Power Flow Analysis for Control Transformers With Off-nominal Turns Ratios

Incorporating control transformers and variable tap changing transformers into the solution to the power flow problem
Overview of Control Transformer representation

- When we have control transformer in the power system the control operations will influence the $Y_{bus}$ matrix since transformers are modeled as branches that interconnect a pair of buses. We have to adjust the $Y_{bus}$ matrix accordingly! There are 2 basic options depending on simplicity, the “$\pi$-Link” and the “impedance” option:

1. **$\pi$-Link** Examine the $Y_{bus}$ matrix and how it is affected by the control transformer represented as a $\pi$-Link

2. **Impedance**. Examine the $Y_{bus}$ matrix with the control transformer as an impedance
A π-Link Control Transformer Branch

Buses

1

2

\( I_1 \)

\( S_1 \)

\( Y_p \)

\( V_1 \)

\( Y_s \)

\( I'_2 \)

\( S_2 \)

\( V'_2 \)

An ideal control transformer \( a:1 \)

A π-Link

The ideal control transformer is lossless but \( a \) is complex:

\[
V'_2 = aV_2
\]

\[
I'_2 = \frac{I_2}{a^*}
\]

\[
V'_2 I'_2^* = V_2 I_2^*
\]
Power Flow Equations for a π-Link Control Transformer Branch

\[ I_1 = \frac{S_1^*}{V_1^*} = V_1 Y_p + \left[ V_1 - a V_2 \right] Y_s \]

\[ I_2' = \frac{I_2}{a^*} = \frac{S_2^*}{V_2^* a} = a V_2 Y_p + \left[ a V_2 - V_1 \right] Y_s \]

\[ I_1 = (Y_p + Y_s) V_1 + \left[ -a Y_s \right] V_2 \]

\[ I_2 = (-a^* Y_s) V_1 + a a^* (Y_s + Y_p) V_2 \]

\[ a a^* = |a|^2 \]

Compare to equations without the transformer!
Accounting for Control Transformers in the $Y_{bus}$ Matrix

Assume we have:

\[ I_1 = V_1y_{11} + V_2y_{12} \]
\[ I_2 = V_1y_{21} + V_2y_{22} \]

\[ y_{11} = Y_s + Y_p \]
\[ y_{22} = |a|^2 (Y_s + Y_p) \]
\[ y_{12} = -aY_s \]
\[ y_{21} = -a^*Y_s \]

- **CONCLUSIONS:** For a π-link, change the $Y_{bus}$ matrix as follows:

  - Change the **diagonal** element $y_{22}$ for one of the 2 buses and multiply it by $|a|^2$. Keep $y_{11}$ unchanged.
  - Change the **off-diagonal** elements $y_{12}$ or $y_{21}$ between the buses in question by multiplying by $a$ or $a^*$. 

A Control Transformer Branch As an Impedance

Buses

An ideal control transformer 1:a

$$Z_s = \frac{1}{Y_s}$$

$$\frac{V_1}{V'_2} = \frac{1}{a}$$

$$aV_1 = V'_2$$
Equations for a Control Transformer Branch As an Impedance

An ideal control transformer \(1:a\)

\[ I_{12} = |a|^2 V_1 Y_s - a^* V_2 Y_s \]
\[ I_{21} = -a V_1 Y_s + V_2 Y_s \]

\[ V_2' = V_1 a = V_2 - Z_s I_{21} \]
\[ I_{21} = \frac{V_2}{Z_s} - \frac{V_1 a}{Z_s} \]

\[ I_{12} = -a^* \rightarrow I_{12} = -a^* I_{21} \]

\[ I_{12} = [a^* (a - 1) Y_s] V_1 + a^* Y_s (V_1 - V_2) \]
\[ I_{21} = a Y_s (V_2 - V_1) + [(1 - a) Y_s] V_2 \]
Algorithm to form the $Y_{bus}$ matrix

