Allocation of Hydroelectric Economic Rent Using a Cooperative Game Theoretic Approach

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Presentation overview

• Introduction
• Model for allocating economic rent
  — HAM1. ~ to the gross energy flow (Kárahnjúkar)
  — Cooperative Game Theory-HAM2-Nucleolus
    • Linear Programming
• Simple computational examples
• Discussion and conclusion
Introduction

• Assume $N$ independent riparian owners of water rights. Their benefit measure is the project economic rent

• Should they build each their own “small” projects or join a coalition, in particular the grand coalition to construct a larger project?

• How should the benefits of coalitions be allocated among the $N$ owners?

• This lends itself to a cooperative game theoretic approach
A general model for a valley

- Territories and estimate points at boundaries
- We have lateral inflow \((a_i)\) in each territory
- Head differences are between boundaries (estimate points)

Legend:
- Boundary with flow estimate point
- Lateral inflow, \(a_n\)
- Theoretical potential
- Owner # \(n\)
- Boundary with flow estimate point
A simplified linear model for a valley

Lateral energy inflows:
Lateral inflows:
River and flow estimate points:
Owner #:
Elevation:
Energy contribution
Stream-flow series
Energy contributions

Energy inflow to zone 1: \( w_1 = k_g \frac{a_1 e_2}{2} \)

Energy inflow to zone 2: \( w_2 = k_g \left( \frac{a_2 e_2}{2} + \frac{a_2 e_3}{2} \right) \)

Energy inflow to zone 3: \( w_3 = k_g a_3 e_3 \)

The energy contribution of each owner is given by, where \( e_1 = 0 \):

Energy contribution of zone 1: \( u_1 = k_g (v_2 + \frac{a_1}{2})e_2 \)

Energy contribution of zone 2: \( u_2 = k_g (v_3 + \frac{a_2}{2})(e_3 - e_2) \)

Zone 3 is “dummy”. It is easy to verify that the total energy in this simple 3 zone case is \( u_1 + u_2 = w_1 + w_2 + w_3 = W \).
Sub/superadditive costs and capacities

Costs:
Grand coalition: $S_N = \{1, 2, 3, \ldots, N\}$ is the set of all projects
Any coalition: $S \subseteq S_N$ is a subset, for instance $S = \{2, 3\}$
The cost of the project built by $S$ is $c(S) = c(\{2, 3\})$
Costs are assumed \textbf{subadditive} $c(S \cup T) \leq c(S) + c(T)$ for any $S \subseteq S_N$ and $T \subseteq S_N$ and $S \cap T = \emptyset$

Capacities
$x(S)$ is the capacity of the project built by coalition $S$,
for instance $x(S) = x(\{2, 3\})$
The capacity of the project for $S$ is $x(S) = x(\{2, 3\})$
Capacities are assumed \textbf{superadditive}, or $x(S \cup T) \geq x(S) + x(T)$
Economic rent $r(S)$:

$$r(S) = p \cdot x(S) - c(S); \text{ The energy price is } p$$

The rent is therefore the annual income in the electricity market, minus the annual cost.

The rent is therefore superadditive $r(S \cup T) \geq r(S) + r(T)$

The benefit, $b(S)$ in joining coalition $S$ is

$$b_x(S) = x(S) - \sum_{i \in S} x(\{i\}) \quad b_r(S) = r(S) - \sum_{i \in S} r(\{i\})$$

$$b_c(S) = \sum_{i \in S} c(\{i\}) - c(S)$$
Perfectly additive energy inflows

Note that the energy inflow in each zone is *perfectly additive*, i.e.

\[ u_{S \cup T} = u_S + u_T \]

for all disjoint sets \( S, T \subseteq S_N \) with \( S \cap T = \emptyset \).
Allocation problem LP formulation

The allocation of Economic rent, $Z$ implies a vector of allocations, $z_i \geq 0, i \in N$, $\forall i$ so $Z = \sum_{i \in N} z_i$. Where $Z$ might be total benefit of the grand coalition

