Defining Optimal Production Capacity in a Purely Hydroelectric Power System

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Abstract — The conventional definition of system and plant firm capacity, in a thermally dominated power system, is often simply the installed MW adjusted by plant availability. A hydro dominated system requires an entirely different and expanded definition due to the complex interactions resulting from temporal reservoir operational decisions. In this paper several economic definitions of capacity are presented and analyzed based on the need for this concept in the operation and expansion of hydro dominated systems.

Index Terms — Hydroelectric-thermal power generation, firm energy, optimal design, optimal project sequencing.

I. INTRODUCTION

Long term simulation/optimization has been extensively used for decades in the expansion and operation planning of hydroelectric systems, as for instance in Norway [1], [2], Sweden [3], Iceland [4], Latin America, [5], [6], France, [7], Japan [8] and last but not least the U.S. [9].

Furthermore, capacity expansion problems have been studied generally [10 - 12], in water resources systems [13], and power generation/transmission systems [14], in particular, as applied to pure hydroelectric systems [15] with heuristics and optimization. Also optimal timing decisions have been studied in general but without the associated allocation and definition of capacity [16].

The conventional definition of system and plant firm capacity, in a thermal power system, is often simply the installed MW adjusted by plant availability. This is based on the term capacity being some measure or index describing an entity’s useful contribution or output in an operational or expansion framework. As opposed to thermal plants, where the fuel input may often be considered unconstrained, this is not the case for hydroelectric stations.

A hydro dominated system calls for an entirely different and expanded firm capacity definition. Systems with hydroelectric generation can use the “free” stored hydro energy in the system reservoirs to meet demand, thus avoiding fuel expenses with thermal units. However, the availability of the hydroelectric energy is limited by reservoir storage capacities which introduces a relationship between the operative decision in a given stage and the future consequences of this decision. Thus, due to the complex interactions resulting from these temporal reservoir operational decisions, capacity is not a straightforward definition in hydro systems as it is in thermal systems. This situation is made more complex by the long and short term variability of inflows to reservoirs and the fact that reservoirs may differ substantially in size across systems. For example, in Iceland, Spain and Norway most reservoirs have a seasonal regulation capacity whereas in Brazil reservoirs have multi-year regulation capability. As a consequence, hydro production can vary a lot from year to year (season to season) and the concept “capacity” needs to reflect the system (or plant) production capability in a sustainable way. Moreover, demand fluctuations call for an energy oriented definition.

The conventional and most widely accepted definition of a hydro system capacity is the so-called “firm” energy. It can roughly be defined as the maximum amount of energy that the hydro system can produce under the most adverse hydrological condition from the historical record, given a demand distribution within the year. Firm energy can be determined by simulating/optimizing the system using the historical record of inflows. In the simulation, the firm energy is based on a within-the-year distribution allocating load to each time step. The annual load is then scaled up and down using this distribution until the conditions for firm energy are met.

Alternatively to the historical simulation scheme, in some other countries (such as Brazil) this concept is extended to a probabilistic basis. In this case, random inflow hydro conditions are generated synthetically by an auto-regressive stream flow model to capture nature’s variability and, in this case, the “assured energy” is defined as the energy that has an x% chance of not being supplied in any given year for the scenarios simulated. In other words, the difference between the two concepts are the degree of reliability: while the “firm energy” guarantees the supply of a given load for the worst hydrological scenario, the “assured energy” determines the equivalent load that can be supplied by the hydro system with a given level of reliability ((1-x)%).

These concepts have been extensively used in most hydro based countries and have an important role in the market design. Usually the “firm energy” is used as “physical” backing for contracts for hydroelectric power systems and for expansion planning studies, for instance in sequencing, sizing and timing of new projects.

