Optimal Design of Interdependent Hydroelectric Schemes Based on Optimal Timing

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Plan of the presentation

- **FRAMEWORK**: The problem framework is defined with projects, schemes and paths for sizing interdependent projects based on optimal timing.

- **MODEL**: An unconstrained minimization problem is developed for optimal timing and sizing in a Dynamic Hydroelectric system (DHS).

- **CASE STUDY**: A numerical case study is presented, illustrating a simple timing and sizing problem.

- **CONCLUSIONS**: Discussions and major conclusions

  (The paper has also a references section)
The problem setting: The expansion process for discreet projects

- A DHS with demand, schemes, expansion paths and projects
- A project corresponds to stages
- Interdependence or independence
- Mutually exclusive alternatives
- Project parameters such as dimensions and volumes
- Cost and benefit functions.
- Optimal timing
Project cost and benefit functions

- Each project has a cost function of parameters
  \[ C_i(u_i) = C_i(u_{i,1}, u_{i,2}, \ldots, u_{i,n_i}) \]

- Each project has a similar benefit or output function
  \[ x_i = g_i(u_i) = g_i(u_{i,1}, u_{i,2}, \ldots, u_{i,n_i}) \]

- Simple “box” constraints may be on the parameters
  \[ u_{i,\text{min}} \leq u_i \leq u_{i,\text{max}} \]

- Accumulated capacity is the sum of outputs of all preceding projects
  \[ \sum_{j=1}^{i-1} g_j(u_j) \]

- Time is capacity divided by demand rate of increase, \( q \). We get a discount factor by an exponential function of time multiplied by the discount rate
  \[ \exp \left( -\frac{\alpha}{q} \sum_{j=1}^{i-1} g_j(u_j) \right) \]
The objective function is the total discounted cost of the sequence

\[ P_1(u_1, u_2, \ldots, u_N) = \sum_{i=1}^{N} C_i(u_i) \exp \left( -\alpha \frac{i-1}{q} \sum_{j=1}^{i} g_j(u_j) \right) \]

\[ + P_{N+1} \exp \left( -\alpha q \left\{ \sum_{j=1}^{N} g_j(u_j) \right\} \right) \]

and the minimizing objective as:

\[ P_1^* = \min_{u_1, u_2, \ldots, u_N} \{ P_1(u_1, u_2, \ldots, u_N) \} \]
The recursion and optimality conditions for in-/interdependent projects

- The recursive equations at each stage in the expansion sequence

\[ P_i(u_i, u_{i+1}, ..., u_N) = C_i(u_i) \]
\[ + P_{i+1}(u_{i+1}, u_{i+2}, ..., u_N) \exp(-\alpha g_i(u_i)/q) \]

- Applying an optimality conditions operator which is as follows

\[ \frac{\partial}{\partial u_i} = \begin{bmatrix} \frac{\partial}{\partial u_{i,1}} & \frac{\partial}{\partial u_{i,2}} & \cdots & \frac{\partial}{\partial u_{i,n_i}} \end{bmatrix}^T = 0 \]

- The result are these optimality conditions at each stage. The output may depend on the next project

\[ \frac{\partial(\alpha C_i)/\partial u_i}{\partial g_i/\partial u_i} = k_i^m = k_{i+1}^r \exp(-\alpha g_i(u_i)/q) \]

\[ \frac{\partial(\alpha C_i)/\partial u_i}{\partial g_i/\partial u_i} = k_i^m = k_{i+1}^r \exp(-\alpha g_i(C_{i+1}(u_{i+1}), u_i)/q) \]
The recursion and optimality conditions for in-/interdependent projects

- Next we assume that capacity depends in next project's cost

\[ x_i(u_i, u_{i+1}) = g_i(C_{i+1}(u_{i+1}), u_i) \]

- We get the following condition by introducing a MAF. Note the change in the **same** project

\[
\frac{\partial (\alpha C_{i+1})/\partial u_{i+1}}{\partial g_{i+1}/\partial u_{i+1}} \beta_{i+1} = k_{i+1}^m = k_{i+2}^r e^{-\frac{\alpha g_{i+1}(u_{i+1})}{q}}
\]

- The **MAF (Marginal Adjustment Factor)** is defined as follows and it may change the optimum solution for the whole sequence

\[
\beta_{i+1} = 1 - \frac{k_{i+1}^r}{\alpha} \cdot \frac{\partial g_i}{\partial C_{i+1}}
\]

