Optimal Sizing of Projects in a Hydro-based Power System.

by

Egill B. Heimsson, Member IEEE
University of Iceland
Reykjavik, Iceland

Abstract
This paper deals with the problem of obtaining the optimal design of hydroelectric power systems with respect to determining each project's optimal size or capacity. The problem of sizing projects is analyzed from a theoretical and economical standpoint and a mathematical model is presented to reflect on project sizing in the general framework of expanding a purely hydroelectric system. The interaction between sizing and sequencing decisions is also discussed. The results offer design rules to choose the optimal size and marginal cost for individual projects, and show the interdependence of scaling and sequencing decisions. These rules are in summary such, that it is optimal to make the current marginal cost (CMC) of each new project equal to the discounted weighted average of the long term marginal unit cost (LRMC) for all future projects.

1. Introduction and general considerations
The expansion process for electrical power systems involves in general selecting and timing generating units from a range of different electricity generating technologies, such as oil based stations, coal fired, nuclear and conventional hydroelectric power stations. Broader economic, political and environmental issues often play an important role in how the different technologies are prioritized.

For instance, a country or a region may decide that it will base its generating capacity on indigenous hydroelectric resources, despite the possibility that oil based generation may be less expensive, at least in the shorter term. Or on the other hand, a nuclear technology may be favoured on the grounds, for instance that it enhances a country's technical basis, despite the existence of indigenous energy resources. Therefore in some instances, one of these technologies stands out as being the favourite candidate for expansion.

In this paper, we concentrate on analysing the expansion process in a power system that is based on hydroelectricity and in this paper such a system will be called a hydro-based system, as opposed to a thermal based system. The importance of the hydroelectric generating technology may come from technical, economic or geopolitical factors, which for the sake of the present discussion may be considered irrelevant.

For the purpose of the present discussion we define a hydro-based system as a power system, where non-hydro generating sources are of secondary nature and thermal backup stations are only used intermittently in dry periods with below normal streamflow accounting for only a negligible fraction of the energy production.

The above approach allows the modelling effort to be tailored to reveal and reflect important trends and factors for such a system, without blurring the picture with complexities arising in a universal model, encompassing all types of generating technologies.

The Icelandic power system is a model system, that is based almost entirely on hydroelectricity, and the modelling approach here is based largely on conditions in the Icelandic system. However, there are of course various other hydro-based systems for which the present discussion should be readily applicable.

2. Special considerations in modelling a hydro-based system.
The principal differences affecting the approach for modelling capacity expansion in a hydro-based system versus a thermal-based system, will now be discussed. These include (1) different methods in defining and determining project size or capacity and (2) different levels of diversity and selection methods for projects.

2.1 Determination of system and project capacity
In this paper the terms size or capacity are interchangeably used meaning a single dimensional measure of a project's output or power or energy production, as defined below.

In a thermal-based system the conventional definition for a project capacity is simply the available megawattage. In a hydro-based system, however, this measure is inadequate. The possibility of low streamflow may result in the system's inability to produce the required megawatts. Therefore, in water resources systems, in general and in hydro-based power systems in particular, the concept of "firm water" or "firm energy" has been used as a measure of size or capacity, meaning the maximum annual energy or water producible from a given system configuration with some prescribed level of reliability and according to some within-the-year distribution in demand.

In the Icelandic hydro-based system, the capacity of a given configuration (or a system) of hydroelectric projects has generally been determined by chronological production simulations, [ref. 13, see also ref. 16] where some optimization procedure is used to operate reservoirs and units of the system. The capacity has been defined as the maximum energy production (for instance measured in GWh/year) of a given system configuration, so that thermal backup stations are only used to produce 0.3% of the energy on the average given, say, a series of 25 years of measured streamflow [13]. Furthermore, capacity of individual projects is defined as the difference in system capacity with and without these projects. The choice of the above factor has in practice been somewhat arbitrary, as is the case with the methodology of selecting the plant factor (megawattage) for each hydro project.

Therefore, for the purpose of the present discussion, capacity for individual projects in a hydro-based system is defined as a single dimensional measure of each project's production output determined by computer simulation and may be considered identical with the concept "firm energy".
In a thermal-based system, the counterpart to a chronological simulation is the production simulation employed in power system planning models, such as WASP-III, EGPAS [6, 17] and other comparable software systems. There it is important to model the availability of individual thermal units, and chronological order is not an important factor. Therefore the load duration curve is often used in place of the within-the-year distribution in demand.

To summarize the above discussion, one can define the capacity of downstream projects in megawattage, but accounting for the availability of individual units by using for instance production simulation technique.

In a hydro-based power system, capacity may be defined as the maximum energy production determined by chronological simulation [16]. Project interdependence, however further complicates this process since for instance the capacity of downstream projects may depend upon whether an upstream project with a reservoir has been realized.

