Abstract - A recursive model is applied in order to estimate the incremental cost of adding energy intensive industries (EII) to a hydro-based power system. The modeling framework assumes available, untapped resources as a set of hydroelectric projects, with a given cost and capacity, to be developed in a sequence. A forecast for Basic Demand (BD) is also assumed. It is shown that economies of scale (EOS) in project cost can be exploited to lower the incremental cost to the EII, especially for larger projects, and in particular, if energy is delivered both to the EII and to serve the Basic Demand (BD). Several different definitions of the incremental cost are proposed depending, for instance, on project selection (sequence). Combining EII and gradually increasing Basic Demand can give economic benefits beyond the classical calculation of a project’s energy cost and lead to the economical exploitation of large hydro resources that could not otherwise have been developed.

I. INTRODUCTION

A recursive optimization model is developed to analyze the expansion process in a hydro-based power system with industrial bulk electricity sales. The model determines the average and incremental or marginal cost of allocating hydroelectric resources to energy intensive industry, such as aluminum smelters. The modeling framework assumes that untapped resources consist of a group of hydroelectric projects, available for future development and the basic cost structure and energy production capacity is known. To reflect actual conditions in a market-based environment, the energy demand is decomposed into (a) a general electricity market-related demand, called Basic Demand (BD) and (b) a long-term bulk demand for energy-intensive industry (EII), called Extra Demand (ED). The former component is assumed to grow gradually with the growth of the general economy, while the industrial component will expand in a stepwise manner with new aluminum smelters and other bulk energy sales. Furthermore, the ED assumes long-term contracts and quantity-based sales.

As exemplified by the case of Iceland, the National Power Co. (Landsvirkjun) law for exploiting hydro-resources states that the price of energy to the EII must never lead to higher prices for the general customers, than would otherwise have been the case. This indicates minimum pricing based in the incremental cost of superimposing the EII demand on top of the Basic Demand (BD). Thus the government has the option to negotiate for new energy intensive industries, if the above conditions are met and therefore the determination of this marginal cost is of great importance.

In previous papers, based partly on the capacity expansion framework of [1], [2] and [3], the problem of optimally sequencing hydroelectric projects without the EII has been addressed [4], optimal sizing of projects, with a given predetermined sequence, also without EII has been determined [5]. General Economies of Scale (EOS) for hydroelectric projects have been investigated and how the EOS in general would impact the selection of projects towards a trade-off between small and large projects. This trade-off is addressed both with and without the EII [6]. With given assumptions on the price to the EII, from a utility with a dual role of serving the BD and the ED, the optimal allocation to the EII has been determined [7] to minimize the net discounted cost.

In this paper, however, we analyze how the Economies of Scale (EOS) for hydroelectric projects can be exploited to lower the marginal costs of the resources and their economical allocation to the EII. Although the problem by nature is a complex mixed integer-constrained optimization problem, some of its practical implications can be focused upon without many such complexities. For instance, if small projects are mainly selected, their utilization will in general be limited to the basic demand (BD) and this development, on the average, may be relatively costly. However, due to discounting all costs and benefits, large projects will not be economical due to the long period of excess capacity, perhaps lasting to satisfy the gradually growing demand for a number of years after a new project is started up, sometimes 5-15 years, unless with Energy Intensive Industry (EII). Thus the exploitation of large projects with implicit economies of scale is only possible through a partial industrial utilization by EII. Therefore questions such as the following are of great importance and will be addressed:

What is the marginal cost of the EII demand, if the project selected is economical even without EII demand, that is, with the
BD only? How will the return on investment be affected, if it is possible to select a large project, which would otherwise not have been selected for the basic demand (BD) only?

The objective of this paper is the development of an analysis tool applicable and useful in the process of developing national policies for resource utilization in hydro-intensive regions and countries. Such a tool should be detailed enough for actual planning while sufficiently simple to reveal important trends and issues.

The paper is organized as follows: In section II, the model is presented, while in section III, the model is applied to data from the Icelandic power system and a resulting case study presented, in particular relating to the present government plans of harnessing the energy resources of the Fljotsdalur projects, (see Figure 5) where start up of construction is planned in 2001 in the eastern part of the country. The apparent lack of synchronization, between available project and industry step sizes, is discussed. This lack exists, in spite of the planned simultaneous start up of the project and the associated aluminum smelter to obtain the lowest possible marginal cost.

