DEVELOPMENT OF A HIERARCHICAL DISTRIBUTION SYSTEM COSTS
MODEL

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ABSTRACT
An electrical power distribution cost model is developed for a power distribution system based on power and energy flow. The model allows for the study of multiple energy classes, such as primary and secondary energy. Embedded costs are used for individual units including feeders, transformers, lines in addition to overhead cost, cost of losses etc. The model is recursive and hierarchical and is applicable to different voltage levels, spatial distribution of equipment, varying load, utilization and coincidence factors and will be applicable in determining distribution tariffs and real costs under various conditions in the system.

KEY WORDS
Distribution systems, Economics, Cost, Hierarchy

1 Introduction
In this paper we present a model, APOWER, for economic analysis of an electrical power distribution system. The system is represented by a hierarchical network of interconnected units as shown in figure 1. In general the network expands as we move into the lower layers of this hierarchy with lower voltage and closer to the retail user to represent an actual distribution network. The model thereby represents interconnections between power system apparatus, such as high voltage lines, feeder, transformer substations, etc.

The whole network is decomposed into units with power/energy flowing between these units and certain costs associated with units. Each unit represents a collection or aggregation of power system components. Therefore the unit represents equipment within a given geographical boundary enclosing an area. The definition of each area within the model depends on the nature and purpose of the modeling effort. Each unit is represented as a "box" in the graphical description of the system topology as shown in figure 1.

The individual units and the model as a whole deal with 2 different types of quantities:

First, power and energy flow, as defined by a separate load flow or measurements based study, is used by the model. Peak power is related to energy by a given load duration curve (LDC) for a certain period, such as a year. These quantities are assumed to flow from an injection point to a load point, where these points can be located anywhere in the hierarchy. A part of the power/energy is, of course, absorbed by the system as losses.

Secondly, the model processes in a hierarchical manner cost variables associated with the power/energy flow. These include total cost and cost per kW or kWh for both power and energy, as defined below. The cost variables can be viewed as a cost flow parallel to the power/energy flow. While power and energy is gradually reduced due to losses when moving from an injection point to a load point, the cost is basically accumulated in each unit along the path of the cost flow, since the unit’s own cost is added in a step-wise manner. Therefore, the network path between injection and load represents a value added chain analogous to a manufacturing process/supply chain. Obviously, since energy/power gradually reduces while cost accumulates along this path, the cost per unit of energy/power will increase substantially in monetary units per kW or kW-hour.

Power distribution costs have been studied extensively [4, 1, 5, 2]. In this paper the model is developed while in a companion paper [3] numerical results and examples are presented by applying the model to a given (generic) rural distribution system in Iceland. The model development represents expansions from [2] such as a comprehensive relation for power an energy losses and their relationship for different classes of energy.

The model is presented in the next section, while the
final section contains a short numerical example and discussion.

2 Model development

First power/energy quantities are defined and developed and then followed by cost quantities. In all cases the input variables are shown in lower case letters while output variables are in capital letters.

2.1 Power and energy flow through individual units

The relevant power/energy quantities are defined here for a unit shown in figure 2. Power and energy enter the unit, represented by a box, from above and leave from below. A distinction is made by the unit’s own load, \( P_{a,i} \), and the power energy passed on to the next unit below in the hierarchy, \( P_{ut,i} \). Losses are defined in the figure as well. We, therefore, have 4 different types of variables/relationships for unit # \( i \): (1) input variables, (2) output variables, (3) the unit’s own variables and (4) how input and output is linked. For instance, we have \( p_i \) as the input peak power in MW for a designated period such as a year and \( w_i \) as the annual input energy, in MWh/year. Similarly, for the unit’s own load: \( P_{a,i} \) and \( W_{a,i} \). Other quantities in figure 2 are analogous.

Relating power between input and output, we get:

\[
P_i = q_{pi} P_i = P_{t,i} + P_i = P_{t,i} + P_{a,i} + P_{ut,i} \quad (1)
\]

where \( P_{t,i} \) represents power losses and \( P_i \) is the output power composed of load, \( P_{a,i} \) and power to the unit below, \( P_{ut,i} \). The ratio \( q_{pi} \) defines a so called \( q \)-power loss factor. Similarly for energy quantities, we get:

\[
w_i = q_{wi} W_i = W_{t,i} + W_i = W_{t,i} + W_{a,i} + W_{ut,i} \quad (2)
\]

where \( W_{t,i} \) represents energy losses and \( W_i \) is the output energy composed of energy load, \( W_{a,i} \) and energy to the unit below in the hierarchy, \( W_{ut,i} \). The ratio \( q_{wi} \) defines similarly a so called \( q \)-energy loss factor.

