Optimal Design of Interdependent Hydroelectric Schemes Based on Optimal Timing

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Abstract—In countries, where electricity generation is based on an active or dynamic expansion of hydroelectric systems, it is important to determine the optimal design of projects and whole project configurations or sequences. This paper looks at this design process assuming interdependent projects as components of schemes or fixed sequences in a defined expansion path. The paper then analyzes economical criteria for the system optimal stage-wise expansion and their impact on individual project design. For instance, dam volume will depend on dam dimensions while the chosen tunnel width could generally be independent of the dam design. Both may impact cost and timing of individual project and thereby the cost of the whole sequence. Optimal timing of projects may also be linked to optimal definition of project output or capacity, based on matching increasing operating cost with time and load to an expansion step. The principal findings are that the optimal size can be adjusted by a marginal adjustment factor (MAF) to take into account the impact of sequential dependencies between adjacent projects. The calculations should be carried out iteratively within larger sequences, in order to reach optimum size based in optimum timing for projects.

I. INTRODUCTION

A. The problem at hand

A Dynamic Hydroelectric System (DHS), [1], is defined as a set of configurations of mutually exclusive, economical, hydroelectric schemes, to be constructed in stages, in a given region, with given environmental and/or political constraints. Assume that such schemes/expansion paths can be followed to a final stage, which occurs when all economically, technically and environmentally viable hydro resources have been exhausted.

Therefore, a DHS is a hydro or renewable based power system, which is being actively expanded. Such systems are well known in Latin America and Asia, since most economical hydro resources are already exhausted in Western Europe or North America or have binding environmental constraints. An exception is Iceland, where large scale active hydro and geothermal is still taking place.

We now discuss the basic characteristics of a DHS. Assume a basic existing hydro and/or renewable based system, a common starting point for the future expansion process. However, the DHS may not be 100% hydro or renewable, and therefore, the stage-wise expansion assumes a set of thermal options for back-up only with variable fuel costs. Even in the extreme case of a 100% hydro, the DHS operation would be based on reliability criteria [1] under dry conditions. Whatever the marginal resource mix, the focus here is on the hydroelectric part.

The DHS is a set of expansion paths, assumed to be available. The problem at hand is choosing paths and designing individual steps. For instance, several small hydro plants or few larger plants can be built in a given valley. Or one can choose between diverting water between valleys, thereby combining flows from multiple rivers into large turbines - or to build separate smaller dams, reservoirs and powerhouses in the separate valleys. Examples of this are the hydroelectric schemes in the eastern part of Iceland, already partially realized, as shown in Figure 1.

Consider breaking down mutually exclusive paths into stages. Such stages may be individual hydro projects or project expansions. In this paper we want to determine optimal design and optimality conditions for project design and how projects may interact. Projects may either interdependent in a given path or independent of each other. They can be mutually exclusive, but the objective is to schedule and design them, as economically as possible, with start-up dates according to demand or market needs.

We will not discuss the engineering design for individual projects. The design is assumed known with cost estimates and energy/power outputs (benefits) for each project. The costs and
outputs may be functions of a given set of parameters, that is cost and benefit functions of continuous or discrete variables, called parameters. Examples of such parameters would include dimensions, such dam heights and width, reservoir size, tunnel width, etc. When parameters depend on each other, for instance when reservoir size depends on dam height, we assume the set of parameters is reduced to an independent set. As projects may or may not be interdependent, cost and benefit functions may also be. For instance, the cost of a project upstream may depend on the head-race level of a project downstream.

The project output function will here be called capacity as defined in [1]. Capacity is simply a single dimensional measure of a project’s benefit. Like the cost, it may depend on other projects’ parameters, (e.g reservoir size). Although capacity allocations [1] are not discussed in this paper, project capacity may be defined as the difference in system capacity with and without the project. Then the project benefit function may depend on reservoir size of another previous project, making the two projects highly interdependent. Generally, projects in a closed interlinked system are likely to be interdependent, but the case of independent projects simplifies the analysis.