The core of the game

$r(\{1, 2, 3\}) = z_1 + z_2 + z_3$

$z_1 \geq r(\{1\}) \quad z_1 + z_2 \geq r(\{1, 2\})$

$z_2 \geq r(\{2\}) \quad z_1 + z_3 \geq r(\{1, 3\})$

$z_3 \geq r(\{3\}) \quad z_2 + z_3 \geq r(\{2, 3\})$
The objective is to maximize the minimum benefit, $\delta$ for each owner to join any coalition. The LP problem is:

Max\(\langle \delta \rangle\)

\[
\begin{align*}
\delta &\leq z_1 - r(\{1\}) \\
\delta &\leq z_2 - r(\{2\}) \\
\delta &\leq z_3 - r(\{3\}) \\
\delta &\leq z_1 + z_2 - r(\{1, 2\}) \\
\delta &\leq z_1 + z_3 - r(\{1, 3\}) \\
\delta &\leq z_2 + z_3 - r(\{2, 3\}) \\
r(\{1, 2, 3\}) &= z_1 + z_2 + z_3 \\
\delta &\geq 0
\end{align*}
\]
### TABLE I: Energy Flow in a River Basin With 3 Zones

<table>
<thead>
<tr>
<th>Zone #</th>
<th>Elevation (m)</th>
<th>Lateral Inflow (G l/Year)</th>
<th>Accumulated Flow</th>
<th>Energy Inflow (GWh/year)</th>
<th>Energy Contribution (%)</th>
<th>Lateral Inflow (M$)</th>
<th>Energy Inflow (M$)</th>
<th>Energy Contribution (M$)</th>
<th>Life Cycle Cost (M$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0</td>
<td>1300</td>
<td>3450</td>
<td>113</td>
<td>487</td>
<td>38%</td>
<td>10%</td>
<td>43%</td>
<td>32,1</td>
</tr>
<tr>
<td>2</td>
<td>80</td>
<td>700</td>
<td>2150</td>
<td>183</td>
<td>313</td>
<td>20%</td>
<td>16%</td>
<td>28%</td>
<td>23,8</td>
</tr>
<tr>
<td>3</td>
<td>160</td>
<td>1100</td>
<td>1450</td>
<td>586</td>
<td>333</td>
<td>32%</td>
<td>52%</td>
<td>29%</td>
<td>28,6</td>
</tr>
<tr>
<td></td>
<td>330</td>
<td>350</td>
<td>350</td>
<td>251</td>
<td></td>
<td>10%</td>
<td>22%</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Total</td>
<td>3450</td>
<td>1133</td>
<td>1133</td>
<td></td>
<td></td>
<td>100%</td>
<td>100%</td>
<td>100%</td>
<td>84,5</td>
</tr>
</tbody>
</table>
A Matlab model using function *linprog*

\[
A = \begin{bmatrix}
-1 & -1 & 0 & 0; & -1 & 0 & -1 & 0; & -1 & 0 & 0 & -1; \\
-1 & -1 & -1 & 0; & -1 & -1 & 0 & -1; & -1 & 0 & -1 & -1,
\end{bmatrix}
\]

\[
b = -\text{xlsread('likan.xls', 1, 'ak17:ak22')}
\]

\[
\text{lb} = [-1000; 0; 0; 0]
\]

\[
\text{ub} = [0; 1000; 1000; 1000]
\]

\[
\text{Aeq} = [0 1 1 1]
\]

\[
\text{beq} = \text{xlsread('likan.xls', 1, 'ak23')}
\]

\[
f = [1 0 0 0]
\]

\[
z = \text{linprog}(f, A, b, \text{Aeq}, \text{beq}, \text{lb}, \text{ub})
\]