Several factors may introduce difficulties in adopting the simple term, firm energy, as the economical definition of a hydro system or project capacity, especially when operations and capacity expansion settings are considered. For example, it may not be economical to restrict the system to be purely hydro and require it to supply its firm energy in the absolute driest year on the historical record. If one thermal plant is present in this system its total firm energy can be increased significantly for a very low ex-
ected cost with the thermal station dispatching, on the average, only a fraction of the increased annual energy. This is made even more interesting if we take into account that a hydropower system usually has some local back-up thermal stations, present in the system usually to increase reliability when the transmission system breaks down curtailing the delivery of the hydropower energy (distant from load centers). If these plants are already installed, and their investment cost is paid, it should be feasible to run these plants to complement the hydro production in dry years, increasing only the expected fuel cost. It may even become economical to run these plants more extensively, to complement the hydro production, and thus increase the system “firm capacity”, since the expected thermal cost over all wet and dry years may still be low. With no thermal plants the previous effect is usually not captured in firm energy evaluations. This calls for an extended definition of “firm capacity” where it is important to recognize the different degrees of reliability that a hydro and a thermal station can provide, with different operating costs.

The objective of this work is to address the proper and meaningful theoretical definition of the concept electricity production capacity of a hydro-based system. We will analyze and define economically the concept capacity for a hydro dominated system based on the usefulness of the concept in capacity expansion settings and showing how the concept capacity can be defined as an expansion planning tool. For example, one requirement is a consistency with optimal timing conditions in expanding the system. Another is simplicity and generality and a third one is to treat appropriately the well known synergy (or superadditivity) effects of hydro systems. The results include a capacity definition considering trade-offs between factors such as interdependence of projects, hydrological risk, timing of new projects and investment and operations costs for hydroelectric expansion sequences.

An underlying assumption of this work is a dynamic hydro-system, which, by definition, is a system being expanded with new hydro plants. This situation is not uncommon in Latin America and Asia. While in most of Western Europe and North America hydro is not being expanded extensively, Iceland is perhaps the exception in this part of the world. A further assumption is that electrical energy is the main output of the hydro system and multipurpose use of water to irrigation, navigation, flood control, etc is not considered.

This work is organized as follows: Section II presents the concepts of system and project capacity in a hydro system. Section III introduces the expansion framework with the associated notation. In section IV several different types of capacity definition are set forth and, finally, Section V discusses these definitions in relation to actual conditions and needs in purely hydroelectric power systems.

II. SYSTEM CAPACITY: ECONOMICAL DEFINITION AND SYSTEM EXPANSION

Some factors introduce difficulties to adopt the simple term firm energy as an economical definition of system capacity.

First, as previously mentioned, it may not be economical to restrict the system to be purely hydro. With one thermal plant, supplementary generation for the driest year is provided, incurring fuel cost with a low probability. The expected cost over, say, 50 years may increase slightly while the annual firm energy figure is increased considerably, since the thermal plant will effectively transform spilled energy to useful energy, at least when extensive storage capacity is not present. Therefore the thermal plant can be an economical way to increase the “firm energy”, despite its high fuel unit cost.

With the load growing each year, more existing thermal plants with different unit costs may be operated in more of the years at the “dry” end of the inflow spectrum. It is economical [16] to do this until the expected annual fuel cost of the thermal component reaches and equals the fixed annual cost of a new hydro project. The new hydro plant will then “off-load” the thermal stations with its “free” water, and thermal cost will be reduced. This work assumes that a new hydro-project is more economical than a continued operation and expansion of the thermal stations in terms of average unit cost for energy and focuses on adding only hydro plants.

The generation expansion cycle is shown in Fig. 1. It is repeated over and over and optimal timing for the “next” project will be when its fixed annual, levellized cost equals the expected variable fuel cost as shown in Fig 1B. This fuel cost depends on the load and which thermal stations are present. The analysis also assumes that decisions are based on expected value. For a risk averse decision maker the probability distribution of inflow scenarios (and operations costs) must be considered when reaching an optimal start-up timing decision.

![](image)

Fig. 1A and 1B. The Stepwise Expansion of Capacity to Meet a Nonlinear Energy Demand.

