A Golden Rule of Depreciation

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ABSTRACT

We derive a Golden Rule for the obsolescence of physical capital. Optimal durability is shown to vary inversely with population growth as well as technological progress. Increased population growth and technological progress accelerate depreciation because providing a rapidly growing and increasingly productive population with high-quality capital is costly in terms of consumption forgone. In the long run, the adverse effect of population growth on the level of output per head is reinforced.

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1. Introduction

This paper makes a simple point. Around the world, differences in the quality of housing, capital and infrastructure are at least as evident as are differences in the quantity of such capital. Comparing the cities of the United States and Mexico, West and East Germany before unification, Austria and Poland, Argentina and Paraguay, Thailand and Laos, and so on, we see vast differences in the quality of housing and other infrastructure. Whether of their own deserts or not, some nations are clearly endowed with physical capital of higher quality than others even if their national income accounts often do not reflect these important differences. How can differences in the quality of capital across countries be explained?

Quality, as we define it, will turn out to be closely related to depreciation – due to economic obsolescence or physical wear and tear. Physical depreciation is a technological phenomenon whereas by economic depreciation we mean obsolescence (see Scott, 1989).[1]

Producers of capital equipment have considerable leeway in deciding the durability of their equipment. Decision-making on the level of planned obsolescence is taught in business schools.

[1] We are not the first to propose an economic theory of depreciation. Galbraith (1958) discussed the wastefulness of changing automobile models. Baumol (1971) analyzed depreciation policy in terms of optimal intertemporal resource allocation. Barro (1972) proposed a theory of depreciation based on an assumption of consumers and producers having different discount rates. Bulow (1986) showed how monopolists may desire uneconomically high rates of depreciation for their produce in order to be able to maintain a high price in future periods; for example, textbook authors revise their books aiming to make earlier editions obsolete. Even so, depreciation has not played a big role in the growth literature. For instance, Aghion and Howitt (1998, p. 111) postulate that increased depreciation will have an ambiguous effect on growth because in the short run it reduces the real rate of interest – which tends to increase the incentive to undertake research – while, on the other hand, it directly reduces the rate of change of the per capita capital stock. Thus, capital accumulation slows down while lower interest rates drive innovators to new highs. Another book by Barro and Sala-i-Martin (1995) does not list depreciation in its index.

We will look at this optimization problem from a macroeconomic standpoint by deriving the optimal level of durability; that is, the one that maximizes consumption in steady state.

2. Durability and Depreciation

When investing, firms decide on the level of costs needed to plan for, organize and ensure the durability of the new capital equipment. By incurring greater costs at the time of investment, they can make the capital last longer. Thus, there arises a trade-off between the costs of investment and future replacement costs.

We measure durability by an index *d* between zero and one and proceed to derive the optimal level of durability using the Solow model. The production function in intensive form is

$$y = f(k) = k^{1-\alpha} \tag{1}$$

were output and capital are normalized by the number of efficiency units of labor: $y = Y/AL$ and $k = K/AL$, with Y denoting output, L labor, K capital, A is the level of Harrod-neutral technology, and $1 - \alpha$ is the elasticity of output with respect to capital.

Physical depreciation δ is a decreasing function of the durability d of the capital stock:

$$\delta = (1-d)^{\beta} \tag{2}$$

where $\beta > 1$ ensures diminishing returns to durability. When d rises from zero to one, δ falls from one to zero. However, durability comes at a cost. A fraction d of total investment expenditures is used to ensure the durability of the installed capital equipment; the rest (i.e., the fraction $1 - d$ of the total) is available for the accumulation of fresh capital.

In the closed economy, saving (sY) is equal to gross investment (I_g) and proportional to output. The dynamics of capital accumulation per capita can now be described as follows:

$$\dot{k} = i_g(1-d) - [\delta(d) + n + \lambda]k \tag{3}$$

Here i_g denotes gross investment per augmented labor unit, n is population growth, and λ is the rate of labor-augmenting technological progress. We use di_g to denote investment necessary to attain a durability level d for new capital $(1-d)i_g$.

In steady state where $\dot{k} = 0$ we have

$$[f(k) - c](1 - d) = [\delta(d) + n + \lambda]k \tag{4}$$

Notice that $c = y - i_g$ is consumption per efficiency unit of labor, or

$$c = f(k) - \left[\frac{\delta(d) + n + \lambda}{1-d}\right]k \tag{5}$$

To find the optimal quantity of capital and its optimal durability we now maximize consumption per unit of augmented labor with respect to k and d. The optimal capital stock k^* is the solution to[2]

$$f_k(k) = \frac{\delta(d) + n + \lambda}{1 - d} \tag{6}$$

The left-hand side of the equation shows the marginal benefit of having one more unit of capital

[2] It can be shown that $\beta \geq 2$ is sufficient but not necessary for the second-order condition for a maximum to be satisfied.

(i.e., the marginal product of capital), while the right-hand side shows the marginal cost of maintaining this extra unit in the face of depreciation, population growth and technological progress. Equation (6) can also be used to derive the optimal saving rate as follows:[3]

$$\frac{f_k k}{y} = \frac{(\delta+n+\lambda)}{1-d}\frac{k}{y} = \frac{i_g}{y} = s \tag{7}$$

or, given equation (1),

$$s = \left(\frac{k}{y}\right)\left[\delta^{1-\frac{1}{\beta}} + (n+\lambda)\delta^{-\frac{1}{\beta}}\right] \tag{7'}$$

In the long run, the optimal saving rate is simply $1 - \alpha$, the standard result. Hence, equation (7') tells us that (i) an increase in n or λ must reduce the capital/output ratio in the long run and (ii) an increase in the depreciation rate δ will similarly reduce the capital/output ratio in the long run as long as $\beta > 1 + (n + \lambda)/\delta$ (more on this condition below). This inverse relationship between the optimal capital/output ratio and depreciation follows from our assumption of diminishing returns to durability.[4]

The golden-rule level of capital k^* depends on both the productivity and durability of capital. The higher is durability, d, the more expensive, in terms of consumption forgone, is the maintenance of the capital stock for a given rate of depreciation. In other words, the more durability, the greater the sacrifice needed to maintain it for given depreciation. This effect appears in the denominator of the right-hand-side term of equation (6) – the higher d, the larger is the ratio and the lower is the optimal capital stock. However, durability also reduces the depreciation rate and hence also the numerator on the right-hand side of the equation. The net effect of durability on the golden-rule capital stock k^* is, therefore, ambiguous.

