Mathematical models of Synchronous machines

Here we continue to analyze the synchronous machine with rotor saliency and different magnetic properties in d and q direction.
The flux linkages can be obtained in a similar manner by only transforming the stator quantities:

\[
\Lambda_{qd0} = KL_{ss} K^{-1} i_{qd0} + KL_{sr} i_r
\]

With 4 windings on the rotor, the resulting equations are after some lengthy algebra for the stator flux linkage quantities in \( qd0 \) coordinates:

\[
\lambda_q = \left[ L_{ls} + \frac{3}{2} (L_0 - L_{ms}) \right] i_q + L_{sg} i_g + L_{skq} i_{kq}
\]

\[
\lambda_d = \left[ L_{ls} + \frac{3}{2} (L_0 + L_{ms}) \right] i_d + L_{sf} i_f + L_{skd} i_{kd}
\]

\[
\lambda_0 = L_{ls} i_0
\]
Therefore, for 4 windings on the rotor these equations are valid in \(qd0\) coordinates \(^1\):

\[
\begin{align*}
\lambda_q &= L_qi_q + L_{sg}i_g + L_{skq}i_{kq} \\
\lambda_d &= L_di_d + L_{sfd}i_f + L_{skd}i_{kd} \\
\lambda_0 &= L_0i_0
\end{align*}
\]

With only 1 winding the equations will reduce to the following: \(^2\):

\[
\begin{align*}
\lambda_q &= L_qi_q \\
\lambda_d &= L_di_d + L_{sfq}i_f \\
\lambda_0 &= L_0i_0
\end{align*}
\]


Mathematical Model of the Wound Rotor Motor (cont’d)

The power into the machine is given by:

\[ P_{in} = v_a i_a + v_b i_b + v_c i_c + v_f i_f + v_g i_g \]

Using the transformations of the stator quantities to the rotor qd0 reference frame, this equation becomes:

\[ P_{in} = \frac{3}{2} \left( v_q i_q + v_d i_d \right) + 3v_0 i_0 + v_f i_f + v_g i_g \]
With further algebraic manipulation and removing the ohmic loss and rate of change of magnetic energy terms, the electromechanical power developed by the motor can be expressed as:

\[ P_{em} = \frac{3}{2} \omega_r (\lambda_d i_q - \lambda_q i_d) \]

For a P-pole motor with rotor speed \( \omega_{rm} \) mechanical radians/sec, we can write:

\[ P_{em} = \frac{3}{2} \frac{P}{2} \omega_{rm} (\lambda_d i_q - \lambda_q i_d) \]
Thus the electromechanical torque provided by the motor is given by:

\[ T_{em} = \frac{P_{em}}{\omega_{rm}} = \frac{3P}{2} \left( \lambda_d i_q - \lambda_q i_d \right) \]
References