Secondly, any capacity determination, whether the pure hydro system has a thermal component or not, will depend on a given set or configuration of projects on
which the calculations are based. In fact, capacity estimates can only be made for a specific configuration. When projects are added to this set, it is observed that the overall firm energy is greater than the sum of the firm energies that would be obtained separately for each hydro plant. In other words, there is a “synergic” increase in the firm energy that results from the cooperative action of all system agents. For example, in Brazil some studies show that there is a difference of almost 8 GW between the sum of “isolated” firm energies and the overall firm energy. This will be called the super-additive characteristic of combining hydro projects.

Consider more closely the capacity expansion process as shown in Fig. 1 for a hypothetical example of 5 projects. The demand function increases over time and capacity (any measure of system output) is expanded stepwise to satisfy this demand when excess capacity is exhausted (shaded areas in Fig. 1A). Each expansion step consists of a specific hydro project and with each step comes a certain excess capacity. The excess capacity is, by definition, at all times positive and has its maximum just after a new project has been started up. When the load increases with time, the excess capacity is gradually reduced to zero. This work aims to define the concepts of system and project “capacity” meaningfully, consistent with the above expansion framework and with rational economic decisions regarding expansion, timing and sequencing of projects.

Consider the operation of the system during any period of excess capacity (Fig. 1A). Fig. 1B shows the associated time dependent costs during this period. The total cost consists of the annual fixed levellized cost of the hydro configuration and the variable thermal cost, increasing with load and time. This total cost is shown by solid curves in Fig. 1B. The optimal timing is when the variable cost equals the fixed levellized costs of the next project in sequence, as previously mentioned. Since the next project will influence the optimal time it will also influence the optimal capacity of a configuration, as discussed in the next 2 sections.

Therefore, the capacity of a given project configuration not only depends on which projects belong to this configuration (set), but also on which project will be the next in sequence after the excess capacity of the current configuration is exhausted. Similarly assume that individual project capacity is defined as the difference in system capacity with and without the project. Then individual project capacity will depend on: (a) The technical characteristics of the current project in question, (b) Configuration of all previous projects (c) The expected annual cost of the next project in sequence after the current project and (d) Which hydrological scenario is selected as a basis for determining capacity as indicated in Fig. 2 showing the framework for optimal timing.

III. MODELING THE EXPANSION PROCESS

This section presents definitions and notation for analytical models to define and determine capacity concepts in the expansion framework as discussed above.

![Fig. 2. Annual Fuel Costs of Thermal System Component. Assume a given hydro-dominated base system consisting of a set of existing hydroelectric stations. This base system – along with any expansion possibilities – may, however, include a set of back up thermal stations, each with its fuel costs, to be used only intermittently, primarily in dry years. For a load light, no thermal generation is needed. As load increases, thermal operation is required and will increase when the annual load is scaled up. Gradually the system will be exhausted with excessive fuel/shortage cost at which time it will become economical (or necessary) to build a new hydroelectric station. In this process the back-up thermal stations will be operated in a merit order until demand is met. Energy shortage occurs when this system is exhausted.]

The model presented below focuses on capacity definitions for hydro projects available for expanding the above base system. Let this set of future projects available for system expansion be \( N = \{1, 2, 3, \ldots, i, j, \ldots, n\} \) and let the discounted cost of project \( i \) be \( C_i \), taken at the project’s start up time. Let the annual levellized cost be \( C_t \). Let \( S \subseteq N \) be any unordered subset of \( N \) and let \( \Delta_N = \{S|S \subseteq N\} \) be the set of all subsets in \( N \). Let \( S - \{i\} \) represent the set \( S \) without project \( i \) and let \( S \cup \{i\} \) (or \( S + i \)) represent the set \( S \) with project \( i \). Let \( s = |S| \) be the number of projects in \( S \), which represents a configuration at stage \( s \) in the expansion process. Let \( N - S \) be the set of remaining projects not included in \( S \).