2.2 Diversity of projects
In a thermal-based system, theoretically at least, new stations may often be replicated at will to form a series of identical expansion steps. Or alternatively, one can select new stations (projects) from a continuum of sizes with respect to design parameters, where factors of design can be varied identically in any or all of the projects in the stepwise expansion process.

Different conditions are, however, present in a hydro-based system. There, nature's diversity creates unique geographical conditions at each potential site for hydroelectric projects. Each project will in general have its unique characteristics that cannot be reproduced elsewhere. However, the design of thermal-based systems and projects sit in the search for the best design parameters, such as height(s) of dam(s), sizes of reservoirs, installed capacity, plant head and several other factors. These factors are all an integral part of the concept we call size or capacity as will be discussed further.

3. Problem Description.
The present paper deals with the optimal design of a given system or sequence of hydroelectric projects. We will not explicitly consider the problem of optimally sequencing hydroelectric projects, since this has been dealt with elsewhere [ref 1-4]. We assume that a given set of hydroelectric projects exists to be realized in a prescribed sequence. We will examine the problem of how to determine the economically optimal scale or size of each project and exemplify using data from the Icelandic power system. To do this, we will formulate a model in the framework of sequencing simple independent projects (for instance ref [7] and [4]). The projects are characterized by a relationship between their cost and capacity and a nondecreasing demand function of time is given. Using a prescribed sequence of projects, we determine what conditions are to be satisfied regarding project scale to minimize the discounted cost of all future development. Similar to the development by Erlenkotter [8], we assume that, when all projects have been constructed, demand will be served by some form of a long term back stop supply. This source could be considered, for instance as thermal generating stations to be constructed, when all "economical" hydroelectric resources have been exhausted.

The model is based on the following assumptions:

1. The definition of capacity or size is single dimensional (i.e. "firm energy") and the single objective is to minimize the total discounted cost of all future development, that satisfies the demand. This is in contrast with various multiobjective water resources, with multidimensional definition of capacity, but is based on conditions in the Icelandic system or a similar system.

2. Demand is projected as continuously nondecreasing over time, and at the beginning of the planning horizon, demand is assumed to equal capacity. A somewhat different problem formulation would result assuming a discrete demand function.

3. The unique cost capacity relationship is assumed to exist for each individual project, where cost is a nondecreasing continuous function of capacity. This cost covers initial investment in construction, and annual operations and maintenance costs are either assumed negligible or to be included in the above construction cost. These costs may also be considered to be proportional to the energy production, and can therefore be excluded from the analysis.

4. Only after all the available hydro-projects have been constructed, a so called long run backstop supply may be used to satisfy additional demand. We assume that all hydro-projects are efficient in a sense defined by Erlenkotter [8], i.e. they are all in some way more economical than the long run backstop supply and therefore should all be constructed prior to using that supply.

Similar to the presentation in reference [4] additional assumptions, set forth below, further define the problem framework.

a) Project costs are assumed to be known and deterministic and are incurred as lump sums at the time of construction. Construction period is taken as being small.

b) There exists a deterministic performance measure on the size or output of a given project. This measure is denoted as capacity and its value can be varied with the design of the project.

c) The discount rate is constant with both respect to the time and magnitude of investment.

d) We assume that secondary energy does not affect the definition of capacity for individual projects or the selection of optimal size.

e) The full capacity is available for use instantaneously upon project completion. Projects, once constructed have infinite lifetime, or else, the discounted cost of replicating each individual project indefinitely is included in the construction cost.

f) Projects are independent, that is capacity or cost of a given project does not depend upon other projects, whether constructed or not.

g) There is an infinite penalty cost incurred, when demand is not satisfied. Or more precisely stated: There is an implicit use of thermal stations in the hydro-based system, regarding definition of the system capacity. Similarly the cost of energy curtailments may be regarded as implicit in that definition. No additional energy deficiencies are allowed.

h) We assume that if a project is to be expanded after its initial construction, such an expansion can be considered equivalent to a new project.

i) Inflation is assumed to have a balancing effect on both cost and revenue and hence can be excluded from further consideration.

It is generally a matter of discussion and engineering judgement to what extent, various deviations from reality in a specific modelling approach will affect the utility of a particular model.
4. Definitions and Notation.