$I_{12} = [a^*(a-1)Y_s]V_1 + a^*Y_s(V_1 - V_2)$
$I_{21} = aY_s(V_2 - V_1) + [(1-a)Y_s]V_2$

or

$I_{12} = |a|^2 V_1 Y_s - a^*V_2 Y_s$
$I_{21} = -a V_1 Y_s + V_2 Y_s$
Algorithm to form the $Y_{bus}$ matrix

$$I_{12} = [a^* (a - 1)Y_s]V_1 + a^* Y_s (V_1 - V_2)$$

$$I_{21} = aY_s (V_2 - V_1) + [(1 - a)Y_s ]V_2$$

1. Define the $Y_{bus}$ matrix so that all control transformers have $a = 1$

2. Add the expression $(|a|^2 - 1)Y_s$ to the diagonal element $y_{11}$ and keep $y_{22}$ unchanged. Then $y_{11}$ will become $(|a|^2 - 1)Y_s + Y_s = |a|^2 Y_s$ as it should in the equation to the upper right

3. Add the expression $-a^*Y_s - (-Y_s) = -(a^* - 1)Y_s$ to the off-diagonal element $y_{12}$. Then $y_{12}$ will become $-(a^* - 1)Y_s + (-Y_s) = -a^* Y_s$ as it should in the equation to the upper right

4. Add $-(a - 1)Y_s$ to the off-diagonal element $y_{21}$. Then $y_{21}$ will become $-(a - 1)Y_s + (-Y_s) = -aY_s$ as it should (above, right)
Software in Power System Analysis (1)

- **Matpower**
  [http://www.pserc.cornell.edu/matpower/](http://www.pserc.cornell.edu/matpower/)
- **Eurostag** [www.eurostag.be](http://www.eurostag.be)
- **EDSA** [www.edsa.com](http://www.edsa.com)
- **ETAP** [www.etap.com](http://www.etap.com)
- **CYME** [www.cyme.com](http://www.cyme.com)
- **ASPEN** [www.aspeninc.com](http://www.aspeninc.com)
- **NEPLAN** [www.neplan.ch](http://www.neplan.ch)
- **Interpss** ([http://www.interpss.org/](http://www.interpss.org/)) open software

- **PSS/E, PSS/O** [http://www.pti-us.com](http://www.pti-us.com)
- **DIGSILENT** [www.digsilent.de](http://www.digsilent.de)
- **QuickStab® Professional** [www.scscc-us.com](http://www.scscc-us.com)
- **CAPE** [www.electrocon.com](http://www.electrocon.com)
- **SKM Power* Tools** [www.skm.com](http://www.skm.com)
- **DINIS** [www.dinis.com](http://www.dinis.com)
- **SPARD®** [www.energyco.com](http://www.energyco.com)
- **ESA Easy Power** [www.easypower.com](http://www.easypower.com)

- **PowerWorld**
  [www.powerworld.com](http://www.powerworld.com)
Software in Power System Analysis (2)

- **DSA PowerTools**  
  www.powertechlabs.com

- **SynerGEE**  
  www.advantica.biz

- **SCOPE**  
  www.nexant.com

- **CDEGS**  
  www.sestech.com

- **ATP/EMTP**  
  http://www.emtp.org

- **EMTP-RV**  
  www.emtp.com

- **PSCAD/EMTDC**  
  www.pscad.com

- **IPSA**  
  www.ipsa-power.com

- **MiPower**  
  www.mipowersoftware.com

- **Distribution Management System (DMS)**  
  http://www.dmsgroup.co.yu/

- **Optimal Aempfast**  
  http://www.otii.com/aempfast.html

- **DEW**  
  http://www.samsix.com/dew.htm

- **Simpow**  
  www.stri.se

- **PSAT (Power System Analysis Toolbox)**  
  http://thunderbox.uwaterloo.ca/~f milano

- **TRANSMISSION 2000**  

- **POM tools (POM, OPM, BOR,...)**  
  for contingency analysis, optimal mitigation measures, Boundary of operating regions, stability,...etc  
  http://www.vrenergy.com/
Software in Power System Analysis (3)

- **Fendi**: (free)  
  [http://www.martinole.org/Fendi/](http://www.martinole.org/Fendi/)

- **Anarede** - load flow, **Anafas** - short circuit, **Anatem** - electromechanical stability (Portuguese)  
  [http://www.cepel.br](http://www.cepel.br)

- **General Electric** - **PSLF**,  

- **Intellicon's Voltage Collapse Diagnostic and Postured Control**  
  [http://www.intellicon.biz](http://www.intellicon.biz)