The number of possible configurations at a stage, \( s \), is clearly \( k_s = \binom{n}{s} = \frac{n!}{s!(n-s)!} \), where \( 0 \leq s \leq n \) and the total number of subsets (configurations) at all stages is \( \sum_{s=0}^{n} k_s = 2^n \). For a given number of projects, \( s \), let \( \Delta_s \) be the set of all subsets, with \( s \) projects, or \( \Delta_s = \{S|S \subseteq N, |S| = s\} \). Therefore the union of these disjoint sets is \( \bigcup_{s=0}^{n} \Delta_s = \Delta_N \) and the number of elements in \( \Delta_s \) is \( k_s \).

No specific sequence has been assumed above. However, in the actual expansion process, projects are constructed in a specific sequence. Therefore, define a certain
permutation of projects in $N$ as an ordered set, $\Theta(n) = \{\theta(1),\theta(2),...,\theta(n)\}$ where $\theta(i)$ is the project in the $i$-th place in the sequence. The number of possible permutations is $n!$. Also define a partial sequence as $\Theta(s) = \{\theta(1),...,\theta(s)\}$ where $s < n$ and let $S(s)$ be the configuration (unordered set) of the first $s$ specific projects in the permutation sequence $\Theta$.

Next consider hydrological uncertainty. In order to account for this, a set, $B = \{h_1,h_2,...,h_n\}$ of hydrological scenarios is introduced. Each scenario has its associated probability, $p_a$, where $\sum_{a=1}^{n} p_a = 1$. Some of these scenarios may be characterized as “dry years” while others may be called “medium years” or “wet years”.

Further assume an annual energy demand variable, $Z$ (in for instance GWh/year) according to a specified within-the-year distribution. This variable is assumed to increase with time, $Z = D(t)$ as a moving annual average (e.g. Fig. 1A).

When a given configuration of future projects, $S$, is subjected to the demand, $Z$, it is possible to analyze its operation by chronological simulation/optimization, where $S$ is superimposed on the base system. For an appropriately defined “dry” hydrological condition, $h_0 \in B$, and with a given $S$, $Z$ can take a certain maximum value $Z_{f,S}$ called the firm energy of $S$, before any thermal stations must be run. For values $Z > Z_{f,S}$, for at least some hydrological scenarios $h, h_2 \in B$, a non-zero, positive annual thermal fuel and operations cost, $c_{f,S}(Z)$, will be incurred, including shortage cost, etc. Therefore for any $S$ and $h$, define the cost function $c_{f,S}(Z)$ and then define the expected cost function, $f_{S}(Z)$, as the expected value of $c_{f,S}(Z)$ for all hydrological scenarios:

$$c_{f}(Z) = f_{S}(Z) = \sum_{a=1}^{n} p_a c_{f,S}(Z) \quad (1)$$

The remainder of the paper assumes solely the expected cost function of (1) while the risk associated with hydrological scenarios awaits further research. Assume that (1) is a non-decreasing function in $Z$. Therefore, for a given $S$, the concept firm energy, $Z_{f,S}$, may be defined as follows:

$$Z_{f,S} = \max \{Z | c_{f}(Z) = 0\} \quad (2)$$

The cost function has by definition the following properties:

$$c_{f}(Z) = 0 \quad \text{if} \quad Z \leq Z_{f,S}$$
$$c_{f}(Z) \geq 0 \quad \text{if} \quad Z > Z_{f,S} \quad (3)$$