The power losses are:

\[
P_{l,i} = p_i \cdot (1 - \frac{1}{q_{pi}}) = r_{pi} \cdot p_i \quad (3)
\]

where \( r_{pi} \) is defined as an \( r \)-power loss factor. Similarly the energy losses are:

\[
W_{l,i} = w_i \cdot (1 - \frac{1}{q_{wi}}) = r_{wi} \cdot w_i \quad (4)
\]

where \( r_{wi} \) is defined as an \( r \)-energy loss factor. The different \( r \) and \( q \) loss factors are related by (3) and (4).

For instance if 5% of the energy input to the unit is lost in the unit, \( r_{wi} = 0.05 \) and \( q_{wi} = \frac{1}{0.95} \approx 1.053 \). We assume that the system is sufficiently disaggregated so that coincidence factors are not needed between the unit’s own \( P_{a,i} \) and \( P_{ut,i} \). Therefore \( P_{a,i} \) and \( W_{a,i} \) represent the system’s internal flow while \( P_{a,i} \) and \( W_{a,i} \) are quantities leaving the system at this unit.

2.2 Cost flow through individual units

Figure 3 represents the model cost quantities in a similar manner as the power/energy quantities in figure 2. We have unit or average costs (\( k \) or \( K \) with the appropriate subscript) or total costs (\( c \) or \( C \) with subscript) for the designated period such as a year. Starting at the unit’s input, the total annual cost is the sum of power and energy cost, or:

\[
c_i = k_{p,i} P_i + k_{w,i} W_i \quad (5)
\]

where \( k_{p,i} \) is the unit cost for power, for instance in $/kW and \( k_{w,i} \) is the unit cost for energy, for instance in $/kWh. We get the average cost at input:

\[
k_{m,i} = \frac{c_i}{w_i} = \frac{k_{p,i} P_i}{w_i} + k_{w,i} \quad (6)
\]

Similarly at the unit’s output the total annual cost is

\[
C_i = c_i + F_i \quad (7)
\]

\[
C_i = K_{p,i} P_i + K_{w,i} W_i \quad (8)
\]

where \( F_i \) is the annual investment and operations cost for all equipment belonging to unit \( i \). The average cost can similarly be calculated at the output of each unit for annual investment and operations cost for all equipment belonging to unit \( i \):

\[
K_{m,i} = \frac{C_i}{W_i} = \frac{K_{p,i} P_i}{W_i} + K_{w,i} \quad (9)
\]

Unit cost of power \( f_i \) can also be defined as:

\[
F_i = \frac{F_i}{P_i} = \frac{F_i}{P_{a,i} + P_{ut,i}} \quad (10)
\]
2.3 Linking of units in an expanding hierarchy

Unit interaction is now considered where one unit may branch to two or several units as shown in figure 4. The bold arrows in the figure show cost flow that continues through the system but, as previously mentioned, we have local outputs such as load or losses. In particular assume an expanding hierarchy and that we have 3 units as in fig 4 where the output of unit 1 branches to units 2 and 3.

We assume continuity in energy and cost flow meaning preservation of these quantities, so that the total output of unit 1 equals the input to unit 2 and 3. However flow of the peak power is diminished by a so called coincidence factor, $s_i$. In this case, therefore, we assume that the peaks at the inputs to units 2 and 3 do not coincide in time and the power peak leaving unit 1 will be less than the sum of the power peaks for units 2 and 3. We therefore have for energy $W_{ut,1} = w_2 + w_3$ while for power we have:

$$P_{ut,1} = s_1(p_2 + p_3) = s_1p_2 + s_1p_3$$  \hspace{1cm} (12)

where $s_1 \leq 1$.