Consider a simple case with a path with a fixed sequence of projects. Now investigate how the following factors influence project optimal design and size:

1) Project interdependence. Theoretically, interdependencies are always present among projects, for instance, the cross correlation in river flow. However, often these are negligible and each project contributes a given benefit at a given cost, irrespective of other projects in the sequence and their parameters. Interdependence may also exist when parameters directly depend on parameters of other projects, as in the case of the head/tailwater levels previously mentioned. In summary, on can distinguish between independent projects, where the cost- and benefit functions depend only on the parameters of the same project and interdependent projects, where the cost- or benefit functions may depend on any other project. For instance, the cost may depend on the dimensions of reservoirs “belonging to” existing nearby projects - or reservoirs to be constructed later.

2) Optimal timing and the optimal capacity of projects. As discussed in [1], optimal timing results in defining optimal capacity. By definition, then, optimal capacity is exhausted at the optimal start-up time for the next project. As shown in [3, 1], optimal time will depend on the annual levelized investment and operations cost of the next project. This added dependency on the next project is of particular interest.

The above issues exemplify the complexities leading to general optimality conditions of a complex hydroelectric scheme as addressed and derived below.

B. A short literature review

Scheduling of projects is discussed in [4, 5, 6], including sequencing, sizing and timing. In [3] the timing is incorporated in an operations cost sub-model showing that optimal timing is reached when the operating cost equals the annual fixed cost of the next project. Optimal sizing is addressed in [7], where a project cost function $C_i(x_i)$ is defined. The cost function with parameters is discussed in [8]. Optimal sequencing by heuristics and Dynamic Programming (DP) is discussed in [9] and randomly generated and discounted costs are in [10].

A wide research base is available on water resources expansion. [11] uses DP to reach a schedule. This author’s previous research started with [12], where optimality conditions lead to optimal size. Genetic algorithms were addressed in [13] and [14]. Bulk energy sales were in [15], with fixed size, but the optimal size was determined with simple optimality conditions. Finally, [16] discusses simple capacity expansion models. [2] is a practical example of designs of hydro schemes, where environmental restrictions play an important role [17].

C. The plan of the paper

The paper is organized as follows: In Section II a model is developed for optimal timing and sizing. In Section III a numerical case study is presented, illustrating a simple timing and sizing problem. Section IV presents major conclusions and discussions. Finally the paper is concluded with a references section.

II. OPTIMAL TIMING AND SIZING OF PROJECTS

A. The general problem framework

We want to construct a fixed sequence, defined by the index $i$, of $N$ hydro projects $\Omega = \{1, 2, 3, ... i, ... N\}$. The task is to determine the design of each project by minimizing an objective function of total discounted cost (TDC). The problem is stated in (1), for a linear demand function $D(t) = q \cdot t$:

\[
\begin{align*}
\text{TDC} &= P_i^* = \min \left\{ \sum_{i=1}^{N} C_i \exp \left[ -\frac{\alpha}{q} \sum_{j=1}^{i} x_j \right] \right. \\
&\quad \left. + P_{N+1} \exp \left[ -\frac{\alpha}{q} \sum_{j=1}^{N} x_j \right] \right\} \\
&\quad \text{where } C_i \text{ is the cost and } x_i \text{ the output of project } i. \\
&\quad \text{The demand growth } q \text{ is constant, } t \text{ is time and } \alpha \text{ is the continuous discount rate. } P_{N+1} \text{ is the TDC of the back-up supply after all hydro resources are exhausted.}
\end{align*}
\]

The quantity inside the braces in (1) can be decomposed into recursive equations, as follows:

\[
\begin{align*}
P_i &= C_i + P_{i+1} \exp(-\alpha x_i / q) \\
&\text{for all } i, \text{ where } P_i \text{ is the TDC at stage } i \text{ in the expansion sequence.}
\end{align*}
\]

To facilitate the minimization of the cost in (1), the variables that can be changed to reach a minimum need to be defined. Therefore, further assume that each project $i$ is characterized
by a set of parameters, defined by a vector of real numbers, $u_i = [u_{i,1}, \ldots, u_{i,n_i}]^T$, where $u_{ij}$ is parameter $i$ and $n_i$ is the number of parameters for project $i$. These parameters describe various project design features and dimensions, such as dam height, reservoir size, tunnel width, etc. as previously discussed.