Furthermore assume the cost function $c_{f}(Z)$ to be convex, as thermal stations are operated in the order of ascending unit cost. With these properties of $c_{f}(Z)$, the inverse function $Z_{f} = f_{S}^{-1}(c_{f})$ exists. For a given project permutation, $\Theta(n)$, and the associated configurations $S(1),S(2),...,S(n)$, we accordingly have a set of cost functions, $c_{f(1)}(Z),c_{f(2)}(Z),...,c_{f(n)}(Z)$ one for each partial sequence, as shown in Fig. 3. (Alternatively we can think of the “cost” functions in Fig. 3 and Fig. 4 as a probability of loss of load functions). In fact, all $2^n$ subsets of projects, $S$, with a designated annual load, $Z_{s}$, have an associated cost function $c_{s} = f_{S}(Z_{s})$ and an associated inverse cost function $Z_{s} = f_{S}^{-1}(c_{s})$, which will be the basis for different capacity definitions, as described below. Note that these functions are based on expected values for all hydrological scenarios, considerations must be given to each hydrological scenario and its corresponding cost function, to treat risk appropriately.

We can now develop different types of capacity definitions for both individual projects and system useful in various purposes in the operations and expansion of hydro systems.

IV. CAPACITY DEFINITIONS

A. Firm energy as a capacity definition

First, the previously mentioned concept firm energy will be used as a basis for defining the system capacity $Z_{f,S}$ of a given configuration $S$. This will be called f-capacity and is defined by (2) for a given $S$. Individual project f-capacity can be defined as the difference between system f-capacity with and without the project, or

$$s_{f}(\{i\},S) = Z_{f,S}-Z_{f,S} \quad (4)$$

For each configuration $S$ there are different individual projects f-capacities and $s_{f}(\{i\},S)$ is of course a function of the characteristics of the project itself.

The problem with using firm energy, as defined by (2), (3) and (4) as a basis for capacity definition is the non-optimal utilization of the hydro resources. With only a small thermal component mixed in a pure hydroelectric system, it is economical to postpone expansion until the demand has outgrown $Z_{f,S}$ by the value $Z_{off} = Z_{econ} - Z_{f,S}$ to the most economical value $Z_{econ}$, because only a fraction of the energy, $Z_{off}$ will be generated in thermal stations. Therefore each KWh generated will be multiplied by an increased utilization of the hydro system and will therefore inherently be very economical as discussed below. This definition of system and project capacity is shown in Fig. 3.

B. Definition of x-capacity

The second definition of capacity, called, x-capacity, is based on a straight line as in Fig. 4, with a fixed slope and its intersection with the cost curves. The interpretation of the line $c_{x} = b-Z_{x}$, with $b$ as a constant slope, means a certain fraction of the load should (in terms of cost) be satisfied by the thermal resources before building a new project. In other words, it comprises the expected amount of thermal resources the decision maker is willing to have running in the system before construction a new hydro
project. The x-capacity, $Z$, is defined as the solution to the following equation:

$$Z = f^{-1}_Z(c_{S(i)}) = f^{-1}_Z(bZ_{1+i})$$

(5)

and is shown as an intersection in Fig. 4. Again, the system x-capacity is a function of system configurations, $S$.

$$x(S) = x(S ∪ {i}) - x(S)$$

(6)

It is a function of system configurations, $S$ and the characteristic of the project itself.

This definition is shown in Fig 4. The primary reason for introducing it is the fact that it was used for many years in practice in the Icelandic power system when expanding the system. Based on experience and empirical data, the factor $b$ was chosen so that operations cost corresponded to operating 0.3% of the load with the least expensive thermal back-up station. This, again, is indicated in Fig. 4 by showing the intersection of the straight line and curves. The advantage of the above definition of x-capacity is its simplicity and consistency while, unfortunately, it does not ensure any optimal timing of projects and therefore not the optimal use of resources.

$$g_{x}(Z) = p_{x}(D^{-1}(Z(S))) = αC_j$$

(9)

The optimal capacity is defined as the inverse function:

$$g_{x}(Z) = g_{x}^{-1}(αC_j)$$

(10)

Further, we define the optimal capacity, $Z_{opt}(S ∪ {i})$, of configuration $S ∪ {i}$ (or alternatively $S+i$) assuming project $j$ to follow project $i$, as a solution to (11):

$$g_{x}(Z) = p_{x}(D^{-1}(Z(S+i))) = αC_j$$

(11)

