Calculations of Capacitance for Transposed Bundled Conductor Transmission Lines
Multi-conductor Lines. An example with a 2 conductor bundle

\( r \): conductor radius, \( d \): distance between conductors of the same phase

Example: \( V_{a,2,II} \) = Voltage of conductor \# 2 in phase “a”, in section II

We calculate the voltage drop of the 1\textsuperscript{st} conductor in phase “a”:

\[
V_{a,1,I} = \frac{1}{2\pi \varepsilon} \left\{ \frac{q_1}{2} \ln \frac{1}{r} + \frac{q_1}{2} \ln \frac{1}{d} + \frac{q_2}{2} \ln \frac{1}{D_{12}} + \ldots \right\}
\]

We then calculate the voltage drop of the 2\textsuperscript{nd} conductor in phase “a”:

\[
V_{a,2,I} = \frac{1}{2\pi \varepsilon} \left\{ \frac{q_1}{2} \ln \frac{1}{r} + \frac{q_1}{2} \ln \frac{1}{d} + \ldots \right\}
\]
Multi-conductor Lines (2)

For phase “a” in a transposed line with sections I, II and III, we calculate the “average voltage” for each section and then the average voltage drop for the whole line (all sections, phase “a”)

\[
V_{a,I} = \frac{V_{a,1,I} + V_{a,2,I}}{2}
\]

\[
V_{a,II} = \frac{V_{a,1,II} + V_{a,2,II}}{2}
\]

\[
V_{a,III} = \frac{V_{a,1,III} + V_{a,2,III}}{2}
\]

\[
V_a = \frac{V_{a,I} + V_{a,II} + V_{a,III}}{3}
\]
We get a combination of factor with logarithms that for instance lead to roots as follows:

\[ h_i = \sqrt[4]{h_a \cdot h_b \cdot \frac{D_a}{2} \cdot \frac{D_b}{2}} \]
Geometric Mean Distances to Images

Similarly for a combination of distances between phases. These lead to roots as follows:

\[ D_{ij} = 4\sqrt{D_a \cdot D_b \cdot D_c \cdot D_d} \]

\[ D'_{ij} = 4\sqrt{D'_a \cdot D'_b \cdot D'_c \cdot D'_d} \]
Effective Radius - Bundled Conductors

Radius of each conductor = \( r \)

The effective radius of each conductor bundle = \( R \). Compare with the GMR

\[
R = \sqrt[4]{r \cdot d \cdot d \cdot d \cdot \sqrt{2}}
\]

\[
R = \sqrt[3]{r \cdot d^2}
\]

\[
R = \sqrt{r \cdot d}
\]

A phase with 4 conductors

A phase with 3 conductors

A phase with 2 conductors

A phase with 1 conductor

\( R = r \)
**Summary of capacitance calculations**

### With earth’s influence:

\[ C_r = \frac{2\pi \varepsilon}{\ln \left( \frac{2h \cdot D}{R \cdot D'} \right)} \]

\[ D = 3\sqrt{D_{12} \cdot D_{23} \cdot D_{31}} \]

\[ h = 3\sqrt{h_1 \cdot h_2 \cdot h_3} \]

\[ R = 4\sqrt{r \cdot d \cdot d \cdot d \cdot \sqrt{2}} \]

### Without earth’s influence:

\[ C_r = \frac{2\pi \varepsilon}{\ln \left( \frac{D}{R} \right)} \]

\[ D' = 3\sqrt{D'_{12} \cdot D'_{23} \cdot D'_{31}} \]

\[ R = 3\sqrt{r \cdot d^2} \]

\[ R = \sqrt{r \cdot d} \]
Capacitance - Inductance Relation
Transposing of lines allows us to form a symmetric circuit model or single phase equivalent which is identical for all phases both regarding reactance and capacitance.

We remember that the earth is conductive while it is not ferromagnetic.