Consider first the case of completely independent projects and some given timing and capacity definition, which may be non-optimal.

### B. Independent projects with fixed characteristics

Define "project fixed characteristics", to designate project parameters $u_i$ so that both capacities $x_i$ and costs $C_i$ are independent of both previous projects $1, 2, \ldots, i - 1$ and future projects $i + 1, i + 2, \ldots, N$ in the sequence. Therefore, both the project cost function $C_i(u_i)$ and benefit function $g_i(u_i)$ depend only on the project’s own parameters. This case has been previously analyzed in [7], but is reviewed here to serve as a base case for the extended development below.

Therefore, assume that each project has a cost function:

$$C_i(u_i) = C_i(u_{i,1}, u_{i,2}, \ldots, u_{i,n_i})$$

and also a corresponding scalar benefit function which expresses the concept capacity as follows:

$$x_i = g_i(u_i) = g_i(u_{i,1}, u_{i,2}, \ldots, u_{i,n_i})$$

according to some definition of that concept [1].

In addition, assume box type constraints on the parameters, defining a feasible range, as follows:

$$u_{i,\text{min}} \leq u_i \leq u_{i,\text{max}}$$

Incidentally, a project’s cost function can be defined by maximizing (3) subject to (4) and (5), see discussion in, for instance [8, 7].

In this case, the TDC (1) for the whole sequence can be rewritten as an unconstrained optimization problem:

$$P_1(u_1, u_2, \ldots, u_N) = \sum_{i=1}^{N} C_i(u_i) \exp \left( -\frac{\alpha}{q} \sum_{j=1}^{i-1} g_j(u_j) \right)$$

$$+ P_{N+1} \exp \left( -\frac{\alpha}{q} \sum_{j=1}^{N} g_j(u_j) \right)$$

and the minimizing objective as:

$$P^*_1 = \min_{u_1, u_2, \ldots, u_N} \{ P_1(u_1, u_2, \ldots, u_N) \}$$

Since the discounted cost at each stage $P_i$ can be expressed as a function of the parameters of all future projects, the recursive equation (2) for any stage $i$ can now also be rewritten as:

$$P_i(u_i, u_{i+1}, \ldots, u_N) = C_i(u_i)$$

$$+ P_{i+1}(u_{i+1}, u_{i+2}, \ldots, u_N) \exp(-\alpha g_i(u_i)/q)$$

With independent projects, TDC at each stage, therefore, depends only on the current project $i$ and future projects $i + 1, i + 2, \ldots, N$.

In order to state necessary optimality conditions, simplify the notation and define the following vector of operators/derivatives to state the necessary optimality conditions:

$$\frac{\partial}{\partial u_i} = \left[ \frac{\partial}{\partial u_{i,1}}, \frac{\partial}{\partial u_{i,2}}, \ldots, \frac{\partial}{\partial u_{i,n_i}} \right]^T = 0$$

if and only if

$$\frac{\partial P_i}{\partial u_{i,j}} = 0 \quad \forall j \in \{1, 2, \ldots, n_i\}$$

where $0 = [0, 0, \ldots, 0]^T$.