In general, utilizing the Economies of Scale (EOS) available at each project site, requires a specific partial utilization of the project capacity to EII to achieve minimum cost. With this arrangement, the marginal cost of the energy intensive industry (EII), as shown, can even be considerably lower than the classical determination of a hydro project's unit cost, determined by dividing the annual cost of a project (investment plus operations) by its annual energy output.

Section IV presents results and conclusions and finally appendices on notation and a references section conclude the paper.

II. THE RECURSIVE INCREMENTAL COST MODEL

Consider the expansion process for a purely hydroelectric power system, first without ED, as shown in Figure 1. An appendix explains the symbols used. The general demand function $D(t)$ for BD increases over time and capacity is expanded stepwise to satisfy this demand. The excess capacity (the difference shown by the shaded areas) is at all times positive and is at its maximum just after a new project has been started up.

We now focus on a linear demand function and expand the concept by adding the ED on top of the BD as shown in Figure 2. While a linear demand is an approximation, it often resembles closely a real situation [6] as shown in Figure 5 and results in a simpler more meaningful model as discussed below. Figure 2 shows conceptually the expansion process in the case of a linear demand function, while Figure 3 shows the same process with the ED added at the start up time of project #2.

It should be noted that the BD consists of gradually increasing demand for energy, which increases smoothly in harmony with the general growth of the economy. However, bulk customers (ED) such as energy intensive industry or export of energy are assumed “large” compared to the size of the market. This would, for instance be the case for the Icelandic electricity sector (See section III) with a new aluminum smelter or in the case of exporting electrical energy through HVDC submarine cable to Scotland or continental Europe.

Figure 3 shows a case when a "slice" of ED is added at the instance when the second project in the sequence is started up. However, additional ‘slices’ of ED can be superimposed on the demand for each project in the sequence, as discussed below. Furthermore, it can be shown that the optimal timing is synchronizing the start up of projects with EII, due to the discounting effect in the calculation of discounted costs and benefits for the whole expansion program.

A. Basic Model without EII

A recursive model describing the hydroelectric system expansion process, in terms of discounted costs, benefits in the form of energy production capacities, and different types of unit costs and marginal costs, will now be presented and expanded from [6] and [7]. We start by a basic model, without EII and expand it later to include the EII. An infinite planning horizon is assumed, which is logical and natural, when there is no specific reason to limit the horizon. The length of the horizon does not alter, however, the results significantly. Hydroelectric projects come on stream, when demand grows to equal existing capacity. The economical lifetime of such projects is in the order of 30-50 years and with discounting rates in the order of e.g. 3-8% per year, anything happening after a project’s lifetime will not much affect the discounted cost. At any rate, we assume that after the project’s lifetime, it will be refurbished repeatedly at some fraction of the original cost. Also, when the hydro-resources are exhausted, long range energy supply with a given price is available to supply all demand [5], [6].

The discounted cost of a single project, refurbished indefinitely, is shown by (1). Reference is made to the Appendix for notations and definitions.

$$C_i = C_{di} + \sum_{j=1}^{\infty} hC_{di} \exp(-j\alpha T) + \sum_{j=1}^{\infty} r_m C_{di} \exp(-j\alpha)$$  \hspace{1cm} (1)

The continuous time interest rate, $\alpha$, is defined as follows, where $r$ is the conventional interest rate in %/year:

$$\alpha = \ln(1 + \frac{r}{100})$$  \hspace{1cm} (2)

Equation (1) leads to:

$$C_i = C_{di} (1 + \frac{r_m}{\alpha} + hD_T)$$  \hspace{1cm} (3)

where

$$D_T = \frac{\exp(-\alpha T)}{1 - \exp(-\alpha T)}$$  \hspace{1cm} (4)

The numerical difference between $C_i$ and $C_{di}$ is insignificant due to the previously mentioned discounting. Equation (5) describes the
With a linear demand function:

\[ D(t) = qt \]  