- **Kína: Power System Analysis Software Package (PSASP)**:  

- **MicroTran** is the electromagnetic transients program (EMTP) version of the University of British Columbia.  
  [http://www.microtran.com](http://www.microtran.com)
Symmetrical Short Circuits

Calculations of short circuit currents and power
Brazilian black-out, November 11, 2009

• Press release 22h14 (Brasilia) There was a large disturbance in the National Integrated System (SIN) that caused a partial disruption of the energy supply in the Southeast and Midwestern equivalent to 28,800 MW. The shutdown of three 750 kV lines and the DC link associated with the Itaipu Binacional plant was identified, with consequent shutdown by action of the protective systems of its generating units.

• The effects of this disturbance were mitigated by the protective systems which minimized the extent of the event – preventing other regions of Brazil from being affected.

• The power system reconnection process started immediately in all areas of SIN, but slowly and in a coordinated manner. Currently, the operating conditions of the system equipment are completely back to normal.

• The Minister of Mines and Energy, Edison Lobão, convened an afternoon meeting of the Monitoring Committee of the electric sector – CMSE in Brasilia, with all the companies involved, to analyze the causes of the occurrence.

Press release, Nov 11, 2009; Translation by: Ray Shoultz
Univ. of Texas at Arlington
The Northeast Blackout of 1965 was a significant disruption in the supply of electricity on November 9, 1965, affecting Ontario, Canada and Connecticut, Massachusetts, New Hampshire, Rhode Island, Vermont, New York, and New Jersey in the United States. Around 25 million people and 80,000 square miles (207,000 km²) were left without electricity for up to 12 hours." (Wikipedia, the Free Encyclopedia)
Faults and blackouts in power systems

- Blackouts are often caused by faults in power systems
- Millions of customers may lose power in a blackout
- The blackout starts with an Initial fault
  - After a fault we have New power flow situation. This situation may cause cascade faults.
  - Loads and generators are disconnected during this process
  - Finally the power is restored during a Restoration process and the whole process may take several hours or days
Examples of fault/blackout: USA August, 14th 2003

- Initiating event
- Generator shutdowns
- Final stage
- Voltage collapse
- 50 Million people affected
- Restoration time up to 30 h
Examples of fault: Italy 28\textsuperscript{th}, September 2003

- Initiating event
- Two lines trip in stormy weather
- All lines into Italy lost one by one
- Final stage unknown
- 58 Million people affected
- Restoration time 16.5 h
Faults in power systems

- The Load Flow analysis in a previous section is a study of the normal operations of the power system.
- Fault analysis, on the other hand, is the study of an abnormal behavior of the power system.
- Faults or short circuits can occur anywhere in a power system which is represented as a network, sometimes consisting of hundreds or thousands of nodes (buses).
- Between these nodes we have branches which are transmission lines or transformers. In spite of the representation of these branches as lumped impedances in pi-links or L-links, the faults can happen anywhere along the continuous length of the power transmission line.
Severity of faults

- When a fault occurs the normal currents and voltages in the system are severely disturbed
  - The voltages at each bus in the system will no longer lie between the normal operating limits (from 95% to 105%). It will most likely temporarily exceed such limits. The voltages may drop significantly or even increase
  - The fault currents in many branches and at the fault location will by many times higher than normal load currents flowing in the system prior to the fault
- The voltage or current sources that feed (or cause) the fault currents and the voltage disturbances, are all the generators running in the system at the time of the fault. The generator that is closest to the fault location has the most impact. This “closeness” is measured in terms of impedance in the network
  - Thus the impedance between the sources and the fault location will for instance determine the magnitude of the fault currents
- It is usually safe to assume that the most severe faults happen at a bus or node, since the additional impedance from the bus to the fault location on the transmission line, will increase the impedance between the fault and the sources
Protection against faults

• Most faults are not permanent. For instance a short circuit to earth due to an insulator break-down due to dirt or other disturbance may make a ionized current path to earth. This path may clear when the voltage on the faulty spot is disconnected and the fault has “disappeared”

• Therefore the power system has an associated relay protection system to disconnect selectively the faulty location when fault occur.