To minimize the function $P_i(u_i, u_{i+1}, \ldots, u_N)$ of (8) the following necessary conditions for a minimum are stated:

$$\frac{\partial P_i}{\partial u_i} = 0 \quad \forall k \in \{i, i+1, \ldots, N\}$$

Applying (11) to (8) first for the case of $k = i$, results in:

$$\frac{\partial P_i}{\partial u_i} = \frac{\partial C_i}{\partial u_i} = P_{i+1} \cdot \frac{\alpha}{q} \frac{\partial g_i}{\partial u_i} \exp(-\alpha g_i(u_i)/q)$$

As discussed in [12] equation (13) is valid:

$$P_i = \frac{q k^r_i}{\alpha^2}$$

which results in:

$$\frac{\partial C_i}{\partial u_i} = P_{i+1} \cdot \frac{\alpha}{q} \exp(-\alpha g_i(u_i)/q)$$

Setting (12) equal to zero, results in:

$$\frac{\partial C_i}{\partial u_i} = P_{i+1} \cdot \frac{\alpha}{q} \exp(-\alpha g_i(u_i)/q)$$

and similarly, using the Long Range Average Cost, LRAC $k^r_i$ and the Long Range Marginal Cost, LRMC $k^m_i$, results in:

$$\frac{\partial (\alpha C_i)}{\partial u_i} = P_{i+1} \cdot \frac{\alpha}{q} \exp(-\alpha g_i(u_i)/q)$$

From [7], the condition (16) leads to the conclusion that each project should be constructed so the current marginal cost (CMC), should equal the LRMC or $k^r_i$ discounted to the start-up time of project $i$.

For $k = i + 1, i + 2, \ldots, N$ minimization of (8) leads to:

$$\frac{\partial P_i}{\partial u_k} = \frac{\partial P_{i+1}}{\partial u_k} \exp(-\alpha g_i(u_i)/q)$$

Therefore it is clear that the necessary conditions (11) also apply, since both factors of (8) are positive.

Next consider the case with defined optimal capacity [1] and how it influences the optimality conditions. This is further illustrated in Figures 1B and 2 in [1].
C.  Interdependent projects with optimal capacity

Previous we assumed that, \( x_i \), depend only on the project’s own parameters. With optimal capacity [1], based on optimal timing, \( x_i \) is a function of \( u_i \), and also of \( C_{i+1}(u_{i+1}) \), since optimal timing is reached when the operating cost function (See Figures 1B and 2 in [1]) for a project or sequence is equal to the annual fixed cost of the next project [3, 1]. Therefore capacity can be written:

\[
x_i(u_i, u_{i+1}) = g_i(C_{i+1}(u_{i+1}), u_i)
\]

(18)

However, for the last project in the sequence, assuming fixed timing criteria, results in a capacity function that is independent of the (non-existing) next project after the last one.

Therefore, the objective function (1) now takes the following form:

\[
P_i^* = \min_{u_i, u_{i+1}, \ldots, u_N} \left\{ \sum_{i=1}^{N} C_i(u_i) \exp \left( -\frac{\alpha}{q} \sum_{j=1}^{i-1} g_j(C_{j+1}(u_{j+1}), u_j) \right) + P_{N+1} \exp \left( -\frac{\alpha}{q} \sum_{j=1}^{N-1} g_j(C_{j+1}(u_{j+1}), u_j) + g_N(u_N) \right) \right\}
\]

(19)

The quantity inside the braces is simply:

\[
P_i = C_1(u_1)e^{-\alpha t_1} + C_2(u_2)e^{-\alpha t_2} + \ldots + C_N(u_N)e^{-\alpha t_N} + P_{N+1}e^{-\alpha t_{N+1}}
\]

(20)

where the optimal construction or start-up time \( t_i \) of each project \( i \) is for a linear demand given by:

\[
t_1 = 0
\]

\[
t_2 = \frac{g_1(C_2(u_2), u_1)}{q}
\]

\[
\vdots
\]

\[
t_{N-1} = \frac{g_{N-1}(C_{N-1}(u_{N-1}), u_{N-2})}{q}
\]

\[
t_N = \frac{g_N(k_{N+1}, u_N)}{q}
\]

(21)

Again, since the expansion process is recursive, and can be decomposed accordingly, the discounted cost at each stage, \( P_i(u_i, u_{i+1}, \ldots, u_N) \), can be expressed as a function of the parameters of all projects \( i, i+1, \ldots, N \) that is current and future projects:

\[
P_i(u_i, u_{i+1}, \ldots, u_N) = C_i(u_i)
\]

\[
+P_{i+1}(u_{i+1}, u_{i+2}, \ldots, u_N) \exp \left( -\frac{\alpha g_i(C_{i+1}(u_{i+1}), u_i)}{q} \right)
\]

(22)

where the difference of (8) and (22) lies in the exponent only, since it is now derived from (21).