(6)

equation (5) reduces to:

\[ P_{di} = C_i + P_{di(i+1)} \exp \left[ -\alpha t_i \right] \]  

(7)

The discounted energy below the linear demand (BD) is defined by (8):

\[ E_i = \int_0^\infty qt \exp \left[ -\alpha t \right] dt + \int_0^{x_i/q} x_i \exp \left[ -\alpha t \right] dt \]  

(8)

while the discounted energy output of a specific project is defined by (9):

\[ e_i = \frac{q}{\alpha^2} \left( 1 - \exp \left[ -\frac{\alpha x_i}{q} \right] \right) \]  

(9)

From (8) the Long Range Marginal Cost (LRMC) of the BD, at each stage in the expansion sequence, is defined by (10) as the ratio of the discounted cost to the discounted energy output of all projects added from a given stage #i in the expansion process:

\[ k_{di}^e = \frac{P_{di}}{E_i} = \frac{P_{di} \cdot \alpha^2}{q} \]  

(10)

Similarly, from (9), the unit cost of energy from the project, based on the linearly increasing demand, called the Actual Unit Cost (AUC), accounting for partial utilization in the period of excess capacity, can be defined as follows:

\[ \text{AUC} = k_{di}^f = \frac{C_i}{x_i} = \frac{\alpha^2 C_i}{q \left( 1 - \exp \left( -\frac{\alpha x_i}{q} \right) \right)} \]  

(11)

For a linear demand function, it was shown in [4], that sequencing the projects by an ascending AUC, would result in an optimal sequence.

With instantaneous full utilization of the project capacity, we get the full utilization unit cost (FUC), simply as the average annual cost divided by the average annual output, as follows:

\[ \text{FUC} = k_{di}^f = \frac{\alpha C_i}{x_i} \]  

(12)

From (7) and (10) the LRMC, \( k_{di}^e \), is connected for adjacent projects in the sequence by (13) similar to the total discounted cost.

\[ k_{di}^e = C_i \cdot \frac{\alpha^2}{q} + k_{di(i+1)} \exp \left[ -\alpha x_i \right] \]  

(13)

From (13) and (11), as shown in [5], the LRMC is the weighted average of the AUCs of future projects, or:

\[ k_{di}^f = k_{di}^f \left( 1 - w_i \right) + k_{di(i+1)}^f w_i \]  

(14)

where

\[ w_i = \exp \left[ -\frac{\alpha x_i}{q} \right] ; \quad 0 < w_i < 1 \]  

(15)

Therefore, for an infinite sequence of identical projects where

\[ C_1 = C_2 = C_3 = \cdots = C \]  

\[ x_1 = x_2 = x_3 = \cdots = x_i = x = x \]  

\[ \Rightarrow P_{d1} = P_{d2} = P_{d3} = \cdots = P_{di} = \cdots = P \]  

(16)

the LRMC equals the AUC by (11) and (14). In such a case:

\[ P = C + P \exp \left[ -\frac{\alpha x}{q} \right] \Rightarrow P = \frac{1}{1 - \exp \left[ -\frac{\alpha x}{q} \right]} \]  

(17)

This could be the approximation of a situation, where there are many available economical hydroelectric projects of approximately equal AUC to choose from. These projects would then be constructed before the more uneconomical part of the resources - consisting of more expensive projects in terms of AUC. If such projects satisfy the BD for a long period, the discounting of future more expensive projects would have insignificant influence on the total discounted cost.

B. Basic Model with Extra Demand (ED) to Serve the EII

Assume now that the ED is superimposed on top of the BD. Figure 3 and Figure 4 show how this affects the expansion sequence, i.e. the projects later in the sequence are shifted forward in time. It is assumed that ED, or bulk sales (y_i), are associated with project # i. The total discounted cost is now connected between adjacent projects in the sequence by (18) instead of (7),

\[ P_{bi} = C_i + P_{bi(i+1)} \exp \left[ -\frac{\alpha (x_i - y_i)}{q} \right] \]  

(18)

\[ 0 \leq y_i \leq x_i \quad i = 1, 2, \ldots, N \]
where the index “b” indicates bulk demand in general.