Prior to formulating the problem mathematically, let us introduce the following notation:

- $i = 1, 2, ..., N$: Index of individual hydroelectric projects. We make the assumption that the projects are to be constructed in the sequence indicated by this index.
- $C_i$: Construction- or investment cost for project number $i$.
- $x_i$: Capacity or size of project number $i$ (expressed for instance in GWh/year).
- $C_i(x_i)$: Construction cost function relating construction cost to capacity of project number $i$.
- $D(t)$: A nondecreasing demand function of time (expressed for instance in GWh/year) an indefinite time into the future. We will devote a special attention to the simple case, when demand is assumed to increase linearly, i.e., $D(t) = qt$, where $q$ is a constant growth in demand.
- $r$: Annual interest rate expressed in percent.
- $\alpha = \ln(1+r/100)$: Discount factor used in continuous discounting.
- $t_i = D_j - D_{j-1}$: Construction- and/or startup date of project number $i$.
- $\Delta t_i = t_{i+1} - t_i$: The period of excess capacity, when total capacity is greater than the demand, during which a new project (number $i$) will become fully utilized.
- $P_i$: Discounted total cost of project number $i$ and all following projects, the time reference of discounting being the startup (construction) date for project number $i$.
- $P_{N+1}$: Total discounted cost of operating the long range back stop supply to be used after all economically efficient hydroelectric resources have been exhausted.

The following quantities will later be defined mathematically:

- $k^*_i$: Long range marginal cost of energy, referred to as LRMC for project number $i$ and all following projects, with a discounting reference to the time of construction of project number $i$. This cost can for instance be expressed in U.S. dollars/MWh (referred to as USD/MWh) or U.S. mills/kWh.
- $k^i$: The Current Marginal Cost (or CMC) associated with the construction of project number $i$ (in units of USD/MWh).
- $k^f_i$: Unit cost of energy (for instance in the units of USD/MWh) from project number $i$ assuming full utilization of the project's capacity, instantaneously after startup of the project. This cost is denoted full utilization cost or FUC.
- $k^a_i$: Actual unit cost (or AUC) of energy for instance in units of USD/MWh, from project number $i$, taking into account the period of excess capacity, during which the project's capacity is only partially utilized.

- $k^r_N$: Long range marginal cost of the long run back stop supply.

5. Problem Formulation and Discussion.

The problem is to determine the optimal size or capacity of individual projects, i.e. to find:

$$x_1, x_2, x_3, ..., x_N$$

to minimize the total discounted cost of development, that is:

$$\min P_1 = \min \sum_{i=1}^{N} C_i(x_i) \exp(-\alpha t_i) + P_{N+1} \exp(-\alpha t_{N+1})$$

Subject to $x_1, x_2, ..., x_N$.

(Eq 1)

Refer to figure 3 for a graphical description of the expansion framework. Since each project's size is not fixed, as opposed to the development given in reference [1] - [4], where the optimal sequence is sought, it becomes necessary here to introduce the second factor in the above equation. This factor accounts for the total cost of developing including the long range back stop supply, after exhausting all hydroelectric resources. The use of this supply is shifted backward or forward in time, if the size of any hydroelectric project is altered.

In a similar manner as in reference [8] equation 1 can be decomposed into the following recursive relationship

$$\min P_1 = \min \left( C_i(x_i) + P_{i+1} \exp(-\alpha t_{i+1}) \right)$$

(Eq 2)

where

$$\Delta t_{i+1} - t_i = D^j \{ x_i, x_{i+1}, ..., x_{N+1} \}$$

(Eq 3)

is the period of excess capacity.

As noted by Erlenkottter [8] the above relationship (equation 2) can be used as a dynamic programming formulation of both the sequencing and sizing problem for hydroelectric projects. We focus here on the project sizing problem leaving out the sequencing problem and limit ourselves to the simple case of assuming a linear demand function, i.e., $D(t) = qt$. We then get the following recursive equation.

$$P_i = C_i(x_i) + P_{i+1} \exp(-\alpha x_i)$$

(Eq 4)

Since in this case, the quantity $P_{i+1}$ depends on the variables $x_{i+1}, x_{i+2}, x_{i+3}, ..., x_N$ we may obtain necessary conditions for an extremum by differentiating equation number 4 to obtain the most economical size for project number $i$ in the sequence:

$$\frac{\partial P_i}{\partial x_i} = \frac{dC_i(x_i)}{dx_i} \cdot \frac{\alpha}{q} P_{i+1} \exp(-\alpha x_i)$$

(Eq 5)

By setting the derivative equal to zero, we get:

$$\frac{dC_i(x_i)}{dx_i} = \frac{\alpha}{q} P_{i+1} \exp(-\alpha x_i)$$

(Eq 6)

Now by defining the following unit cost:

$$k^r_i$$

called the long range marginal cost (or LRMC) of energy, in such a way that:

$$P_i = k^r_i \int_{t_i}^{\infty} \left( D(t) - D(t_i) \right) \exp(-\alpha t(t_i)) dt$$

(Eq 7)
or for a linear increase in demand

\[ P_i = k_i \int_0^\infty q \exp(-\alpha t) \, dt = \frac{k_i^r}{\alpha} q \]  

(Eq 8)

one can obtain an interesting interpretation of the extremum conditions as set forth in equation 6.