The above preservation is valid for cost flow as well, i.e., $C_{ut,1} = c_2 + c_3$. Since $C_{ut,1} = K_{p,1}P_{ut,1} + K_{w,1}W_{ut,1}$ we have:

$$K_{p,1}P_{ut,1} + K_{w,1}W_{ut,1} = (k_{p,2}p_2 + k_{w,2}w_2) + (k_{p,3}p_3 + k_{w,3}w_3)$$ \hspace{1cm} (13)

Using (12) we get:

$$K_{p,1}(s_1p_2 + s_1p_3) + K_{w,1}(w_2 + w_3) = (k_{p,2}p_2 + k_{w,2}w_2) + (k_{p,3}p_3 + k_{w,3}w_3)$$ \hspace{1cm} (14)

This gives (15) which links the unit cost at the output of unit 1 to the unit cost at the input of units 2 and 3.

$$K_{w,2} = k_{w,2} = K_{w,1}$$ \hspace{1cm} (15)

By defining weights, $\mu_2$ and $\mu_3$ where

$$\mu_2 = \frac{w_2}{w_2 + w_3} = \frac{w_2}{W_{ut,1}}$$ \hspace{1cm} (16)

and

$$\mu_3 = \frac{w_3}{w_2 + w_3} = \frac{w_3}{W_{ut,1}}$$ \hspace{1cm} (17)

and $\mu_2 + \mu_3 = 1$ we get:

$$K_{m,1} = \mu_2k_{m,2} + \mu_3k_{m,3}$$ \hspace{1cm} (18)

where the average cost, $k_{m,1}$, is defined by (6) and where $K_{m,1} = \frac{P_{ut,1}}{W_{ut,1}} = \frac{K_{p,1}P_{ut,1} + K_{w,1}W_{ut,1}}{W_{ut,1}}$. If $k_{m,2}$ is defined as the cost at the input to unit 3, we of course get $K_{m,1} = k_{m,2} = k_{m,3}$.

Therefore (19) summarizes the above discussion:

$$W_{ut,1} = w_2 + w_3 \hspace{1cm} P_{ut,1} = s_1(p_2 + p_3) = s_1p_2 + s_1p_3$$ \hspace{1cm} (19)

Therefore (15), (18) and (19) along with figure 4 define how an expanding hierarchy treats power, energy and cost variables.

2.4 Linking of units in an contracting hierarchy

The interface between units is now considered where the outputs of two or several units are combined into a single unit as shown in figure 5. This is a contracting hierarchy. As before, we assume that energy and cost flow is preserved from one level to the next, while coincidence factors affect how the peak power outputs are summed up. There are no losses at the interface of units neither in figure 5 or anywhere; losses only occur within the units themselves. From the figure we see that the total output cost from units 1 and 2 equals the input cost to unit 3, or $c_3 = C_{ut,1} + C_{ut,2}$, where $C_{ut,1} = K_{p,1}P_{ut,1} + k_{w,1}W_{ut,1}$ and $C_{ut,2} = K_{p,2}P_{ut,2} + k_{w,2}W_{ut,2}$. The same goes for the energy, so we have $w_3 = W_{ut,1} + W_{ut,2}$. However, the coincidence factor, $s_3$ means that $p_3 = s_3(P_{ut,1} + P_{ut,2})$. From above we have

$$K_{p,1}P_{ut,1} + K_{w,1}W_{ut,1} + K_{p,2}P_{ut,2} + K_{w,2}W_{ut,2} = k_{p,3}p_3 + k_{w,3}w_3$$ \hspace{1cm} (20)

Distinguishing between power and energy quantities, and using (20), we get
Figure 5. An contracting hierarchy

\[ k_{p,3} = \frac{K_{p,1}P_{ut,1} + K_{p,2}P_{ut,2}}{w_3} = \frac{K_{p,1}P_{ut,1} + K_{p,2}P_{ut,2}}{W_{ut,1} + W_{ut,2}} \]

We can define the weights \( \phi_1 \) and \( \phi_2 \) where

\[ \phi_1 = \frac{W_{ut,1}}{W_{ut,1} + W_{ut,2}} = \frac{W_{ut,1}}{w_3} \]

and

\[ \phi_2 = \frac{W_{ut,2}}{W_{ut,1} + W_{ut,2}} = \frac{W_{ut,2}}{w_3} \]

and since \( \phi_1 + \phi_2 = 1 \) we get from (21):

\[ k_{w,3} = \phi_1 K_{w,1} + \phi_2 K_{w,2} \]

We also get from (20) for the average cost per kWh:

\[ k_{m,3} = \phi_1 K_{m,1} + \phi_2 K_{m,2} \]

Therefore (24) and (25) can be used to calculate the unit cost at the input to unit 3 in figure 5 from the weighted average of output costs from units above. Therefore it is possible to go through the hierarchy in an iterative manner and calculate new unit and average costs when these costs are known at an adjacent hierarchy level.