We therefore assume, in general, that projects later in the sequence may have ED associated with them. The discounted cost with ED can now be restated as follows:

\[ P_{b}(y_{1}, y_{i+1}, y_{i+2}, \ldots, y_{N}) = C_{i} + P_{b(i+1)} \exp \left[ -\frac{\alpha(x_{i} - y_{i})}{q} \right] \]  \hspace{1cm} (19)

In particular, for the first project in the sequence, \((i=1)\) (20) describes the total discounted cost of the development, ED may be associated with all projects:

\[ P_{b(1)}(y_{1}, \ldots, y_{N}) = C_{i} + P_{b(2)}(y_{2}, \ldots, y_{N}) \exp \left[ -\frac{\alpha(x_{i} - y_{i})}{q} \right] \]  \hspace{1cm} (20)

The discounted energy output of each project to serve the ED is

\[ f_{i} = \int_{0}^{y_{i}} \exp \left[ -\alpha x \right] dt = \frac{y_{i}}{\alpha} \]  \hspace{1cm} (21)

while the total discounted energy of the ED with all projects, at the start-up time of project \( #i \), is:

\[ F_{i} = f_{i} + F_{i+1} \exp \left[ -\frac{\alpha(x_{i} - y_{i})}{q} \right] \]  \hspace{1cm} (22)

C. Definitions of Incremental Unit Cost

Basically the most general incremental unit cost (IUC) definition in a hydroelectric power system is along the following lines: Define the optimal expansion sequence and size of all projects without any Extra demand (ED) and calculate its discounted cost. Define, as well, the optimal expansion sequence and size of all projects with the ED and calculate its discounted cost. The ED can be served from any number of future projects. The IUC is then the difference between the above two discounted costs divided by the discounted energy of the ED. Relaxations from this definition include project sequence and number of projects with ED, as shown below.

We therefore proceed to propose 5 different definitions of IUC, i.e. IUCa, IUCb, IUCc, IUCd and IUCe. The difference between these definitions is the degree of generality, where IUCd and IUCe are the most practical, and will be applied in a case study in Section III. IUCd is the only definition, assuming a different project sequence, with and without the ED.

We therefore proceed to propose 5 different definitions of IUC, i.e. IUCa, IUCb, IUCc, IUCd and IUCe. The difference between these definitions is the degree of generality, where IUCd and IUCe are the most practical, and will be applied in a case study in Section III. IUCd is the only definition, assuming a different project sequence, with and without the ED.

Starting with a multidimensional definition, IUCa, recognizing that ED can be associated with any or all of the future projects, we get the following definition of IUCa using (7), (19) and (22). For a given set of energy outputs, from each project allocated to ED or \( y_{1}, y_{2}, \ldots, y_{N}, \) IUCa associated with ED will be defined as:

\[ \text{IUCa} = k_{bi} = \frac{P_{bi} - P_{b0}}{F_{i}} = k_{bi}^{d}(y_{1}, y_{i+1}, \ldots, y_{N}) \]  \hspace{1cm} (23)

Therefore, IUCa is the incremental discounted cost of adding the ED to the expansion sequence, divided by the discounted energy allocated to the ED. This incremental discounted cost is the difference between the discounted cost of the expansion sequence with and without the ED as defined by (23).

By (23), the IUCa is a function of all energy allocations from future projects to the EII, or the vector \( y_{1}, y_{2}, \ldots, y_{N} \).

The development of IUC assumes that we allocate a specified amount of the resources to EII, as economically as possible, by distributing this given energy among future projects. Therefore from (23) we get the following definition for IUCb:

\[ \text{IUCb} = k_{bi}^{d}(y) = \min_{y_{1}, y_{i+1}, \ldots, y_{N}} k_{bi}^{d}(y_{1}, y_{i+1}, \ldots, y_{N}) \]  \hspace{1cm} (24)

The 3rd definition of the incremental unit cost, called IUCc, assumes that all ED allocated to EII is delivered from the start of the planning horizon, i.e. all future projects are shifted forward in time, so all ED energy can be delivered at the start up date of the first project. This means that any project in the sequence is fully allocated to the EII, before any other project later in the sequence is allocated.