Note that if capacity \( C_{i+1} \), then \( \beta_{i+1} = 1 \).
Example of project configurations for options against Kárahnjúkar project in eastern Iceland
Assume we have a simple case of $N = 2$ projects, each with one parameter, that is $\mathbf{u}_1 = [u]$ and $\mathbf{u}_2 = [v]$. Furthermore, the convex cost functions are $C_1 = u^2$ and $C_2 = v^3$ and the concave output functions are $x_1 = g_1(u) = 1 - (1 - u)^2$ and $x_2 = g_2(v) = 1 - (1 - v)^3$ with box constraints $0.5 \leq u \leq 1$ and $0.5 \leq v \leq 1$. These functions are shown in Figure 2. All quantities are measured in some arbitrary "units" and we assume a linear growth rate $q = 0.4$ and an interest rate of 3% per unit of time, i.e. $\alpha = 0.0296$. Finally, assume a LRAC of the back-up supply to be $k_3^r = 3$ units.
The 2nd project. Figure 3 illustrates the FUC capacity of the first project and how these will change in the 2nd case, when the back-up supply to be per unit of time, i.e. assume a linear growth rate. All quantities are measured in some arbitrary "units" and we use in the numerical case study. The unshaded (white) area represents the feasible region.

Note the box contraints, illustrated by the hatched areas.
Figure 2. Examples of convex cost functions used in the numerical case study. The unshaded (white) area represents the

\[ x_1 = g_1 (C_2(u_2), u_1) = 1 - (1 - u)^2 + \frac{(C_2 - C_{2,0})}{C_{2,0}} \cdot k_v \]

\[ C_2 = v^3 \]

\[ x_1 = g_1 = 1 - (1 - u)^2 + \frac{(v^3 - v_0^3)}{v_0^3} \cdot k_v \]

\[ \beta_2 = 1 - \frac{k_r^r}{\alpha} \cdot \frac{\partial g_1}{\partial C_2} = 1 - \frac{k_r^r}{\alpha} \cdot \frac{k_v}{C_{2,0}} = 1 - \frac{2.7877}{0.0296} \cdot \frac{0.02}{0.6141} \]

\[ C_2^+ = 0.7811 \quad x_2^+ = 0.9995 \]

- AUC and FUC costs are plotted in the next slide
Actual Unit Cost (AUC) and Full Utilization Cost (FUC) functions for the numerical case study,

\[
\text{FUC} = \alpha C_i / x_i \\
\text{AUC} = \alpha^2 C_i / (q(1 - \exp(-\alpha x_i / q)))
\]
Cost and capacity interdependence between adjacent projects

- Next we assume that capacity of the 1st project depends in next project’s cost (Last term)

\[ x_1 = g_1 (C_2 (u_2), u_1) = 1 - (1 - u)^2 + \frac{(C_2 - C_{2,0})}{C_{2,0}} \cdot k_v \]

- This leads to the following condition:

\[ x_1 = g_1 = 1 - (1 - u)^2 + \frac{(v^3 - v_0^3)}{v_0^3} \cdot k_v \]

- This leads to the **(limited) change** in the optimal capacity of the 2nd project and a corresponding change for the 1st project both in terms of the parameters, size and costs. This may be attributed to combined expected sharp concavity of the output function near \( x = 1 \) \((u = 1, v = 1)\)

\[ x_2^* = 0.9992 \text{ to } x_2^+ = 0.9995 \text{ units}, \]
Figure 4. Plots of the TDC functions $P_i$ from the numerical case study, where $i = 2$ at left and $i = 1$ at right. The unshaded (white) area represents the feasible region.
Conclusions (1)

• First, a single project has been analyzed with linking to a recursive model of an expansion sequence.

• Secondly, the design parameters have been shown to lead to similar results for a sequence of projects.

• This resulted in conditions for optimal size

• Third, dependent parameters between adjacent projects lead to similar results as independent parameters in terms of optimality conditions linking adjacent projects where dependency is explicitly accounted for by an MAF
Conclusions (2)

- **BASIC RESULTS**: The *marginal cost* of each kilowatt-hour in any project *should be equal* with respect to the marginal cost of all parameters.

- *(Otherwise it would be economical to substitute these incremental capacities with each other).*

- **Interdependence of projects does not basically change this conclusion but with an MAF**.

- **EXPANSION**: The model should be *tested* on *realistic project sequences*, since it has here been applied only to a simple test case.

- This paper has analyzed the important *project interaction* with marginal price signals, dictating a balance in terms of marginal cost.
THANK YOU!!!