• We will primarily study faults which are symmetric. This means that the same fault occurs in all 3 phases. This is the most sever type of fault. If the system can withstand such fault it should be able to withstand other types of faults, such as 1-phase or 2-phase faults.
“Smart Grids”

• “...In a Smart Self Healing Grid the transmission grid will separate into islands with matching generation and load in order that the total interconnection does not fail....”

• Then in the transmission islands that do fail, Smart Micro or Distribution level grids will serve the most critical loads with the local distributed generation and storage.

• “....It must be remembered that the major production centers are far from large centers of consumption and, therefore, it is necessary to transport energy over long distances. In addition, due to technical and economic reasons, systems are increasingly interconnected allowing them to be more exploited to there limits, often leading to lack of investment in production. Thus, given the current state, it seems impractical, technically and financially, that any concept of smartgrid avoid situations like this...”.

• ...Blackouts will always exist since the grid is exposed to nature and its information and control systems will be built by humans and will therefore have errors...”
Faults

- **Selectivity** means that we want to disconnect only the fault location from the system but continue to supply electric power to the remainder of the system as much as possible.
- The **relay protection** system gives signal to the appropriate circuit breakers to disconnect the fault location.
- Often after disconnecting the fault, the relay protection gives signals to **reclose** the circuit breakers to see if the fault has cleared. Sometimes there can be a series of reclosures.
Introduction to power system faults

• Types of Faults
  – Over Voltage
  – Open Conductor
  – Time overcurrent
  – Instantaneous overcurrent.

  • 3 phase fault
  • 3 phase to ground fault
  • single line to ground fault
  • line to line fault
  • double line to ground fault
Causes and protections against faults

• Causes of Faults
  - Lightning
  - Plant/Animal Life
  - Equipment failures
  - Human Error

• Protection Against Faults: The relay protection system
  - Fuses
  - Circuit Breakers
  - Relay protection system: To measure and monitor the power system for faults and deliver signals to isolate and disconnect the faulted part
Short Circuits

- One-phase short circuit to earth (70%)
- Short circuit between phases (15%)
- Short circuit between 2 phases and earth (10%)
- 3 phase symmetrical short circuit (5%)

- The 3 phase symmetrical short circuit is the most severe type of fault. Therefore we originally concentrate on this type of fault. If the system withstands a 3 phase balanced fault it will likely withstand other types.
One Phase Short Circuit to Ground

The location of the fault in the power system is a specific 3 phase "connection" point. (for instance in or near a bus, line or transformer)

$I_f$ is the short circuit or fault current

$Z_f$ is the short circuit impedance
Short Circuit Between 2 Phases

The power system

The location of the fault in the power system

\( I_f \) is the short circuit or fault current.

\( Z_f \) is the short circuit impedance
Simultaneous Short Circuit Between Phases and to Ground

The location of the fault in the power system

$I_f$ is the short circuit or fault current.

$Z_f$ is the short circuit impedance
A Symmetric 3 Phase Short Circuit to Ground (Worst Case)

The power system

\[ I_f \] is the short circuit or fault current
\[ Z_f \] is the short circuit impedance

\[ Z_{fa} = Z_{fb} = Z_{fc} = Z_f \]
Fault analysis versus load flow analysis

• When performing fault analysis on the power system the system is modeled in a similar way as when we are performing load flow analysis.

• However we have to treat generators differently, they are the exception.

• We have to take into account the internal reactance of the generators since these will impact the fault currents significantly.
  – We must bear in mind that in load flow, we have viewed the generators only as injection units which are injecting quasi static currents into the corresponding bus.

• In fault analysis generators are modeled as Thevenin equivalent circuits with a voltage source behind a transient reactance.
  – Furthermore, regarding this reactance we must bear in mind that transient behavior of generators is different from steady state behavior. This means that the generator transient internal reactance is different from the steady state reactance.

• The determination of transient reactance of generators is beyond the scope of an introductory course in power system analysis.
Models for generators in power systems analysis

- A single-phase model for a generator in power flow analysis as injecting current into a bus or a simple voltage source behind a synchronous reactance.