### 2.5 Disaggregation of average costs at the output of units

The total annual cost at the output of a unit is composed of the unit’s input cost and the unit’s own investment and operations cost (7). The cost of losses does not seem to enter as a cost factor. However the losses constitute a part of the cost per kWh since there are fewer kWh at output than at input due to losses. Therefore from (6), (7) and (9) we get \( K_{m,i}W_i = k_{m,i}w_i + F_i \) and furthermore \( K_{m,i}W_i = k_{m,i}W_{t,i} + F_i \). Dividing by \( W_i \) we get:

\[ K_{m,i} = k_{m,i} + \frac{k_{m,i}W_{t,i}}{W_i} + \frac{F_i}{W_i} \]

Therefore \( K_{m,i} \), the average cost at unit output, is composed of 3 factors:

- The average cost, \( k_{m,i} \), at input from units above,
- Average cost of losses, \( K_{loss,i} \)
- Operations and investment cost, \( K_{f+r,i} \)

The last cost factor, \( K_{f+r,i} \), can be further expanded by considering 3 factors, or \( K_{f+r,i} = K_{invest,i} + K_{oper,i} + K_{indirect,i} \), where \( K_{invest,i} \) is the average annual cost for investment, \( K_{oper,i} \) is the average annual operations cost and \( K_{indirect,i} \) is the annual indirect cost. Therefore we have:

\[ K_{m,i} = k_{m,i} + K_{loss,i} + K_{invest,i} + K_{oper,i} + K_{indirect,i} \]

Similarly we can decompose the total output cost from a unit to cost however decompose the system cost for all units into 3 categories: a) investment and operations b) Energy/power losses and c) Injection of generated power/energy.

We get from (2) and (7)

\[ C_i = c_i(W_{t,i} + F_i) \]

or

\[ C_i = c_{ad,i} + C_{t,i} + F_i \]

where \( c_{ad,i} \) is the adjusted cost at input. The results are similar to (26) where the cost is logically composed of 3 factors: (a) input of energy, (b) energy losses and (c) investments/operations/indirect cost. We can calculate the adjusted cost \( c_{ad,i} \) where we have accounted for the losses or:

\[ c_{ad,i} = c_i \cdot \frac{W_i}{w_i} = c_i \cdot (1 - r_{wi}) \]

\[ C_{t,i} = c_i \cdot \frac{W_{t,i}}{w_i} = c_i \cdot r_{wi} \]

which helps to go through the stages in the expanding or contracting hierarchy. Finally from (3) the cost of losses can be defined as:

\[ C_{t,i} = c_i \cdot \frac{W_{t,i}}{w_i} = k_{p,i}P_i + k_{w,i}W_{t,i} \]

or

\[ C_{t,i} = c_i \cdot \frac{W_{t,i}}{w_i} = k_{p,i} + k_{w,i} \]

From (3) and (4), the total cost of losses is defined by (32):

\[ C_{t,i} = c_i \cdot \frac{W_{t,i}}{w_i} = k_{p,i}P_{t,i} + k_{w,i}W_{t,i} \]

The factor \( g_i \) is defined below in (33).

### 2.6 A cost example for total cost evolving through a hierarchy

Expressions for decomposing cost of each kWh are defined by (26) (27) and total cost in (29). Disaggregation can be made from the load extraction at the lower level in the hierarchy up to the injection points at the upper levels. Figure
1 shows an example with 6 units, reflecting different contracting and expanding positions. The load extracted from unit 6, $W_{a,6}$ is examined and how it is disaggregated up through the hierarchy.