First Equation 7 can be interpreted as the definition of the long run marginal cost (LRMC) of energy, where the cost of project number 1 and the cost of all projects in the sequence after number 1 including the effects of the long run back stop supply, have been accounted for in this unit cost. By substituting equation 8 into eq. 4, we get the following recursive relationship for the LRMC:

\[ k_i^r = C_i^r \frac{q}{q_i} \exp \left( -\frac{\alpha q}{q_i} \right) \]  

(Eq 9)

Similarly, by substituting equation 8 into equation 6 one gets the following conditions for an extremum (minimum) cost:

\[ \frac{\Delta C_i^r(x_i)}{\Delta x_i} = k_i^r + \frac{d}{dx_i} \exp \left( -\frac{\alpha x_i}{q_i} \right) \]  

(Eq 10)

or:

\[ \frac{d}{dx_i} C_i^r(x_i) = k_i^r + \frac{d}{dx_i} \exp \left( -\frac{\alpha x_i}{q_i} \right) \]  

(Eq 11)

Equation 11 is the definition of the current marginal cost (CMC) and has the natural and interesting interpretation, that the resources at each project site should be utilized to such a degree, that the CMC of additional capacity for the current (next) project should equal the long range marginal cost (LRMC) of meeting all future demand after the current project, but with a discounting of this cost to the start up date of the current project.

This constitutes the economical rule that should be used in the design process of each new project to obtain the optimal utilization of the resources at the project site.

One can obtain a new recursive expression for the long range marginal cost by defining the following unit cost quantity. We shall call this the actual unit cost or AUC, which is obtained by dividing the discounted cost for each project by the discounted energy output, or:

\[ AUC = k_i^a = \frac{C_i^a(x_i)}{E_i} \]  

(Eq 12)

where

\[ E_i = \int_0^{x_i/q} q \exp(-\alpha t) \, dt + \int_{x_i/q}^{\infty} x_i \exp(-\alpha t) \, dt \]

is the discounted value of all future energy output of the project, assuming, as before, a linear increase in demand.

By evaluating the integrals, one gets the following expression for the AUC.

\[ AUC = k_i^a = \frac{2 C_i^a(x_i)}{q \left( 1 + \exp \left( -\frac{\alpha x_i}{q} \right) \right)} \]  

(Eq 13)

This quantity can be interpreted as the actual cost of energy from the project but accounting for the real limited utilization of the project's capacity during the period of excess capacity. This is in contrast with the full utilization cost (FUC) assuming instantaneous full utilization, that is:

\[ FUC = k_i^f = \frac{\alpha C_i^f(x_i)}{x_i} \]  

(Eq 14)

which is an expression for the annual cost of the project divided by its annual output. Equation 13 becomes identical with equation 14, when demand increases faster, that is:

\[ \lim_{q \to \infty} \left( k_i^a \right) = k_i^f \]  

(Eq 15)

which can be evaluated by L'Hopital's rule.

By substituting the expressions in equation 13 into equation 9, the following version of the recursive relationship is obtained.

\[ k_i^r = k_i^a (1 - \omega_i) + k_i^r + \frac{d}{dx_i} \exp \left( -\frac{\alpha x_i}{q_i} \right) \]  

where

\[ \omega_i = \exp \left( -\frac{\alpha x_i}{q_i} \right) \]  

(Eq 17)

is the factor of discounting. Equation 16 has the interesting interpretation that the long range marginal cost (LRMC) can be determined by a backward recursive computation, where the LRMC is computed as the weighted average of the AUCs of all future projects.

Further assume that the projects are sequenced in an optimal manner and we have a sequencing with the least total discounted cost of all future development. This can be accomplished with a linear increase in demand by sequencing the projects in a sequence with ascending value of the AUC. [ref 1-4].

That is, we assume a sequence, where:

\[ k_i^a \leq k_i^a \]  

(Eq 18)

Then it can be shown, (by using equation 16), that

\[ k_i^r \leq k_i^r \]  

(Eq 19)

and

\[ k_i^r \geq k_i^a \geq k_i^f \]  

(Eq 20)

using equations 13 and 14.

The above inequalities and reasoning can be interpreted in the following way:

The projects are sequenced in the order of ascending AUC (Eq. 18), which indicates a utilization of the resources moving from the most economically favourable power water sites early in the expansion process to the least economically favourable sites late in the process, when the economical resources have been exploited and exhausted. Similarly the LRMC (Eq. 19) will have an ascending value, a relatively low value early in the process and larger value late in the process.

Finally, the LRMC at each stage in the expansion process is always greater than the AUC at this same stage, which in turn is greater than the FUC at the same stage. (Eq 20)

However it is not possible to determine in general whether the CMC to be used in the optimal allocation of resources (Eq. 11) is greater than or less than the FUC.