More precisely, we can show by taking the total differential of equation (6) that increased durability will raise the optimal capital stock if the following condition holds:

$$\beta > 1 + \frac{n+\lambda}{\delta} \tag{8}$$

A high value of β means that with a more durable capital stock there is less need for replacement investment, making it less costly to maintain a given stock of capital. This increases the optimal stock of capital. However, increased durability comes at a cost. First, it costs more to replace the units of capital that do depreciate in spite of greater durability and the first term on the right-hand side of inequality (8) captures this. So, in the absence of population growth and technological progress we would need $\beta > 1$ for more durability to increase the optimal capital stock. With population growth and technological progress, however, increased durability makes it more costly to produce capital equipment to satisfy a growing and increasingly productive population, and this is captured by the second term on the right-hand side.

From equation (5) we can similarly derive the first-order condition for optimal durability d^* as:

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[3] This can be rewritten as

$$\frac{f_k k}{y} = \frac{profits}{y} = s$$

which gives the Golden Rule of saving as stated by Phelps: "Save profits and consume wages."

[4] More precisely, our assumption of diminishing returns to durability is a necessary but not sufficient condition for a negative long-run equilibrium relationship between the depreciation rate and the optimal capital-output ratio as shown in equation (7').

$$\frac{(1-d)^{\beta}+n+\lambda}{(1-d)^2}k \;=\; \frac{\beta(1-d)^{\beta-1}}{1-d}k \tag{9}$$

The left-hand side shows the marginal cost of increasing durability *d*. This is the increase in the cost of replacement investment – units of output used up in building up durability – that is needed every year. The right-hand side represents the marginal benefit from less depreciation in long-run equilibrium. So, with a more durable capital stock, there are fewer units of capital that need to be replaced, but replacing each unit is more costly in terms of consumption forgone.

The marginal benefit in equation (9) depends on the parameter β that shows the effect of durability on the depreciation rate; see equation (2). The greater the effect of investing in durability on depreciation, the higher is the optimal level of such investment. Notice also that the capital stock appears on both sides of equation (9). Therefore, the optimal level of durability does not depend on the capital stock, and is given by

$$d = 1 - \left[\frac{n+\lambda}{\beta-1}\right]^{\frac{1}{\beta}} \tag{10}$$

As long as $\beta > 1$, the optimal level of durability varies inversely with population growth and technological progress. Hence, as $n + \lambda$ rises, the optimal rate of depreciation also rises:

$$\delta = \frac{n+\lambda}{\beta-1} \tag{11}$$

Given our assumption that $\beta > 1$, we have here a positive relationship between optimal depreciation and long-run economic growth. When the rate of population growth is high or the rate of technological progress is high, it is costly to maintain a high-quality capital stock as each unit of capital costs more to install. This is also the reason why both rapid population growth and rapid technological progress cause the optimal stock of capital (per unit of augmented labor) to be low. It follows that increased population growth or technological progress causes both the quantity and quality of capital to drop in the long run.[5]

The total effect of a change in population growth or technological progress on the optimal stock of capital now consists of both the direct effect on the quantity of capital *k* and the indirect effect through durability. By taking the total differential of equation (6) we see that the indirect effect vanishes when $\beta = 1 + (n + \lambda)/\delta$ (optimal depreciation), reinforces the direct effect when $\beta > 1 + (n + \lambda)/\delta$ (too much depreciation) and offsets the direct effect when $\beta < 1 + (n + \lambda)/\delta$ (too little depreciation). Under certain conditions – namely, a high value of β – the total effect of a rise in population growth or technological progress on the level of steady-state capital per person

[5] We could also let technological progress influence depreciation – or obsolescence – directly by replacing equation (2) by

$$\delta = (1+\lambda-d)^{\beta}.$$

Then equation (9) becomes

$$\frac{(1+\lambda-d)^{\beta}+n+\lambda}{(1-d)^2}k \;=\; \frac{\beta(1+\lambda-d)^{\beta-1}}{1-d}k\;.$$

Now a change in λ increases not only the marginal cost of durability on the left-hand side of the new version of equation (9) like before but also the marginal benefit of durability on the right-hand side of the equation. Therefore, the net effect of technological progress on durability and hence also on depreciation is ambiguous in this case.

is larger than the direct effect because the indirect effect operating through the depreciation rate reinforces the direct effect.

3. Concluding Remarks

Increased population growth accelerates depreciation given our assumption of diminishing returns to durability because providing a rapidly growing population with high-quality capital is costly in terms of consumption forgone. As a result, economic growth slows down in the medium term more than it would if depreciation were exogenous. In the long run, the adverse effect of population growth on the level of output per head is reinforced.

More rapid technological progress also accelerates depreciation for an analogous reason and thereby stimulates medium-term growth less than it would if depreciation were exogenous. This means that more rapid technological advance increases the level of output per capita less than it would if depreciation were exogenous, even if long-run per capita growth remains unchanged and equal to the rate of technological progress.

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