We assume that the average cost $K_{m,6}$ is given in the model at the output of unit #6 as well as the cost of losses $K_{f,r,6}$ and the unit’s own cost, $K_{f,r,6}$. Then we have $K_{m,6} = K_{m,6} + K_{f,r,6} + K_{f+r,6}$ by (26). Assume also that power and energy flows are given. Then we calculate $K_{m,6} = K_{m,5}$ since units #5 and #6 are vertically linked. We assume the same utilization time for the loads $W_{a,5}$ and $W_{ut,5}$. Therefore we again have $K_{m,5} = K_{m,2} + K_{f,r,2} + K_{f+r,5}$ which we can insert in the previous equation. The cost at the input to unit 5 is dependent on the output cost from the unit above or $K_{m,5} = K_{m,3}$ if we assume the same utilization time is into units 4 and 5 and depends on (18).

The input cost at unit 3 is $K_{m,3} = K_{m,3} + K_{f,r,3}$. Proceeding up the hierarchy and joining units 1 and 2 we get $K_{m,3} = \phi_1 K_{m,1} + \phi_2 K_{m,2}$ where $\phi_1$ and $\phi_2$ are given weighting factors. The discussion shows how it is possible to trace the cost variables from the bottom up through the hierarchy and further numerical examples are given in [3].

### 2.7 Relation between power and energy losses

Energy and power losses are related and can be calculated if the temporal distribution of load is known. If we define “power” as the annual peak power, it is related to energy at the same location through a load duration curve (LDC). Figures 6 show the general form of the LDC on the left, while the right part of the figure shows the case where the LDC is represented by a straight line (Linear LDC). We assume 2 classes of energy (See [3]) and first examine the class of primary energy and later will look at the secondary power. The location under examination can be anywhere in the system.

Here we derive how the loss factors defined earlier in (3) and (4), $r_{pi}$ and $r_{wi}$ are interrelated, based on the LDC. We write their ratio as:

$$g_i = \frac{r_{pi}}{r_{wi}}$$

(33)

Therefore we want to determine $g_i$ from the LDC at a certain location. It is assumed that power at the input of unit $i$ changes every hour of the year or is a function of the time variable, $x$ which measures the fraction of the year as by figure 6. The peak power at $x = 0$ is $p_i = p_i(0)$ and, in general, we have a decreasing function for the LDC, $p_i(x)$.

The annual energy, $w_i$ is:

$$w_i = \int_0^{h_m} p_i(x)dx$$

(34)

where $h_m = 8760$ is the number of hours/year. The utilization time is then defined:

$$h_i = \frac{1}{p_i} \int_0^{h_m} p_i(x)dx$$

(35)

Power losses, $P_{t,i}(x)$ can be treated in a similar way, where $P_{t,i} = P_{t,i}(0)$ are the peak losses which relate to peak power by (3).

The energy losses can be treated as in (34) or:

$$W_{t,i} = \int_0^{h_m} P_{t,i}(x)dx$$

(36)

It is well known from power systems analysis that power losses in a unit can be represented as a 2nd degree polynomial of the power flow or:

$$P_{t,i}(x) = a_i p_i^2(x) + b_i p_i(x) + c_i$$

(37)

where $a_i$, $b_i$ and $c_i$ are known constants. For the peak losses and peak load we have:

$$P_{t,i} = a_i p_i^2 + b_i p_i + c_i$$

(38)

Using (34), (36) and (37) we get

$$W_{t,i} = \int_0^{h_m} \left\{ a_i p_i^2(x) + b_i p_i(x) + c_i \right\} dx$$

$$= a_i \int_0^{h_m} p_i^2(x) dx + b_i \int_0^{h_m} p_i(x) dx + c_i \int_0^{h_m} dx$$

$$= a_i w_{2,i} + b_i w_{i} + c_i h_m$$

(39)

where we define $w_{2,i}$ by (40):

$$w_{2,i} = \int_0^{h_m} p_i^2(x) dx$$

(40)

Then we get for the loss factor, $r_{pi}$ from (3):

$$r_{pi} = \frac{P_{t,i}}{p_i} = a_i p_i + b_i + c_i$$

(41)

and similarly from (4) for the loss factor, $r_{wi}$

$$r_{wi} = \frac{W_{t,i}}{w_i} = a_i w_{2,i} + b_i + c_i h_m$$

(42)

From (41), (33) and (42) we get:

$$g_i = \frac{r_{pi}}{r_{wi}} = \frac{a_i p_i + b_i + c_i}{a_i w_{2,i} + b_i + c_i h_m}$$

(43)
To simplify (43) further, 2 assumptions are made, common in power systems modeling:

1. Power loss constants in (37) are assumed zero, or \( b_i = 0 \) and \( c_i = 0 \) and then the losses can be represented by \( P_i(x) = a_i p_i^2(x) \)

2. The LDC according to figure 6b can be represented by a straight line, or

\[
p_i(x) = p_i - \frac{p_i - p_{i, \text{min}}}{h_m} \cdot x
\]

From (43) the former assumption leads to:

\[
g_i = \frac{w_i p_i}{w_{2,i}}
\]

We have for the energy from fig 6b: \( w_i = \frac{p_i + p_{i, \text{min}}}{2} h_m \) and therefore \( p_i, \text{min} = \frac{2w_i}{h_m} - p_i \). Since utilization time is \( h_i = \frac{w_i}{p_i} \), we get from (44):

\[
p_i(x) = p_i (1 - A_i x)
\]

where the constant \( A_i \) is defined:

\[
A_i = 2 \cdot \frac{1 - h_i/h_m}{h_m}
\]

We can now calculate \( w_{2,i} \) using (40) and (46):

\[
w_{2,i} = p_i^2 \cdot \int_0^{h_m} (1 - 2A_i x + A_i^2 x^2) dx
\]

From (48) we therefore get:

\[
w_{2,i} = \frac{B_i p_i^2}{A_i}
\]

where the constant \( B_i \) is defined

\[
B_i = h_m - A_i h_m^2 + \frac{A_i^2 h_m^3}{3}
\]

Finally we can calculate the factor \( g_i \) by using (49) in (45).

\[
g_i = \frac{w_i p_i}{w_{2,i}} = \frac{w_i p_i}{B_i p_i^2} = \frac{w_i}{B_i} = \frac{h_i}{B_i}
\]

3 Numerical example, discussions and conclusions

Table 1 shows a numerical example with energy losses (33) and a linear LDC, see (51), (50) and (47). For instance if we assume that in a certain unit energy losses are 4 %, we see from the table that power losses with utilization time of 5000 h/year are 47.4 % higher or 5.89 %. Further numerical results are given in [3] where the model is applied to a generic rural distribution system resembling that of a part of the system in Iceland.

In this paper the basic model, APOWER, has been presented without assuming explicitly any class distinction, for instance between primary or secondary energy classes. The model can be used to meeting different needs and applications, such as tariff studies. Generally different classes would share both the cost and energy and power losses, according to some prescribed rules. In [3] we will examine a practical application of the model with numerical assumptions and numerical results to a system different energy classes with costs that are analogous to actual distribution system costs. In this context, marginal losses will be analyzed with respect to the role of each class in loss sharing and with respect to the actual applications and system needs in the planning and operations of Icelandic rural power distribution system.

Table 1. Energy to power loss ratios, \( g_i \), and the associated utilization time

<table>
<thead>
<tr>
<th>( h_i ) (hours)</th>
<th>( A_i )</th>
<th>( B_i )</th>
<th>( g_i )</th>
<th>( g_i(%) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>4000</td>
<td>0.124</td>
<td>2.689</td>
<td>1.488</td>
<td>48.8%</td>
</tr>
<tr>
<td>4500</td>
<td>0.111</td>
<td>3.002</td>
<td>1.499</td>
<td>49.9%</td>
</tr>
<tr>
<td>5000</td>
<td>0.098</td>
<td>3.392</td>
<td>1.474</td>
<td>47.4%</td>
</tr>
<tr>
<td>5500</td>
<td>0.085</td>
<td>3.858</td>
<td>1.426</td>
<td>42.6%</td>
</tr>
<tr>
<td>6000</td>
<td>0.072</td>
<td>4.399</td>
<td>1.364</td>
<td>36.4%</td>
</tr>
<tr>
<td>6500</td>
<td>0.059</td>
<td>5.017</td>
<td>1.295</td>
<td>29.5%</td>
</tr>
<tr>
<td>7000</td>
<td>0.046</td>
<td>5.711</td>
<td>1.226</td>
<td>22.6%</td>
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<tr>
<td>7500</td>
<td>0.033</td>
<td>6.482</td>
<td>1.157</td>
<td>15.7%</td>
</tr>
<tr>
<td>8000</td>
<td>0.020</td>
<td>7.328</td>
<td>1.092</td>
<td>9.2%</td>
</tr>
</tbody>
</table>

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