Total construction costs and unit costs can usually be determined by a set of curves as shown in figures 1 and 2, for a hydroelectric power project or a more general production facility.

By utilizing the following theorem [see for instance ref 9 and 10] the optimal size of a hydroelectric project does not
necessarily correspond to the minimum of the FUC curve but the optimal size may be on either side of this minimum.

The theorem is as follows using the present terminology: The CMC curve intersects the FUC curve at the minimum of the FUC curve. If the FUC is rising the CMC lies above it. If the FUC is constant the CMC is equal to it. If the FUC is falling the CMC lies below it. This theorem is demonstrated in fig. 1.

This concludes our discussion on the theoretical aspects of optimally sizing hydroelectric projects in a hydro-based system.

6. Applications of the sizing procedure.

In this section we will first illustrate the above optimization procedure by applying it to a simple sizing problem, and then illustrate the above concepts and show the interdependence of sizing and sequencing decisions by using actual data from the Icelandic power system.

Assume that we have a simple sizing problem with only 2 projects (N=2). After these 2 projects have been exhausted, the long run backstop supply has to be used to satisfy the linear demand. Further assume that the FUC unit cost function can for each project be approximated by a 2nd order polynomial, where the approximation is valid in a certain capacity range between a lower and an upper limit. In this case the approximation will exactly represent cost estimates for 3 different sizes of a given hydroelectric project. References [14] and [15] exemplify such cost estimates with curves analogous to those in figure 1. In particular assume the following form of the FUCs for the 2 projects (i = 1, 2):

$$FUC = k_i^f = \frac{aC_i(x_i)}{x_i} = a_i x_i^3 + b_i x_i^2 + c_i$$

(Eq 21)

Then the construction cost function can be written:

$$C_i(x_i) = \frac{1}{\alpha} \left( a_i x_i^3 + b_i x_i^2 + c_i x_i \right)$$

(Eq 22)

Assume that 3 distinct cost estimates for different project sizes for FUCs have been obtained for each project by the following table:

<table>
<thead>
<tr>
<th>Project number</th>
<th>i = 1</th>
<th>i = 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>x_i (GWh) FUC (GWh)</td>
<td>150 30</td>
<td>150 30</td>
</tr>
<tr>
<td>(USD/MWh)</td>
<td>200 45</td>
<td>200 45</td>
</tr>
</tbody>
</table>

Then in this example the coefficients, a_i, b_i, and c_i can be determined for instance by solving 2 sets of 3x3 simultaneous linear equations, by substituting the values in the above table into Eq. 21, and we get the following values:

<table>
<thead>
<tr>
<th>Project number</th>
<th>i = 1</th>
<th>i = 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>Coefficient</td>
<td>a_i</td>
<td>0.0014</td>
</tr>
<tr>
<td>b_i</td>
<td>-0.73</td>
<td>-0.6375</td>
</tr>
<tr>
<td>c_i</td>
<td>108</td>
<td>137.5</td>
</tr>
</tbody>
</table>

Figures 1 and 2 show the plotting of corresponding cost curves to equations 21 and 22 with the values from the above table inserted. In addition the CMC is shown in figure 1 according to the following equation obtained by differentiating eq. 22:

$$CMC = \frac{\alpha dC_i(x_i)}{dx_i} = 3a_i x_i^2 + 2b_i x_i + c_i$$

(Eq 23)

Figure 1

Current Marginal Cost (CMC) and Full Utilization Cost (FUC) for 2 projects in the simple sizing problem

<table>
<thead>
<tr>
<th>CMC #1</th>
<th>CMC #2</th>
<th>FUC #1</th>
<th>FUC #2</th>
</tr>
</thead>
<tbody>
<tr>
<td>Unit costs (USD/MWh)</td>
<td>100.0</td>
<td>100.0</td>
<td>100.0</td>
</tr>
<tr>
<td>Project Capacity (GWh/year)</td>
<td>0.0</td>
<td>100.0</td>
<td>150.0</td>
</tr>
</tbody>
</table>

Figure 2

Construction Cost of a Function of Capacity for the 2 projects in the simple sizing problem.

<table>
<thead>
<tr>
<th>Constr. Costs (Million USD)</th>
<th>Project #1</th>
<th>Project #2</th>
</tr>
</thead>
<tbody>
<tr>
<td>Project Capacity (GWh/year)</td>
<td>0.0</td>
<td>100.0</td>
</tr>
</tbody>
</table>

To solve this simple sizing problem, use a backward recursive procedure in time by starting with i=3, that is the long run backstop supply, and continuing calculations for i=2 and then finally i=1. Assume the following rather arbitrary value for the LRMC for the long run backstop supply:

$$k_i^r = 70 \text{ USD/MWh}$$

Also assume a linear growth in demand with q=150 GWh/year and an annual interest rate of 7%. Substitute eq. 23 into eq. 11 and we get:

$$3a_i x_i^2 + 2b_i x_i + c_i = k_i^r e^{\frac{x_i}{q}}$$

(Eq 24)