Therefore 3-phase transmission lines with equi-distant conductors (located at the corner of a triangle with equal sides) will ensure a symmetric model regarding inductance while the earth will influence its capacitance.

This is because the conductor closest to the ground has a different geographical relation than the other conductors - to the earth but not to the other conductors.
Inductance for a system of parallel conductors without considering the internal inductance

\[
\Delta \bar{u} = \begin{bmatrix}
\Delta u \\
\Delta u \\
\vdots \\
\Delta u_n
\end{bmatrix} = j\omega \bar{L}
\begin{bmatrix}
I_1 \\
I_2 \\
\vdots \\
I_n
\end{bmatrix}
\]

\[
\bar{L} = \frac{\mu_0}{2\pi} \begin{bmatrix}
\ln \frac{1}{r_1} & \ln \frac{1}{D_{12}} & \ldots & \ln \frac{1}{D_{1n}} \\
\ln \frac{1}{D_{21}} & \ln \frac{1}{r_2} & \ldots & \ln \frac{1}{D_{2n}} \\
\vdots & \vdots & \ddots & \vdots \\
\ln \frac{1}{D_{n1}} & \ldots & \ldots & \ln \frac{1}{r_n}
\end{bmatrix}
\]
Capacitance Matrix - Beta Matrix

We can now compare the previous matrices regarding both inductance and capacitance. In both cases these matrices can not exist physically, although mathematically there is no problem. This is because each element in these matrices is a logarithm of a factor which has a dimension of \( m \)!!
Therefore the Beta-matrix shown here is not physically possible since each element is a logarithm of the quantity $1/\text{length}$ where the length is measured in $m$. 

\[
\tilde{\beta} = \frac{1}{2\pi\varepsilon} \begin{bmatrix}
\ln\frac{1}{r_1} & \ln\frac{1}{D_{12}} & \cdots & \ln\frac{1}{D_{1n}} \\
\ln\frac{1}{D_{21}} & \ln\frac{1}{r_2} & \cdots & \ln\frac{1}{D_{2n}} \\
\vdots & \ddots & \ddots & \vdots \\
\ln\frac{1}{D_{1n}} & \cdots & \cdots & \ln\frac{1}{r_n}
\end{bmatrix}
\]
Capacitance - Inductance

We now consider the product of these 2 matrices shown to the right. The result is that the product of the capacitance matrix and the inductance matrix is constant for a system of thin conductors.

\[
\frac{\vec{L}}{\mu / 2\pi} = \vec{C}^{-1} \cdot 2\pi\varepsilon
\]

\[
\vec{L} \vec{C} = \mu\varepsilon \cdot \vec{E} = \frac{1}{c^2} \vec{E}
\]

\[
\vec{E} = \begin{pmatrix}
1 & \ldots & 0 \\
\vdots & \ddots & \vdots \\
0 & \ldots & 1
\end{pmatrix}
\]

is the unit matrix.
The inductance for a power line lies in the range of 0.3-0.4 ohm/km.
## ACSR Table Data