The formal definition for IUCc would be as follows, assuming project #1:

\[ \text{IUCc} = k_{bi}^{c}(y) = k_{bi}^{d}(y_{1}, y_{2}, \ldots, y_{N}) \]

The following special case of (25), called IUCd, is considered, where the ED is limited by the capacity of project \#i, and defined as follows:

\[ y_{i} = \begin{cases} 0 & \text{if} \ y \leq 0 \\ y & \text{if} \ 0 < y < x_{i} \\ x_{i} & \text{if} \ y > x_{i} \end{cases} \]  \hspace{1cm} (25)

The following special case of (25), called IUCd, is considered, where the ED is limited by the capacity of project \#i, and defined as follows:

\[ y_{i} = \begin{cases} 0 & \text{if} \ y \leq 0 \\ y - \sum_{j=1}^{i-1} y_{j} & \text{if} \ 0 < y - \sum_{j=1}^{i-1} y_{j} < x_{i} \\ x_{i} & \text{if} \ y - \sum_{j=1}^{i-1} y_{j} > x_{i} \end{cases} \]  \hspace{1cm} (25)

The following special case of (25), called IUCd, is considered, where the ED is limited by the capacity of project \#i, and defined as follows:

\[ y_{i} = \begin{cases} 0 & \text{if} \ y \leq 0 \\ y_{i} & \text{if} \ 0 < y < x_{i} \\ x_{i} & \text{if} \ y > x_{i} \end{cases} \]  \hspace{1cm} (26)

This means that the ED is partially associated with only the first project and not with any later projects in the sequence.

Thus, the incremental unit cost, called IUCd, is defined by (27) according to (7), (18) and (21):

\[ \text{IUCd} = k_{bi}^{d}(y) = \frac{P_{bi} - P_{b0}}{F_{i}} \]  \hspace{1cm} (27)

Since ED is only associated with project \#i, (18) becomes

\[ P_{bi} = C_{i} + P_{a(i+1)} \exp \left[ -\frac{\alpha(x_{i} - y_{i})}{q} \right] \]  \hspace{1cm} (28)

Using (7), (28), (21) and (10), we get:

\[ \text{IUCd} = k_{bi}^{d} = \frac{q\alpha x_{i}}{\alpha y_{i}} \exp \left[ -\frac{\alpha x_{i}}{q} \right] \left[ \exp \left( \frac{x_{i}}{q} \right) - 1 \right] \]  \hspace{1cm} (29)

For an infinite sequence of identical projects, (16) where the 1st project is completely allocated to the ED or \( y_{1} = x \), (29) reduces to:

\[ \text{IUCd} = k_{bi}^{d} = \frac{\alpha C}{x} \]  \hspace{1cm} (30)
which is identical to the FUC according to (12).

It is also possible to define the incremental unit cost, IUCd, as a fraction of the Long Range Marginal Cost (LRMC), as defined by (10), and this results in:

$$\beta_b = \frac{k_d^{(i)}}{k_d^{(i+1)}} = \exp(-\alpha x_i \frac{k_d^{(i)}}{q} \exp(\alpha y_i \frac{k_d^{(i)}}{q} - 1 \] (31)

Defining the ED as a fraction $b_i$ of capacity, or $y_i = b_i x_i$, results in:

$$\beta_b(b_i, x_i) = \frac{k_d^{(i)}}{k_d^{(i)}} = \exp(-\alpha x_i \frac{k_d^{(i)}}{q} \exp(\alpha b_i x_i \frac{k_d^{(i)}}{q} - 1 \] (32)

Equation (32) therefore, using (7) and (28), states the incremental cost of serving the extra demand (ED) partially associated with project $#i$ - the same project with and without the ED. This is the cost of adding the ED to a project sequence that is otherwise suitable for serving the basic demand. Table 1 shows the IUCd according to (32) for 6% annual interest rate and $q = 50$ GWh/year/year (see Section III)

For IUCd, it may not be possible to get the benefit from the Economies of Scale (EOS) of large projects, since they may be too large for serving the BD only.

Next, we will therefore examine the incremental cost (IUCe) of adding the ED when the first project of the planning period (stage # i in the expansion sequence) is a "large" project.