It is readily observed that applying the optimality conditions (9) to (22), similar results as before are obtained. For \( k = i \) this leads to:

\[
\frac{\partial P_i}{\partial u_i} = \frac{\partial C_i}{\partial u_i} - P_{i+1} \frac{\alpha}{q} \exp(-\alpha g_i(C_{i+1}(u_{i+1}), u_i)/q) \frac{\partial g_i}{\partial u_i} = 0
\]

(23)

which using (13) for all \( i \) results in:

\[
\frac{\partial (\alpha C_i)/\partial u_i}{\partial g_i/\partial u_i} = k_i^m = k_{i+1} \exp \left( -\alpha g_i(C_{i+1}(u_{i+1}), u_i)/q \right)
\]

(24)

leading to the conclusion that a project should be constructed so the CMC at left in (24) should equal the LRMC \( k_i^m \).

The optimality conditions (11) are applied to (22) for other values of \( k \). In the case of \( k = i + 1 \), it leads to:

\[
\frac{\partial P_i}{\partial u_{i+1}} = \frac{\partial P_{i+1}}{\partial u_{i+1}} \exp(-\alpha g_i(C_{i+1}(u_{i+1}), u_i)/q) - P_{i+1} \frac{\alpha}{q} \exp(-\alpha g_i(C_{i+1}(u_{i+1}), u_i)/q) \frac{\partial g_i}{\partial u_{i+1}} = 0
\]

(25)

which, using (13), leads to

\[
\frac{\partial P_{i+1}}{\partial u_{i+1}} = \frac{\partial g_i}{\partial u_{i+1}} = \frac{\partial g_i}{\partial C_{i+1}} \frac{\partial C_{i+1}}{\partial u_{i+1}}
\]

(26)

It is now possible to multiply (14) for \( i + 1 \) by \( \alpha \) and substitute into the left side of (26) and it leads to:

\[
\frac{\partial (\alpha C_{i+1})}{\partial u_{i+1}} - \frac{\partial g_{i+1}}{\partial u_{i+1}} e^{-\alpha g_{i+1}(u_{i+1})/q} = \frac{k_{i+1}}{\alpha} \frac{\partial g_i}{\partial C_{i+1}} \frac{\partial C_{i+1}}{\partial u_{i+1}}
\]

(27)

which can be rewritten as

\[
\frac{\partial (\alpha C_{i+1})}{\partial u_{i+1}} \beta_{i+1} = \frac{k_{i+1}^m}{\alpha} \frac{\partial g_{i+1}}{\partial C_{i+1}} e^{-\alpha g_{i+1}(u_{i+1})/q}
\]

(28)

or rearranging

\[
\frac{\partial (\alpha C_{i+1})/\partial u_{i+1}}{\partial g_{i+1}/\partial u_{i+1}} \beta_{i+1} = k_{i+1}^m = k_{i+2}^m e^{-\alpha g_{i+1}(u_{i+1})/q}
\]

(29)

where

\[
\beta_{i+1} = 1 - \frac{k_{i+1}^m}{\alpha} \frac{\partial g_{i+1}}{\partial C_{i+1}}
\]

(30)

is here defined as a marginal adjustment factor (MAF) which impacts the optimal marginal cost to the output ratio on the left hand side in (28). Note that iteration between stages of the recursion may be required.

Therefore (28) gives a condition for optimal sizing of project \( i + 1 \), for minimizing \( P_i \) when the output of of project \( i \) depends on the cost of project \( i + 1 \). It is analogous to (16), except for the MAF or \( \beta_{i+1} \) in (30). Note that if capacity \( x_i = g_i(C_{i+1}(u_{i+1}), u_i) \) is independent of the "next" cost \( C_{i+1} \), then \( \beta_{i+1} = 1 \).