\[
\text{xlswrite('likan.xls', z, 1,'an17:an19')}
\]
Matlab computation

\[
A = \begin{bmatrix}
-1 & -1 & 0 & 0 \\
-1 & 0 & -1 & 0 \\
-1 & 0 & 0 & -1 \\
-1 & -1 & -1 & 0 \\
-1 & -1 & 0 & -1 \\
-1 & 0 & -1 & -1 \\
\end{bmatrix}
\]
\[
b = \begin{bmatrix}
-23.8609 \\
-9.0783 \\
-18.7924 \\
-61.1033 \\
-72.9709 \\
-60.9484 \\
\end{bmatrix}
\]
\[
lb = \begin{bmatrix}
-1000 \\
0 \\
0 \\
0 \\
\end{bmatrix}
\]
\[
ub = \begin{bmatrix}
0 \\
1000 \\
1000 \\
1000 \\
\end{bmatrix}
\]
\[
Aeq = \begin{bmatrix}
0 & 1 & 1 & 1 \\
\end{bmatrix}
\]
\[
beq = \begin{bmatrix}
145.5817 \\
\end{bmatrix}
\]
\[
f = \begin{bmatrix}
1 & 0 & 0 & 0 \\
\end{bmatrix}
\]

Optimization terminated.

\[
z = \begin{bmatrix}
-30.3862 \\
54.2471 \\
39.9023 \\
51.4323 \\
\end{bmatrix}
\]
### TABLE II: COALITIONS COSTS AND OUTPUTS IN THE 3 ZONE EXAMPLE

<table>
<thead>
<tr>
<th>Coalition</th>
<th>Projects in S</th>
<th>Number of projects in coal.</th>
<th>Coalition energy factor</th>
<th>Coalition energy firm energy</th>
<th>Individual project firm energy</th>
<th>Benefit in joining coalition</th>
<th>Coalition Cost</th>
<th>Individual project cost</th>
<th>Benefit in joining coalition</th>
<th>Unit cost</th>
</tr>
</thead>
<tbody>
<tr>
<td>S</td>
<td>α(S)</td>
<td>x(S)</td>
<td>Σx_i</td>
<td>b_x(S)</td>
<td>β(S)</td>
<td>c(S)</td>
<td>Σc_i</td>
<td>b_c(S)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>1 {1}</td>
<td>1</td>
<td>46%</td>
<td>224</td>
<td>224</td>
<td>0</td>
<td>100%</td>
<td>32.1</td>
<td>32.1</td>
<td>0.0</td>
<td>143</td>
</tr>
<tr>
<td>2 {2}</td>
<td>1</td>
<td>42%</td>
<td>131</td>
<td>131</td>
<td>0</td>
<td>100%</td>
<td>23.8</td>
<td>23.8</td>
<td>0.0</td>
<td>181</td>
</tr>
<tr>
<td>3 {3}</td>
<td>1</td>
<td>57%</td>
<td>190</td>
<td>190</td>
<td>0</td>
<td>100%</td>
<td>28.6</td>
<td>28.6</td>
<td>0.0</td>
<td>151</td>
</tr>
<tr>
<td>4 {1,2}</td>
<td>2</td>
<td>56%</td>
<td>448</td>
<td>355</td>
<td>93</td>
<td>91%</td>
<td>50.9</td>
<td>55.9</td>
<td>5.0</td>
<td>114</td>
</tr>
<tr>
<td>5 {1,3}</td>
<td>2</td>
<td>62%</td>
<td>508</td>
<td>414</td>
<td>95</td>
<td>89%</td>
<td>54.1</td>
<td>60.7</td>
<td>6.7</td>
<td>106</td>
</tr>
<tr>
<td>6 {2,3}</td>
<td>2</td>
<td>66%</td>
<td>426</td>
<td>321</td>
<td>105</td>
<td>87%</td>
<td>45.6</td>
<td>52.4</td>
<td>6.8</td>
<td>107</td>
</tr>
<tr>
<td>7 {1,2,3}</td>
<td>3</td>
<td>75%</td>
<td>849</td>
<td>545</td>
<td>304</td>
<td>79%</td>
<td>66.8</td>
<td>84.5</td>
<td>17.8</td>
<td>79</td>
</tr>
</tbody>
</table>
TABLE III: The Calculation of Economic Rent and Results From the HAM2 Allocation