This nonlinear equation, which expresses the necessary conditions for the optimal size, can be solved for the unknown variable x_i (optimal capacity) for i=2 by instance by numerical iteration method, for instance the Gauss-method. We get the values x_i=412.58 GWh/year and from equation 22 we get: C_2 =142.67 Million USD. These values can be substituted into equation 9 and we get:

$$k_2^r = 62.47 \text{ USD/MWh}$$

We can now proceed for the case of i=1. By substituting this value into equation 24, we get by solving again by iteration x_i=505.92 (GWh/year) as the optimal capacity for project number 1 and C_1=70.99 Million USD as the
corresponding construction cost. These optimal values are indicated in figure 2 and it is noted that they are slightly on the right side of the minimum for the FUC-curve. Although this simple sizing problem is presented mainly for demonstration purposes, in practice any FUC cost curve could be approximated by a second order polynomial, conceptually as a Taylor series expansion. This concludes our discussion of the simple project sizing problem.

Next the above concepts will be illustrated with data from the Icelandic power system for 19 actual projects and these data are used to exemplify the hydroelectric system expansion process (figure 3) and show the interaction between sequencing and sizing. These projects have all been examined, to obtain an estimate of their construction cost and capacity, by consulting engineering companies [12,14,15]. In many cases various configurations have been studied to obtain the best design from an engineering point of view. Thus in a number of cases, an estimate exists of the construction cost function of the form comparable to the curve in figure 2. In other cases a single target estimate has been presented. A summary of the design and cost estimates have been performed by YST Consulting Engineers, Reykjavik, Iceland [Ref. 12, 14, 15]. In all cases the selection of the “optimal” design has been done by using a somewhat arbitrary level of the CMC.

We will examine numerically an example of how the FUC, the AUC, the LRMC and the CMC will evolve in an optimal sequence of these projects and how these quantities are affected by a deviation from an optimal sequence.

Tables 1 and 2 demonstrated the above data for 19 Icelandic projects. [See ref 12]. Also a linear increase in demand is assumed, and the projects are sequenced in the order of ascending AUC. Interest rate in both cases is 5% p.a. and the unit cost of the back stop supply is assumed here the rather arbitrary value of 50 USD/MWh. Table 1 shows the development with an optimal sequence, while table 2 indicates the results with a nonoptimal sequence. In both tables the energy demand is assumed to grow at a rate of 150 GW/year (g = 1.50 GW/year). The price levels are approximately 1992 prices.

TABLE 1
Marginal Cost data for 19 Icelandic hydroelectric projects using an optimal sequence.

<table>
<thead>
<tr>
<th>Name of Project</th>
<th>Capacity (GW/year)</th>
<th>Construction Cost (USD/MWh)</th>
<th>AUC (USD/MWh)</th>
<th>FUC (USD/MWh)</th>
<th>LRMC (USD/MWh)</th>
<th>CMC (USD/MWh)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Blinda</td>
<td>700</td>
<td>172.1</td>
<td>9.55</td>
<td>8.84</td>
<td>11.18</td>
<td>8.53</td>
</tr>
<tr>
<td>Kvikavatn</td>
<td>820</td>
<td>142.2</td>
<td>9.65</td>
<td>8.46</td>
<td>11.18</td>
<td>8.57</td>
</tr>
<tr>
<td>Vilvöðli</td>
<td>750</td>
<td>135.1</td>
<td>9.76</td>
<td>8.66</td>
<td>11.86</td>
<td>9.13</td>
</tr>
<tr>
<td>Víflungas</td>
<td>100</td>
<td>39.1</td>
<td>10.04</td>
<td>12.18</td>
<td>11.45</td>
<td></td>
</tr>
<tr>
<td>190</td>
<td>74.6</td>
<td>11.00</td>
<td>10.40</td>
<td>12.30</td>
<td>10.97</td>
<td></td>
</tr>
<tr>
<td>873</td>
<td>178.1</td>
<td>11.41</td>
<td>9.93</td>
<td>12.45</td>
<td>9.37</td>
<td></td>
</tr>
<tr>
<td>Búðardalur</td>
<td>550</td>
<td>120.7</td>
<td>10.69</td>
<td>12.79</td>
<td>10.70</td>
<td></td>
</tr>
<tr>
<td>Nógrós</td>
<td>525</td>
<td>118.1</td>
<td>10.94</td>
<td>13.01</td>
<td>10.87</td>
<td></td>
</tr>
<tr>
<td>Lagarfljót</td>
<td>305</td>
<td>70.8</td>
<td>11.33</td>
<td>13.21</td>
<td>11.96</td>
<td></td>
</tr>
<tr>
<td>Hólmavík</td>
<td>375</td>
<td>87.1</td>
<td>11.33</td>
<td>13.35</td>
<td>11.81</td>
<td></td>
</tr>
<tr>
<td>Ásbrúavatn</td>
<td>1350</td>
<td>254.2</td>
<td>12.08</td>
<td>13.52</td>
<td>9.00</td>
<td></td>
</tr>
<tr>
<td>1323</td>
<td>270.9</td>
<td>12.28</td>
<td>9.92</td>
<td>14.24</td>
<td>9.25</td>
<td></td>
</tr>
<tr>
<td>790</td>
<td>180.6</td>
<td>12.65</td>
<td>11.15</td>
<td>15.29</td>
<td>11.83</td>
<td></td>
</tr>
<tr>
<td>385</td>
<td>93.1</td>
<td>12.55</td>
<td>11.80</td>
<td>14.07</td>
<td>14.18</td>
<td></td>
</tr>
<tr>
<td>470</td>
<td>115.1</td>
<td>12.88</td>
<td>11.95</td>
<td>16.54</td>
<td>14.19</td>
<td></td>
</tr>
<tr>
<td>280</td>
<td>77.6</td>
<td>14.15</td>
<td>13.52</td>
<td>17.34</td>
<td>15.65</td>
<td></td>
</tr>
<tr>
<td>1550</td>
<td>353.9</td>
<td>14.18</td>
<td>11.74</td>
<td>17.43</td>
<td>10.53</td>
<td></td>
</tr>
<tr>
<td>Fjöraður</td>
<td>105</td>
<td>36.5</td>
<td>17.25</td>
<td>19.56</td>
<td>18.00</td>
<td></td>
</tr>
<tr>
<td>Austuravík</td>
<td>850</td>
<td>104.8</td>
<td>17.62</td>
<td>19.64</td>
<td>12.22</td>
<td></td>
</tr>
</tbody>
</table>