The cases for \( k = i + 2, i + 3, \ldots, N \) will not be discussed further, see (17), but the above concepts will be illustrated with a case study with two projects, each with simple cost and output functions of one parameter, both where capacity may or may not depend on the "next" cost.
A. General Assumptions

Assume we have a simple case of $N = 2$ projects, each with one parameter, that is $u_1 = |u|$ and $u_2 = |v|$. Furthermore, the convex cost functions are $C_1 = u^2$ and $C_2 = v^3$ and the concave output functions are $x_1 = g_1(u) = 1 - (1 - u)^2$ and $x_2 = g_2(v) = 1 - (1 - v)^3$ with box constraints $0.5 \leq u \leq 1$ and $0.5 \leq v \leq 1$. These functions are shown in Figure 2. All quantities are measured in some arbitrary “units” and we assume a linear growth rate $q = 0.4$ and an interest rate of 3% per unit of time, i.e. $\alpha = 0.0296$. Finally, assume a LRAC of the back-up supply to be $k_5 = 3$ units.

We want to calculate the optimal cost and size for each project and how these will change in the 2nd case, when the capacity of the first project $x_1$ may depend on the cost $C_2$ of the 2nd project. Figure 3 illustrates the FUC = $\alpha C_i / x_i$ and AUC = $\alpha^2 C_i / (q(1 - \exp(-\alpha x_i / q)))$, as defined in [12], for $i = 1, 2$, as functions of the parameters $u$ or $v$.

B. Capacity independent of project cost

Consider first a case when the two projects are independent. Then capacity of the 1st project is independent of the 2nd project’s cost. The expression for TDC, that is $P_2$ at stage 2 in this case will be from (8):

$$P_2(v) = v^3 + \frac{q k_5^*}{\alpha^2} e^{-\frac{q}{\alpha}(1-(1-v)^3)}$$

which for the given numerical values results in:

$$P_2(v) = v^3 + \frac{0.4 \cdot 3}{0.0296^2} \exp\left(-\frac{0.0296}{0.4} \cdot (1 - (1 - v)^3)\right)$$

where $P_3 = q k_3 / \alpha^2 = 0.4 \cdot 3/0.0296^2 = 1369.6$. Now start a backward recursion and calculate the optimal value $v = v^*$ and the optimal size $x_2^* = g_2(v^*)$ first for project 2 by using (16). With $C_2' = 3v^2$ and $g_2' = 3(1-v)^2$, this leads to:

$$\frac{\partial (\alpha C_2)}{\partial g_2} = \frac{3 \cdot 0.0296 v^2}{3(1-v)^2} = \frac{k_2^*}{q}$$

$$k_2^* \exp(-\alpha g_2(v^*)/q) = 3 \exp\left(-\frac{0.0296(1 - (1-v)^3)}{0.4}\right)$$

Equation (33) can be solved numerically and (31) can be shown to have a local minimum within the feasible range at $v^* = 0.90656$. Therefore, we have the optimal value $C_2^* = (v^*)^3 = 0.7451$ for the cost and $x_2^* = 1 - (1 - v^*)^3 = 0.9992$ for the capacity. Then from (31) we have $P_2^*(v^*) = 1272.7$. From (13) we get $k_2^* = P_2^* \cdot \alpha^2 / q = 1272.7 \cdot 0.0296^2 / 0.4 = 2.7877$ units.

Next calculate the same quantities for project 1 using (2). This gives (34), analogous to (31):

$$P_1(u) = u^2 + P_2^* e^{-\frac{q}{\alpha}(1-(1-u)^2)}$$

To find the optimum value $u^*$, use (16) again, which will be:

$$\frac{\partial (\alpha C_1)}{\partial g_1} = \frac{0.0592 u}{2(1-u)} = 3 \exp\left(-\frac{0.0296(1 - (1-u)^2)}{0.4}\right)$$

This results in the solution $u^* = 0.98869$ which substituted into (34) gives $P_1^*(u^*) = 1182.9$. Figure 4 shows the plots for $P_1$ and $P_2$ in the neighborhood of the respective minima.

