such is not the case. The Anglo-Italian critics of orthodox capital theory located the problems of building a long-run theory of the rate of interest in the market equilibrium conditions for the demand for saving in the form of capital goods. The true problem, as I have argued, is to build a theory of the long-run supply of savings. For that problem the Anglo-Italians had nothing to offer.

APPENDIX: MULTIPLE DIAMOND-MODEL SOLUTIONS

A very simple two-period model can be solved, to show how multiple solutions can arise. A consumer born at the start of period t plans life-time consumption to maximize:

$$[c_t]^\sigma + \beta[c_{t+1}]^\sigma \qquad \text{(A1)}$$

where $\beta < 1$ reflects discounting of future consumption. This same maximization applies for any value of t.

The consumer supplies one unit of labour in the first period, earns a wage w, and receives a gross return $1+r$ on first-period saving. The budget constraint is:

$$c_t + \frac{1}{1+r} c_{t+1} \leq w. \qquad \text{(A2)}$$

Solving for consumptions in the two periods yields:

$$c_t = \frac{w}{1+\beta^{\frac{1}{1-\sigma}}(1+r)^{\frac{\sigma}{1-\sigma}}} \qquad \text{(A3)}$$

$$c_{t+1} = \frac{w \cdot [\beta(1+r)]^{\frac{1}{1-\sigma}}}{1+\beta^{\frac{1}{1-\sigma}}(1+r)^{\frac{\sigma}{1-\sigma}}}. \qquad \text{(A4)}$$

The gross-output production function is:

$$Y = K + F(K,L). \qquad \text{(A5)}$$

This strange-looking function is in fact quite standard. First note that it is strictly concave in both K and L if $F(.)$ is a standard concave production function. Also if $F(0,L)=0$, then output is zero if $K=0$. If $F(K,0)=0$ and $L=0$, then output is equal to the capital input K. This is only to say that capital is storable. If no labour is supplied, no net production arises, but the capital remains for the next period.

Substituting in marginal productivity conditions for w and r and assuming a steady-state solution gives:

$$\frac{f(k)}{k} - f_1(k) = \beta^{\frac{1}{\sigma-1}}(1+f_1(k))^{\frac{\sigma}{\sigma-1}} + 1. \qquad \text{(A6)}$$

The right-hand side of (A6) increases with k. In general, the left-hand side is not monotonic with k. This opens up the possibility of multiple equilibria.

The left-hand side of (A6) may be rewritten as:

$$f_1(k)\left[\frac{f(k)}{kf_1(k)} - 1\right]. \qquad \text{(A7)}$$

This declines with k in the Cobb–Douglas case—when the term in square brackets is constant. In general, multiple solutions require that the elasticity of substitution should oscillate above and below unity—an untidy possibility, but not strictly impossible.

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