D. Incremental Unit Cost of ED with a large project

The following question will be addressed: What is the incremental cost of inserting a "large" project, and allocating a part of its energy to ED, into a series of smaller projects, constructed and suitable to meet the BD only?

<table>
<thead>
<tr>
<th>TABLE 1</th>
<th>IUCd COMPARED TO THE LRMC FOR DIFFERENT PROJECT CAPACITIES (SIZES) AND DIFFERENT SIZE OF THE ED</th>
</tr>
</thead>
<tbody>
<tr>
<td>Capacity (GWh/year)</td>
<td>The size of the ED as a fraction of project size ($b_i$) in 10%, 30%, 50%, 70%, 90%, 100%</td>
</tr>
<tr>
<td>----------------</td>
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</tr>
<tr>
<td>200</td>
<td>80.1% 82.0% 84.0% 86.0% 88.1% 89.2%</td>
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<tr>
<td>300</td>
<td>71.7% 74.3% 77.0% 79.9% 82.8% 84.4%</td>
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<td>64.2% 67.3% 70.7% 74.2% 78.0% 79.9%</td>
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<tr>
<td>500</td>
<td>57.5% 61.0% 64.8% 68.9% 73.4% 75.8%</td>
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<tr>
<td>600</td>
<td>51.5% 55.3% 59.5% 64.1% 69.2% 71.9%</td>
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<td>700</td>
<td>46.1% 50.1% 54.6% 59.6% 65.3% 68.4%</td>
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<td>41.3% 45.4% 50.1% 55.5% 61.7% 65.0%</td>
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<td>29.6% 33.8% 38.9% 44.9% 52.2% 56.4%</td>
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<td>1200</td>
<td>26.5% 30.7% 35.8% 41.9% 49.5% 53.8%</td>
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<td>19.0% 22.9% 27.8% 34.1% 42.3% 47.2%</td>
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<td>13.7% 17.1% 21.7% 27.9% 36.4% 41.8%</td>
</tr>
<tr>
<td>1900</td>
<td>12.2% 15.5% 20.0% 26.2% 34.7% 40.2%</td>
</tr>
</tbody>
</table>

Let $P$ be the discounted cost, without the ED, of the project sequence suitable (optimal) for the BD only. Let $k_i$ be the LRMC corresponding to $P$.

Consider now the cost of inserting a "large project", in front of other projects, with an investment cost $C_i$ in order to benefit from its EOS by allocating some of its capacity, $y_i$, to the ED. Define the total discounted cost in this case to be $P_i$. The total discounted cost with the ED and with this "large" project as the $i$th project is therefore:

$$P_i = C_i + P \cdot \exp(-\alpha (x_i - y_i))$$ (33)

The incremental cost, IUCe, will be:

$$\Delta k = \frac{\Delta P}{y} = \frac{P_i - P}{y}$$ (34)

If, again, $y = b x_i$ defines the fraction of the project’s energy allocated to ED, we get:

$$\Delta k = \frac{1}{b} k^f - k_i = \frac{1}{\alpha b x_i} \left[1 - \exp\left(-\alpha (x_i (1-b))\right)\right]$$ (35)

where the Full Utilization Cost (FUC) of the $i$th project is:

$$k^f = \frac{C_i}{x_i}$$ (36)

as also defined by (12). Assume that the FUC of the “large project” is a certain fraction, $c$, of the future LRMC or $k^f = c k_i$. Then the following expression, (37), for the IUCe is obtained:

$$IUCe = \Delta k = \frac{c}{b} \frac{1 - \exp\left(-\alpha (x_i (1-b))\right)}{\alpha b x_i}$$ (37)

E. Optimal Sizing of Projects with a Specific ED

Finally, it is of interest to determine what is the most economical size of the “large project”, assuming a certain allocation of its energy to EII, by finding the minimum IUCe. Therefore, with a fixed $y$, we can determine this optimal size $x_i$ by differentiating the incremental cost as follows.