LRMC backcast: 50.00

Table 2 shows the deviation with a nonoptimal sequence, if the largest project (Austuravíkur), which occupies the last place in the optimal sequence, is moved forward. The influence of the long run back-stop supply is now heavier for future projects with regard to the LRMC and the CMC. It is interesting to note, that due to the unrealistically long period of excess capacity (8530/150=57 years) for this large project, the CMC will be very low, when it is constructed which is because investment in the project has to be weighted against the discounting effect of saving the funds for the next project(s).

If the period of excess capacity is made equal for the projects by for instance a partial instantaneous utilization with for instance energy intensive industry, the CMC should be an ascending quantity, as the LRMC, but the optimal sequence would probably change.

TABLE 2
Marginal Cost data for 19 Icelandic hydroelectric projects using a nonoptimal sequence.

<table>
<thead>
<tr>
<th>Name of Project</th>
<th>Capacity (GW/year)</th>
<th>Construction Cost (USD/MWh)</th>
<th>AUC (USD/MWh)</th>
<th>FUC (USD/MWh)</th>
<th>LRMC (USD/MWh)</th>
<th>CMC (USD/MWh)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Blinda</td>
<td>700</td>
<td>135.1</td>
<td>9.55</td>
<td>8.16</td>
<td>10.75</td>
<td>8.83</td>
</tr>
<tr>
<td>Kvikavatn</td>
<td>820</td>
<td>142.2</td>
<td>9.64</td>
<td>8.46</td>
<td>11.18</td>
<td>8.57</td>
</tr>
<tr>
<td>Vilvöðli</td>
<td>750</td>
<td>133.1</td>
<td>9.76</td>
<td>8.66</td>
<td>11.86</td>
<td>9.13</td>
</tr>
<tr>
<td>Víflungas</td>
<td>100</td>
<td>39.1</td>
<td>10.04</td>
<td>12.18</td>
<td>11.45</td>
<td></td>
</tr>
<tr>
<td>190</td>
<td>74.6</td>
<td>11.00</td>
<td>10.40</td>
<td>12.30</td>
<td>10.97</td>
<td></td>
</tr>
<tr>
<td>873</td>
<td>178.1</td>
<td>11.41</td>
<td>9.93</td>
<td>12.45</td>
<td>9.37</td>
<td></td>
</tr>
<tr>
<td>Búðardalur</td>
<td>550</td>
<td>120.7</td>
<td>10.69</td>
<td>12.79</td>
<td>10.70</td>
<td></td>
</tr>
<tr>
<td>Nógrós</td>
<td>525</td>
<td>118.1</td>
<td>10.94</td>
<td>13.01</td>
<td>10.87</td>
<td></td>
</tr>
<tr>
<td>Lagarfljót</td>
<td>305</td>
<td>70.8</td>
<td>11.33</td>
<td>13.21</td>
<td>11.96</td>
<td></td>
</tr>
<tr>
<td>Hólmavík</td>
<td>375</td>
<td>87.1</td>
<td>11.33</td>
<td>13.35</td>
<td>11.81</td>
<td></td>
</tr>
<tr>
<td>Ásbrúavatn</td>
<td>1350</td>
<td>254.2</td>
<td>12.08</td>
<td>13.52</td>
<td>9.00</td>
<td></td>
</tr>
<tr>
<td>1323</td>
<td>270.9</td>
<td>12.28</td>
<td>9.92</td>
<td>14.24</td>
<td>9.25</td>
<td></td>
</tr>
<tr>
<td>790</td>
<td>180.6</td>
<td>12.65</td>
<td>11.15</td>
<td>15.29</td>
<td>11.83</td>
<td></td>
</tr>
<tr>
<td>385</td>
<td>93.1</td>
<td>12.55</td>
<td>11.80</td>
<td>14.07</td>
<td>14.18</td>
<td></td>
</tr>
<tr>
<td>470</td>
<td>115.1</td>
<td>12.88</td>
<td>11.95</td>
<td>16.54</td>
<td>14.19</td>
<td></td>
</tr>
<tr>
<td>280</td>
<td>77.6</td>
<td>14.15</td>
<td>13.52</td>
<td>17.34</td>
<td>15.65</td>
<td></td>
</tr>
<tr>
<td>1550</td>
<td>353.9</td>
<td>14.18</td>
<td>11.74</td>
<td>17.43</td>
<td>10.53</td>
<td></td>
</tr>
<tr>
<td>Fjöraður</td>
<td>105</td>
<td>36.5</td>
<td>17.25</td>
<td>19.56</td>
<td>18.00</td>
<td></td>
</tr>
<tr>
<td>Austuravík</td>
<td>850</td>
<td>104.8</td>
<td>17.62</td>
<td>19.64</td>
<td>12.22</td>
<td></td>
</tr>
</tbody>
</table>