- A model for a generator in short circuit analysis. The machine short circuit reactance $x_{\text{short circuit}}$ depends on time frame:
  - transient reactance
  - subtransient reactance
Aggregating the influence of all generators

- As previously mentioned, all the generators in the system feed fault current into the fault location. In order to analyze the network and find the combined current caused by all these generators we use the **Thevenin's theorem** for electrical networks.
....”The changes that take place in a network voltages and currents, due to the addition of an impedance between two network nodes are identical with those voltages and currents that would be caused by an electromotive force (EMF) placed in series with the impedance and having a magnitude and polarity equal to the pre-fault voltage that existed between the nodes in question and all other active sources being zeroed”.....
Thevenin’s theorem (2)

- If you connect an impedance, $Z$, between 2 nodes in a network, certain changes in voltages and currents take place.

- These changes are equal to the voltages and currents that arise when:
  - All active sources are deleted
  - A new voltage source, $E$ is connected between the 2 nodes in series with the new impedance
  - The magnitude of $E$ is the voltage that existed between the 2 nodes, before $Z$ was connected
1. Define the power system and its operational conditions including the fault location

2. Calculate the pre-fault power flow and calculate pre-fault voltages and currents. Calculate the internal impedance and pre-fault voltage (The “Thevenin equivalent”) at the fault location

3. Calculate the fault currents (i.e. system wide changes of currents)

4. Calculate the post fault currents as the sum of the pre-fault currents and the fault currents (by the superposition principle)
Short circuit calculations

• Every location in the power system has a certain **input impedance**
• For instance: each **bus (ai)** has an input impedance, \( Z_{i,in} \)
• \( Z_{i,in} \) in this circuit could be measured by an “ohm meter” if we zeroed all active sources
• Each bus has a pre-fault voltage, \( V_0 \)
A Power System Model for a Power Flow Circuit Study

- Assume that we have a power system with \( n \) buses as shown in the figure. Assume that generators are at bus #1 and #2 and a fault occurs at bus #3. The following equations apply to this system in steady state load flow situation prior to the fault:

\[
\begin{bmatrix}
I_1 \\
I_2 \\
\vdots \\
I_n \\
\end{bmatrix} = \begin{bmatrix}
y_{11} & y_{12} & \cdots & y_{1n} \\
y_{21} & y_{22} & \cdots & y_{2n} \\
\vdots & \vdots & \ddots & \vdots \\
y_{n1} & y_{n2} & \cdots & y_{nn} \\
\end{bmatrix} \begin{bmatrix}
V_{1,0} \\
V_{2,0} \\
\vdots \\
V_{n,0} \\
\end{bmatrix}
\]

\[
I_{\text{bus}} = Y_{\text{bus}} \cdot V_{\text{bus},0}
\]

Generators are injecting current or loads are drawing current
A Power System Model for a Short Circuit Study

- In a fault situation each generator is a Thevenin circuit - a voltage source behind a reactance - instead if a current/power source.
- Remember, we have to account for the transient reactances of generators as shown in the figure. These must now be a part of a modified $Y_{bus}$ matrix.
- Assume a short circuit at bus #3.
A Power System Model for a Short Circuit Study

- By Thevenin’s theorem, we can remove all voltage sources from generators.
- We instead have to insert a single source at the fault location (bus #3) as we close the breaker (fault occurs).
- The voltage of this new source is the pre-fault voltage at bus #3.

Short circuit, transient reactances, $X_d$, $X_d'$, $X_d''$.
A Power System Model for a Short Circuit Study

- The new modified $Y_{bus}'$ matrix must in a fault situation account for the transient reactances of generators.
- This is in addition to all the branches and pi-links as before in the load flow situation.

$$
\Delta V_1 = \begin{bmatrix}
    y_{11}' & y_{12}' & \cdots & y_{1n}' \\
    y_{21}' & y_{22}' & \cdots & y_{2n}' \\
    \vdots & \vdots & \ddots & \vdots \\
    y_{n1}' & y_{n2}' & \cdots & y_{nn}'
\end{bmatrix} \Delta V_{fault}
$$

Short circuit, transient reactances, $x_d, x_d', x_d''$
Overview

• Assume we want, for instance, to calculate the short circuit current to ground at a given location (node) in the network.

• As we shall see, the network input impedance at this node is the key factor that determines the magnitude of this current.