C. Capacity depends on project cost

Consider next the case when capacity of the 1st project depends on the 2nd project’s cost. This may constitute a chain of dependencies requiring iteration. In particular, use all assumptions as in Section III-B, except the output (18) now has a cost term, the last in (36).

$$x_1 = g_1(C_2(u_2), u_1) = 1 - (1 - u)^2 + \frac{(C_2 - C_{2,0})}{C_{2,0}} \cdot k_v$$

In (36) the last term is the proportional deviation of $C_2$ from its nominal value $C_{2,0}$. Assume that in Section III-B
The output was based on this value $C_{2,0}$. Now we adjust the output to $C_2$ by (28). The proportionality factor $k_v$ reflects the optimal capacity based on optimal timing, as shown in Figure 5 and reflects how $x_1$ changes relative to $C_2$.

Since $C_2 = v^*$, we get:

$$x_1 = g_1 = 1 - (1 - u)^2 + \left(\frac{v^3 - v_0^3}{v_0^3}\right) \cdot k_v$$  \hspace{1cm} (37)

In particular, assume $v_0 = 0.85$ leading to $C_{2,0} = 0.6141$ and use, for instance, $k_v = 0.02$, since $k_v$ is small as mentioned, due to the general sharp rise in the curve in Figure 5. We recall from Section III-B that $v^* = 0.90656$, which means that the capacity there was based on the cost $C^*_2 = (v^*)^3 = 0.7451$ units.

Now calculate the MAP or $\beta_{i+2}$ for $i = 1$ using $\partial g_1/\partial C_2 = k_v/C_{2,0}$

$$\beta_2 = 1 - \frac{k_v^2}{\alpha} \cdot \frac{\partial g_1}{\partial C_2} = 1 - \frac{k_v^2}{\alpha} \cdot \frac{k_v}{C_{2,0}} = 1 - \frac{2.7877}{0.0296} \cdot \frac{0.02}{0.6141}$$ \hspace{1cm} (38)

This results in the numerical value $\beta_2 = 0.6933$

Similarly, as in (33), we then substitute $\beta_2$ into (28) and get:

$$\frac{3 \cdot 0.0296v^2}{3(1 - v)^2} = \frac{3}{0.6933} \cdot \exp(-0.0296(1 - (1 - v)\cdot(1 - v^3)/0.4)$$ \hspace{1cm} (39)

Within the feasible range (39) has the solution $v^+ = 0.92096$ leading to $C^*_2 = 0.7811$ and $x_2^+ = 1 - (1 - v^+)^3 = 0.9995$ units. We see that the change in size $x_2$ is minimal or from $x_2^* = 0.9992$ to $x_2^* = 0.9995$ units, which may be attributed to combined expected sharp concavity of the output function near $x = 1$ (Figure 2 at right) and the sharp increase or convexity in cost relative to output (Figure 5).

IV. CONCLUSIONS AND DISCUSSIONS

This paper has expanded on optimal design of hydro systems:

1) First, a single project has been analyzed with linking to a recursive models of an expansion sequence.

2) Secondly, the design parameters have been shown to lead to similar results as the previous models, where project cost was a function of the output [12].

3) Third, dependent parameters lead to the same results as independent parameters in terms of linking adjacent projects with a discount factor.

The marginal cost of each kilowatt-hour in any project should be equal with respect to the marginal cost of all parameters. Otherwise it would be economical to substitute these incremental capacities with each other. Interdependence of projects does not basically change this conclusion.

In design of hydro systems, assumptions may be used with a wide time frame, of years or decades. This paper has analyzed the project interaction with marginal price signals, dictating a balance in terms of marginal cost.

REFERENCES