From (34) the incremental cost of the ED can be visualized as a function of the size of the ED:

$$\Delta k(x_i) = \frac{C_i(x_i) + P \cdot \left[\exp\left(-\alpha (x_i - y)\right) - 1\right]}{y}$$ (38)

By differentiating (38) we get the following condition for a minimum:

$$\frac{d\Delta k(x_i)}{dx_i} = \frac{\alpha}{y} \left[\frac{dC_i(x_i)}{dx_i} - P \cdot \alpha \frac{\exp\left(-\alpha (x_i - y)\right)}{q}\right] = 0$$ (39)

From (39) and (10) we get the current marginal cost (CMC) [5]:

$$CMC = \alpha \frac{dC_i(x_i)}{dx_i} = k_i \cdot \exp\left(-\alpha (x_i - y)\right)$$ (40)
This result simply states: The optimal size of the project corresponds to the LRMC of future projects, discounted to the current start up date. This optimal size directly depends on how much energy, \( y \), is allocated to the EII. If \( y \) is large the the CMC is high, and vice versa.

III. AN ICELANDIC POWER SYSTEMS CASE STUDY

In this section we will apply the recursive model to data from the Icelandic power system [6]. (See Figure 5) The data consists of cost and capacity estimates of the remaining and already harnessed hydroelectric resources available.

In particular, the previously defined incremental costs, IUCd and IUCe, will be calculated based on the system conditions associated with the Fljotsdalur hydroelectric development in eastern Iceland.

The Fljotsdalur project (210 MW, 1400 GWh/year) is an example of a large project -- too large for the BD only. The BD in this small power system increase about 50 - 70 GWh/year/year (\( q = 50-70 \)) and, as shown in [6], the suitable (optimal) size of projects is in the range of 300-800 GWh/year or about half the size of the Fljotsdalur project. Furthermore, the period of excess capacity would be in the range of 20-28 years. Therefore the construction of the Fljotsdalur project is synchronized with a new aluminum smelter in Reydarfjordur with an initial production capacity of 120,000 tons and using approximately 1700 GWh/year

A. Incremental cost to EII with an invariant sequence of projects

An examination of IUCd will first be presented. This means the same sequence of projects with and without the ED. Table 1 shows this incremental cost as compared to the LRMC of future projects. It is interesting to note that the cost is independent of the project’s construction cost, but depends on the LRMC, however. Assume, for instance, that the size of the first project is 700 GWh/year. The table shows IUCd of 65% of the LRMC assuming full allocation of the project’s capacity, or \( b = 100\% \). This corresponds to the Full Utilization Cost (FUC) of the project. Landsvirkjun, (The National Power Co.) has estimated the LRMC to be approximately 2.30 kr/kWh (1) which equals about 32 U.S.$/MWh. Therefore the incremental cost, IUCd is about 21 U.S.$/MWh in this case.

The full allocation of a project’s capacity to EII, such as aluminum smelters, is analogous to a case, where an aluminum company owns a hydroelectric station producing energy solely to serve the aluminum smelter. If, however only a part of the capacity is allocated to the aluminum smelter, the incremental cost is lower as seen by Table 1.

Therefore, an important conclusion is the greater economics of allocating only a part of a project’s capacity to energy intensive industry, rather than the full capacity of a project, even for an invariant sequence of projects. The incremental unit cost to EII can even be lower than the classical FUC of energy from the project.

Therefore, the reduction of the period of excess capacity and the increased utilization of the project’s capacity following the start-up of the project is not the primary reason for the lower energy cost to EII. Rather, the explanation lies in the delay of all projects that follow. This delay brings savings, due to the time value of money, and is especially economical, if the first project is fairly large, with a period of excess capacity of say 10 - 30 years, such as the Fljotsdalur project.

B. IUC of energy to EII allocated from a "large" project

The incremental unit cost (IUCe), with a change in the 1st project in the sequence, is shown in Figure 6 according to (37). The Figure assumes a “large project”, Fljotsdalur, with a partial allocation to EII from the project, with the percentage shown on the horizontal axis. The LRMC is 2.25 kr/kWh according to a Landsvirkjun estimate, as previously discussed, while the FUC is assumed to be 1.15 kr/kWh [6] on a comparable price level. This is in accordance with cost estimates for this project. The interest rate is 6% and the growth of the demand is, again, 50 GWh/year/year.