<table>
<thead>
<tr>
<th>AG</th>
<th>AH</th>
<th>AI</th>
<th>AJ</th>
<th>AK</th>
<th>AL</th>
<th>AM</th>
<th>AN</th>
<th>AO</th>
</tr>
</thead>
<tbody>
<tr>
<td>9</td>
<td>Coalition</td>
<td></td>
<td></td>
<td>Coalition Economic Rent</td>
<td></td>
<td></td>
<td>Allocation</td>
<td></td>
</tr>
<tr>
<td>10</td>
<td>Number of projects in coalition</td>
<td></td>
<td></td>
<td>Coa- lition Economic Rent</td>
<td></td>
<td></td>
<td>Benefit in joining coalition</td>
<td>Allocated Economic Rent</td>
</tr>
<tr>
<td>11</td>
<td>Projects in coalition</td>
<td></td>
<td></td>
<td>Economic Rent</td>
<td></td>
<td></td>
<td>Benefit in joining coalition</td>
<td>Benefit joining grand coalition</td>
</tr>
<tr>
<td>12</td>
<td>Number in coalition</td>
<td></td>
<td></td>
<td>r(S)</td>
<td></td>
<td></td>
<td>Economic Rent</td>
<td></td>
</tr>
<tr>
<td>13</td>
<td>#</td>
<td></td>
<td></td>
<td>Σr_i</td>
<td></td>
<td></td>
<td>grand coalition</td>
<td></td>
</tr>
<tr>
<td>14</td>
<td>S</td>
<td>Market price ($/MWh)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>15</td>
<td>r(S)</td>
<td></td>
<td></td>
<td>b_r(S)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>16</td>
<td>Σr_i</td>
<td></td>
<td></td>
<td>z_i</td>
<td></td>
<td></td>
<td>z_i - r_i</td>
<td></td>
</tr>
<tr>
<td>17</td>
<td>1</td>
<td>{1}</td>
<td>1</td>
<td>250</td>
<td>23.9</td>
<td>23.9</td>
<td>0.0</td>
<td>54.2</td>
</tr>
<tr>
<td>18</td>
<td>2</td>
<td>{2}</td>
<td>1</td>
<td>250</td>
<td>9.1</td>
<td>9.1</td>
<td>0.0</td>
<td>39.9</td>
</tr>
<tr>
<td>19</td>
<td>3</td>
<td>{3}</td>
<td>1</td>
<td>250</td>
<td>18.8</td>
<td>18.8</td>
<td>0.0</td>
<td>51.4</td>
</tr>
<tr>
<td>20</td>
<td>4</td>
<td>{1,2}</td>
<td>2</td>
<td>250</td>
<td>61.1</td>
<td>32.9</td>
<td>28.2</td>
<td>94.1</td>
</tr>
<tr>
<td>21</td>
<td>5</td>
<td>{1,3}</td>
<td>2</td>
<td>250</td>
<td>73.0</td>
<td>42.7</td>
<td>30.3</td>
<td>105.7</td>
</tr>
<tr>
<td>22</td>
<td>6</td>
<td>{2,3}</td>
<td>2</td>
<td>250</td>
<td>60.9</td>
<td>27.9</td>
<td>33.1</td>
<td>91.3</td>
</tr>
<tr>
<td>23</td>
<td>7</td>
<td>{1,2,3}</td>
<td>3</td>
<td>250</td>
<td>145.6</td>
<td>51.7</td>
<td>93.9</td>
<td>145.6</td>
</tr>
</tbody>
</table>
Discussions and conclusions

• The **independent ownership** of private owners of water rights assumes that the owners have and interest in developing their resources.

• By definition they cannot compare all options of joining various coalitions unless **cost and firm output estimates** are available and updated with the development of the river basin.

• These possibilities should be **weighted against the grand coalition** whether or not imposed by the government on the private owners.

• This methodology should be beneficial in the **basic debate whether water rights are public or private goods** and how private ownership plays a role in the borderline between the deregulated market environment and government imposing public interest on these owners.
Thank you!