Similarly to (10) we get for the optimal capacity of configuration $S+i$, with project $j$ to follow, as:

$$Z_{opt}(S+i) = g_{x}^{-1}(αC_j)$$

(12)

Fig. 3 shows the framework for the above optimal definitions of capacity. Finally, from (10) and (12) we can define optimal project capacity $x_{opt}(S, j)$ of project $i$, as an expansion project added to configuration $S$, but assuming project $j$ to follow project $i$. It is the difference in optimal system capacities with and without the project:

$$x_{opt}(S, j) = Z_{opt}(S) - Z_{opt}(S)$$

(13)

Optimal system capacity, by (10) is, therefore, defined for each possible set $S$ and each project $j$ in the set of $N-S$, the remaining projects. Formally it is a function on the cardinal product $Δ_×(N-S) → R$.

$$Δ_×(N-S) × N -(S ∪ {i}) → R$$

(12)

Optimal project capacity, by (13) is, however, defined for each possible set, $S$, each project $i$ in the set: $N-S$, and each project $j$ in the set: $N -(S ∪ {i})$. We call this project $j$ an Associated Following Project. Formally optimal project capacity is therefore a function on the cardinal product $Δ_×(N-S) × N -(S ∪ {i}) → R$. 
D. Dimensions and representations of the solution space

Since the number of elements in $A_s$ is $k_s$, and the number of elements in $N-S$ is $n-s$, the number of different values of optimal system capacity at each stage, $s$ is:

$$n! / (n-s)! (n-s)! (n-s-1)!$$

(14)

Fig. 5 gives an example of the search tree involved in the optimal timing and the simultaneous determination of the optimal capacity, for the case of 2, 3 or 4 available projects. The number of the projects in the partial sequence is shown in each case with the number of the Associated Following Project in parentheses. For instance, with $n = 4$ and $s = 1$ or 2, equation (14) results in 12 which is the number of nodes at the 1st and 2nd stage in Fig 5, case III. The actual optimization process on this tree, however, is beyond the scope of this paper.

![Figure 5](image-url)  
Fig. 5. States and Stages in the Dynamic Programming (DP) Formulation of Interdependent Projects with an Associated Following Project in the case of 2, 3 or 4 Projects (cases I, II or III).

V. CONCLUSION AND DISCUSSION

This work has analyzed theoretically the expansion framework for hydroelectric systems and definition of capacities, showing the intricate nature of the general interdependencies among hydroelectric projects and several different economic-based concepts for firm capacity of a hydro system. An important question is how significant these interdependencies become in the practical planning process of hydroelectric systems. Further linkages are brought into the picture trough the design process and sizing of projects [17]. Further research with appropriate case studies and numerical examples is however needed to determine the importance of these factors.

Moreover, the existence of “synergic” benefits in the firm energy production immediately leads to the question of their allocation among the hydro plants. This allocation has great commercial importance in many countries. For example, in the Brazilian system the firm energy right of each plant defines its contracting limit, which in turn has a direct effect on the plant’s revenues. The problem of allocating firm energy rights is an example of the general problem of allocation of costs and benefits among agents that cooperate on the construction of a shared resource, such a road or a transmission network. Such a problem is studied in coalition (cooperative) game theory, [18] and allocating the firm capacity rights on an incremental basis must be subject of future research.

VI. REFERENCES


VII. BIOGRAPHIES

Egil Benedikt Hreinsson has the MSc in Electrical Engineering from the University of Lund, Sweden and the MSc in Industrial and Systems Engineering from Virginia Tech, Blacksburg, VA. In 1972 he joined, the National Power Co., Iceland working on SCADA/EMS, power system planning and analysis of hydroelectric system expansion. In 1982 he joined the Department of E&C Engineering, University of Iceland, where he is currently a professor. His research interests include power system analysis, economics and operations planning.

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