### Table A8.1: Bare Aluminum Conductors, Steel Reinforced (ACSR) Electrical Properties of Multilayer Sizes (Cont'd)

<table>
<thead>
<tr>
<th>Code Word</th>
<th>Size (kcmil)</th>
<th>Stranding Al./St.</th>
<th>Number of Aluminum Layers</th>
<th>Resistance ac-60 Hz</th>
<th>Phase-to-Neutral, 60 Hz Reactance at One ft Spacing</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>DC 20°C (Ohms/Mile)</td>
<td>25°C (Ohms/Mile)</td>
</tr>
<tr>
<td>Flicker</td>
<td>477</td>
<td>24/7</td>
<td>2</td>
<td>0.1889</td>
<td>0.194</td>
</tr>
<tr>
<td>Hawk</td>
<td>477</td>
<td>26/7</td>
<td>2</td>
<td>0.1883</td>
<td>0.193</td>
</tr>
<tr>
<td>Hen</td>
<td>477</td>
<td>30/7</td>
<td>2</td>
<td>0.1869</td>
<td>0.191</td>
</tr>
<tr>
<td>Osprey</td>
<td>556.5</td>
<td>18/1</td>
<td>2</td>
<td>0.1629</td>
<td>0.168</td>
</tr>
<tr>
<td>Parakeet</td>
<td>556.5</td>
<td>24/7</td>
<td>2</td>
<td>0.1620</td>
<td>0.166</td>
</tr>
<tr>
<td>Dove</td>
<td>556.5</td>
<td>26/7</td>
<td>2</td>
<td>0.1613</td>
<td>0.166</td>
</tr>
<tr>
<td>Eagle</td>
<td>556.5</td>
<td>30/7</td>
<td>2</td>
<td>0.1602</td>
<td>0.164</td>
</tr>
<tr>
<td>Peacock</td>
<td>605</td>
<td>24/7</td>
<td>2</td>
<td>0.1490</td>
<td>0.153</td>
</tr>
<tr>
<td>Squab</td>
<td>605</td>
<td>26/7</td>
<td>2</td>
<td>0.1485</td>
<td>0.153</td>
</tr>
</tbody>
</table>

- **GMR**
- **Inductance and Capacitance**

**Code words**
Typical values of overhead line characteristics at 50 Hz

<table>
<thead>
<tr>
<th>Operating voltage (kV)</th>
<th>SIL (MW)</th>
<th>Line charging (Mvar/km)</th>
<th>X (Ohm/km)</th>
<th>X/R</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.4</td>
<td>-</td>
<td>-</td>
<td>0.40</td>
<td>0.5</td>
</tr>
<tr>
<td>10</td>
<td>-</td>
<td>-</td>
<td>0.40</td>
<td>0.5</td>
</tr>
<tr>
<td>130</td>
<td>50</td>
<td>0.05</td>
<td>0.40</td>
<td>3</td>
</tr>
<tr>
<td>220</td>
<td>130</td>
<td>0.14</td>
<td>0.40</td>
<td>6</td>
</tr>
<tr>
<td>400</td>
<td>550</td>
<td>0.6</td>
<td>0.33</td>
<td>15</td>
</tr>
<tr>
<td>750</td>
<td>2200</td>
<td>2.3</td>
<td>0.28</td>
<td>30</td>
</tr>
</tbody>
</table>

Typically for voltages below 60 kV line charging may be ignored. For extra high voltages (400 kV+) line charging must be carefully analyzed.
Typical values for underground cable characteristics at 50 Hz

<table>
<thead>
<tr>
<th>Operating voltage (kV)</th>
<th>SIL (MW)</th>
<th>Line charging (Mvar/km)</th>
<th>X (Ohm/km)</th>
<th>X/R</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.4</td>
<td>-</td>
<td>-</td>
<td>0.97</td>
<td>0.3</td>
</tr>
<tr>
<td>10</td>
<td>3</td>
<td>0.01</td>
<td>0.10</td>
<td>0.4</td>
</tr>
<tr>
<td>130</td>
<td>500</td>
<td>2</td>
<td>0.15</td>
<td>2</td>
</tr>
<tr>
<td>220</td>
<td>1000</td>
<td>4</td>
<td>0.18</td>
<td>6</td>
</tr>
<tr>
<td>400</td>
<td>3200</td>
<td>13</td>
<td>0.20</td>
<td>9</td>
</tr>
</tbody>
</table>

For underground cables SIL exceeds the thermal rating which means that underground cable connections are always net producers of reactive power.
Capacitances (nF/km)

Figure 8.19
Effective capacitance $C'_b$ in nF/km per conductor for operating voltages up to 150 kV
Additional Transmission topics

• **Ground wires:** Transmission lines are usually protected from lightning strikes with a ground wire. This topmost wire (or wires) helps to attenuate the transient voltages/currents that arise during a lighting strike. The ground wire is typically grounded at each pole.