LRMC backcast: 50.00

Interest rate 5.00%
Exp. int. rate 4.88%
Market growth (GW/year/year) 150

As indicated in the table, the LRMC is slightly higher than the AUC for all projects and is ascending, as we move towards the less favourable sites. The CMC however shows variations, due to the different size of the projects, and is not necessarily in any direct relation of the FUC or the LRMC.
7. Conclusions and discussion

In this paper a model has been presented for the project sizing problem which involves determining the optimal size of each project in a given sequence of hydroelectric projects. The problem is solved by the determination of the optimal current marginal cost (CMC), and is readily solved in the case of a linearly increasing demand with a simple recursive procedure backwards in time. The model has been applied to a simple sizing problem and exemplified using data for as specific model power system to show the interdependence of sequencing and sizing.

The model should be useful to planners of hydro-based power systems, where a scenario has been defined for a development involving a sequence of several projects perhaps decades into the future when the costs of future projects are known or estimated. With this model it is possible to determine the optimal size of each project and the associated marginal cost (CMC). This is important since it determines how far the resources at each project site should be exploited, for instance regarding design parameters of the project such as what new diversions from different catchment areas are to be included, sizes of seasonal reservoirs, height of dams, determination of plant heads, or forebay level, tailrace level, etc.

In practice it is likely that both estimates of costs and capacities are inaccurate for projects to be constructed in the future, and even the sequence is not known or decided. Rather it is likely that all these factors are parts of an iterative planning process, where the different factors interact with each other, and the focus is on developing more information as a basis for decisions about the near future. The present model is aimed at such a methodology and energy policy for exploiting a region's water resources.

Further research could be directed towards solving the problem in the general case of a nonlinear demand function and investigating the interaction between the Project Sequencing Problem and the Project Sizing Problem. However the simple case of a linear demand, which in fact is often a reasonably good approximation to a comprehensive demand forecast, function is of considerable interest as a screening method in the early stages of the planning process of hydroelectric systems.

8. References


Egil B. Heiinsson (Member) received the Master's Degree in Electrical Engineering from the University of Lund, Lund, Sweden in 1972 and the Master of Science Degree in Industrial Engineering and Operations Research from the Virginia Polytechnic Institute & State University, Blacksburg, VA, in 1980. In 1972 he joined the National Power Co., Reykjavik, Iceland with duties specializing in computer control, SCADA systems and telecommunication and telemetering systems. From 1977-1982 he was a Division Engineer with the Engineering and Planning Department., The National Power Co specializing in power system planning and analysis of hydroelectric system expansion alternatives and studies relating to the expansion of Energy Intensive Industries. From 1982 to the present he has been an associate professor at the University of Iceland, Department of Electrical Engineering, with teaching and research in the fields of power systems analysis, optimization and economics. Mr. Heinsson has been a consultant to various Power Generation and Distribution Utilities, on issues relating to power system planning and computer applications in power systems.