  – 1\textsuperscript{st} Question: How do we calculate the input impedance and fault current at each node?

  – 2\textsuperscript{nd} Question: How do we calculate the voltages in the network before and after the fault (pre-fault, post-fault voltages)

• We will try first to answer the 2\textsuperscript{nd} Q and then the 1\textsuperscript{st} Q.
Short Circuit (Fault) Analysis Using Matrix Methods

Using matrix calculations to determine short circuit currents (also called fault currents) and voltages
Calculating the voltage changes and post fault voltages at each node in the power system (assuming first that the fault current is known)
Overview

- We will find useful both matrices, the $Y_{bus}$ matrix and the $Z_{bus}$ matrix - which is the inverted $Y_{bus}$ matrix.

\[
I_{bus} = Y_{bus} \cdot V_{bus} \quad \longleftrightarrow \quad V_{bus} = Z_{bus} \cdot I_{bus}
\]

\[
Y_{bus} = Z_{bus}^{-1}
\]

- We will find later that it is useful to have a $Z_{bus}$ matrix building algorithm to make $Z_{bus}$ directly from the branch components, (or “from scratch” so to say) in stead of the inversion of the $Y_{bus}$ matrix.
A Power System Model for a Short Circuit Study

- The voltages after the short circuit occurred are the sum of the pre-fault voltages and the voltage changes, or

\[
\begin{bmatrix}
V_1 \\
V_2 \\
\vdots \\
V_n
\end{bmatrix} = \begin{bmatrix}
V_{1,0} \\
V_{2,0} \\
\vdots \\
V_{n,0}
\end{bmatrix} + \begin{bmatrix}
\Delta V_1 \\
\Delta V_2 \\
\vdots \\
\Delta V_n
\end{bmatrix}
\]

\[V_{bus} = V_{bus,0} + \Delta V_{bus}\]
A Power System Model for a Short Circuit Study

• The pre-fault voltages can be determined by a pre-fault load flow analysis

• By Thevenin’s theorem, the voltages changes can be found by the following matrix equation, if we know the fault current

\[
\begin{bmatrix}
\Delta V_1 \\
\Delta V_2 \\
\vdots \\
\Delta V_n
\end{bmatrix} =
\begin{bmatrix}
z'_{11} & z'_{12} & \cdots & z'_{1n} \\
z'_{21} & z'_{22} & \cdots & z'_{2n} \\
\vdots & \vdots & \ddots & \vdots \\
z'_{n1} & z'_{n2} & \cdots & z'_{nn}
\end{bmatrix}
\begin{bmatrix}
0 \\
o \\
0 \\
I_{\text{fault}}
\end{bmatrix}
\]

• The current column vector has zero elements for all buses except at the bus (#3) where the fault occurs. This element is the fault current

• The Z'_{bus} matrix is the inverted Y'_{bus} has been modified and in matrix notation the equation becomes:

\[
\Delta V_{\text{bus}} = Z'_{\text{bus}} \cdot I_{\text{bus,f}}
\]
A Power System Model for a Short Circuit Study

Therefore, by combining the 2 previous equations we can calculate the post-fault voltages at all buses in the power system.

\[ V_{bus} = V_{bus,0} + \Delta V_{bus} \]

\[ \Delta V_{bus} = Z'_{bus} \cdot I_{bus,f} \]
Transient synchronous machine short circuit currents

- Subtransient, $x_d''$ and transient reactances, $x_d'''$ are smaller than the steady state synchronous reactances, $x_d''$
- A DC component is present in all 3 phases
- The exact shape of the short circuit current will depend on the phase instant of the short circuit and is therefore different in the 3 phases

DC component

sub-transient-áhrif

transient-áhrif

phase “a”

phase “b”

phase “c”
Fault currents calculations

- Prefault currents $I_0$ (load currents) are usually in phase or near in phase with the voltage.
- The fault currents $I_f$ (the changes) are primarily inductive, i.e. out of phase with the voltage.
- Short circuit currents are much greater than the load currents.
- Power system components are reactive.
- Load components are resistive.
- The total current is the sum of the pre-fault current and the changes due to fault.
- Pre-fault load currents can often be ignored.