• **Corona discharge:** Due to high electric fields around lines, the air molecules become ionized. This causes a crackling sound and may cause the line to glow!
Resistance of transmission lines and transmission real losses
Factor Influencing Line Resistance

Because ac current tends to flow towards the surface of a conductor, the resistance of a line at 60 Hz is slightly higher than at dc. Resistivity and hence line resistance increase as conductor temperature increases (changes is about 8% between 25°C and 50°C)

\[ R_{dc} = \frac{\rho \cdot \ell}{A} \]

- Skin effect (0-5%)
- Temperature
- Conductor winding (Spiraling effect) (0-5%)
Real losses – an example

Line resistance per length, $l$, is given by

$$ R = \frac{\rho l}{A} $$

where $\rho$ is the resistivity

Resistivity of Copper = $1.68 \times 10^{-8}$ $\Omega$-m

Resistivity of Aluminum = $2.65 \times 10^{-8}$ $\Omega$-m

Example: What is the resistance in $\Omega$ / mile of a 1" diameter solid aluminum wire (at dc)?

$$ A = \pi r^2 = 3.1416 \times (0.0127)^2 \text{ m}^2 $$

$$ R = \frac{2.65 \times 10^{-8} \text{ $\Omega$-m}}{A} \times 1609 \frac{m}{\text{ mile}} = 0.084 \frac{\Omega}{\text{ mile}} $$
Effective Resistance (ohm/km)

Effective resistance $R'$ (ohm/km per conductor) at 20 °C mean conductor temperature for conductors manufactured in accordance with DIN 48 201 Part 1 (copper), Part 5 (aluminium), and Part 6 (E-AlMgSi)

<table>
<thead>
<tr>
<th>Nominal cross-sectional area $a_N$ (mm$^2$)</th>
<th>Conductor diameter $d = 2r$ (mm)</th>
<th>Copper $R'_{20\degree C}$ (Ω/km)</th>
<th>Aluminium $R'_{20\degree C}$ (Ω/km)</th>
<th>E-AlMgSi (Aldey) $R'_{20\degree C}$ (Ω/km)</th>
</tr>
</thead>
<tbody>
<tr>
<td>10</td>
<td>4.10</td>
<td>1.806</td>
<td>—</td>
<td>—</td>
</tr>
<tr>
<td>16</td>
<td>4.10</td>
<td>1.139</td>
<td>1.802</td>
<td>2.090</td>
</tr>
<tr>
<td>25</td>
<td>4.10</td>
<td>0.746</td>
<td>1.181</td>
<td>1.370</td>
</tr>
<tr>
<td>35</td>
<td>4.10</td>
<td>0.527</td>
<td>0.834</td>
<td>0.967</td>
</tr>
<tr>
<td>50</td>
<td>4.10</td>
<td>0.366</td>
<td>0.579</td>
<td>0.621</td>
</tr>
<tr>
<td>70</td>
<td>4.10</td>
<td>0.276</td>
<td>0.437</td>
<td>0.507</td>
</tr>
<tr>
<td>95</td>
<td>4.10</td>
<td>0.195</td>
<td>0.309</td>
<td>0.358</td>
</tr>
<tr>
<td>120</td>
<td>4.10</td>
<td>0.155</td>
<td>0.246</td>
<td>0.285</td>
</tr>
<tr>
<td>150</td>
<td>4.10</td>
<td>0.124</td>
<td>0.196</td>
<td>0.227</td>
</tr>
<tr>
<td>185</td>
<td>4.10</td>
<td>0.100</td>
<td>0.159</td>
<td>0.184</td>
</tr>
<tr>
<td>240</td>
<td>4.10</td>
<td>0.075</td>
<td>0.119</td>
<td>0.138</td>
</tr>
<tr>
<td>300</td>
<td>4.10</td>
<td>0.061</td>
<td>0.097</td>
<td>0.112</td>
</tr>
<tr>
<td>400</td>
<td>4.10</td>
<td>0.046</td>
<td>0.072</td>
<td>0.084</td>
</tr>
<tr>
<td>500</td>
<td>4.10</td>
<td>0.037</td>
<td>0.058</td>
<td>0.067</td>
</tr>
<tr>
<td>625</td>
<td>4.10</td>
<td>—</td>
<td>0.046</td>
<td>0.054</td>
</tr>
<tr>
<td>800</td>
<td>4.10</td>
<td>—</td>
<td>0.036</td>
<td>0.042</td>
</tr>
<tr>
<td>1000</td>
<td>4.10</td>
<td>—</td>
<td>0.029</td>
<td>0.034</td>
</tr>
</tbody>
</table>
Circuit models for short transmission lines
Transmission Capacity
One phase equivalent model for a short line