IV. RESULTS AND CONCLUSIONS

The main conclusions of the paper deal with the incremental or marginal cost of adding optional energy intensive industry (EII) customers to a hydroelectric power system. These customers are an option for the utility in a system serving other gradually increasing demand of general residential and industrial customers.

The main conclusions of the paper are:

- The incremental energy cost for the energy intensive industry can be considerably lower than the long range marginal cost:
  - if the timing of new EII customers is synchronized with start up of hydroelectric projects

\[ I = 100\% \]
if projects are larger in size or energy output than the demand by the EII customers, so the projects can supply simultaneously both the EII demand and the basic general demand.

- if economies of scale are utilized to benefit the EII customers with large projects that otherwise would not have been economical for the basic general demand (BD) only.

- Energy Intensive Industry demand (ED) constitutes an opportunity to harness large hydroelectric resources. These resources would be difficult to harness without such demand, as described in this paper.

- When constructing hydroelectric stations to supply energy intensive industry customers, these stations should be designed or selected to exceed the step size of these customers in capacity. With such an arrangement both types of demand can and should be served simultaneously.

- The classical approach of building hydro-stations exclusively for power intensive industry, separate from the public or basic demand, is therefore not optimal for power intensive industry, separate from the public or basic demand. The decision maker has often the option to serve the ED consisting of energy sales to EII or bulk energy export.

REFERENCES


APPENDIX -- DEFINITIONS AND NOTATION

BD Basic Demand which must be satisfied at all times, consisting of residential and commercial customers and “light” industry as exemplified by the case of Iceland.

\[ D(t) \] A nondecreasing demand function of time (e.g. in GWh/year) an infinite time into the future describing the BD. In particular the case of linear increase, i.e. \[ D(t) = qt \], where \( q \) is a constant growth factor is examined.

ED Extra Demand, ladder or step-function shaped, superimposed on the basic demand. BD. The decision maker has often the option to serve the ED consisting of energy sales to EII or bulk energy export.

EOS Economies of Scale

EEI Energy or power intensive industry

LRMC Long range marginal cost

IUC Incremental unit cost

CMC Current marginal cost

\[ i = 1, 2, ..., N \] Index of individual hydroelectric projects. The projects are assumed to be constructed in the sequence indicated by this index.

\[ C_{di} \] Construction or investment cost of project \#i

\[ C_i \] Construction or investment cost of project \#i with embedded operations and refurbishing costs

\[ r_s \] Operations cost as a fraction of investment cost

\[ h \] Refurbishing cost as a fraction of investment cost.

\[ T \] Project life-time

\[ E_{i} \] Discounted energy of the BD to the start-up date of project \#i.

\[ F_{i} \] Discounted energy of the future ED to the start-up date of project \#i

\[ e_{i} \] Discounted energy output of project \#i allocated to the BD.

\[ x_{i} \] Capacity or size of project number \( i \) (e.g. in GWh/year).

\[ x \] Capacity of any project in an infinite sequence of identical hypothetical projects

\[ k'_{ai} \] Long Range Marginal Cost (LRMC) of the expansion sequence for project number \( i \) and all projects later in the sequence. No ED is assumed

\[ r \] Annual interest rate (%).

\[ q \] Constant growth rate of linear demand function (GWh/year/year).

\[ \alpha \] Continuous time interest or discount rate.

\[ y_{i} \] Amount of capacity or size of project \#i to serve ED (in GWh/year).

\[ f_{i} \] Discounted energy from project \#i allocated to the ED.

\[ k'_{ai} \] Actual Unit Cost (AUC) of project \#i, representing the unit cost from this project accounting for the period of excess capacity.

\[ R_{ai} \] Full Utilization Cost (FUC) of project \#i, or the unit cost from the project with instantaneous full utilization of the project capacity.

\[ P_{ai} \] Discounted total cost of project number \( i \) and all following projects, the time reference of discounting being the start-up date for project number \( i \). No energy generation for bulk sales (ED) assumed.

\[ P_{bi} \] Discounted total cost of project number \( i \) and all following projects, the time reference of discounting being the start-up date for project number \( i \). This is with energy generation for bulk sales (ED).

\[ P \] Discounted cost of an infinite sequence of identical hypothetical projects.