\[ R + jX \]

\[ \frac{C}{2} \]

[Diagram of a one-phase equivalent model for a short line with symbols and equations]
One phase equivalent model for a short line (2)

Equivalent circuit of a short line.
One phase equivalent model for a long line

Equivalent circuit for a long transmission line.
One phase equivalent model for a long line (3)

Equivalent circuit for a long transmission line.

• Series L draws reactive power
  \[ Q_L = wLI^2 \]  – decreases V along line

• Line charging C generates reactive power
  \[ Q_C = wCV^2 \]  – increases V along line
Voltage balance along the line

- $Q_L << Q_C$
  - Light load – voltage increases along line
- $Q_L >> Q_C$
  - Heavy load – voltage decreases

What happens if: $Q_L = Q_C$?
Transmission capacity definitions

• **Thermal limits:**
  - With $V$ being constant, $I$ is the limiting factor ($I_{\text{max}}$)

  $$ P = \sqrt{3} \cdot V \cdot I \cos \phi $$

• **Steady State Stability Limits:**

  $$ P_{\text{max}} = \sqrt{3} \cdot V \cdot I_{\text{max}} \cos \phi $$

• **Natural Loading**
  - Surge impedance Loading or (SIL)

  $$ P = \frac{V_1 \cdot V_2}{X} \sin \delta \quad P_{\text{max}} = \frac{V_1 \cdot V_2}{X} $$

  $$ Z_c = \sqrt{\frac{L}{C}} \quad S_{\text{SIL}} = \frac{V^2}{Z_c} * = \frac{V^2}{\sqrt{\frac{L}{C}}} (= P_{\text{SIL}}) $$
Surge Impedance Loading (SIL)

- SIL is reached, when the generated reactive power equals the consumed power in the high voltage line.
- SIL is not maximum loading but a “characteristic loading”

\[
Q_{\text{consumed}} = X_L |I|^2 = \omega L |I|^2
\]

\[
Q_{\text{generated}} = \frac{|V|^2}{X_c} = \frac{|V|^2}{\frac{1}{\omega C}} = \omega C |V|^2
\]

\[
Q_{\text{generated}} = Q_{\text{consumed}}
\]

\[
\omega L |I|^2 = \omega C |V|^2
\]

\[
\frac{|V|^2}{|I|^2} = \frac{L}{C} = Z_c^2
\]

\[
Z_c = \sqrt{\frac{L}{C}} = \sqrt{\frac{X}{\omega C}}
\]
Surge Impedance Loading

- **Surge Impedance:**
  - Also called characteristic impedance. This is the impedance with which you can insert a surge at the sending end of the line and not get any reflection back at the receiving end.

- **X** is the reactance of the line
  - (in Ohm/km or in Ohm)

- **B** is the susceptance of the line
  - (in Siemens/km or in Siemens)

- $Z_c = \sqrt{\frac{L}{C}} = \sqrt{\frac{X}{\omega C}} = \sqrt{\frac{X}{B}}$
  - ($\approx 250 - 400$ ohm)
Ractive power balance of a transmission line

- For light loading the line produces more Mvars than it consumes
- For heavy loading the line is a net consumer of reactive power