08.73.11 Algorithms, Logic and Complexity

Final exam **December 12th, 2006**

Teacher: Hjálmtýr Hafsteinsson **Time: $9^{00} - 12^{00}$**

All problems have the same value. ***You only have to solve 5 problems out of 6. The five best problems count.*** *All written material and a calculator are allowed.*

- Note that an answer without justification is worthless. Therefore justify all answers and remember that it is not necessary to write up definitions that are in the book.
- You can answer in either English or Icelandic.

1. Show that if you only allow one accept state in deterministic finite automata (DFA) then that model can not accept all regular languages. To solve this problem, show a regular language that can not be accepted with a DFA that only has one accept state.

2. Use the pumping lemma for context-free languages to show that the following language is not context-free

$$A = \{ a^i b^j c^k \mid i, j, k \geq 0 \text{ and } k = \min(i, j) \}.$$

3. For each of the following statements say whether it is true or false. Give a short proof (or a counterexample) in each case.

a) If neither A nor B are regular languages, then $A \cap B$ is not a regular language.

b) Let A be a decidable language and B be a Turing-recognizable language. Then $A - B$ is Turing-recognizable.

c) If $A \leq_m B$ then $\overline{A} \leq_m \overline{B}$.

d) If the language B is NP-complete and $A \leq_p B$, for A in P, then P = NP.

4. Prove that the language $ONE_{DFA} = \{ \langle D \rangle \mid D \text{ is a DFA and } |L(D)| = 1 \}$ is decidable.

5. Define $COMP_c = \{ x \mid$ the string x is *c-compressible*, i.e. $K(x) < |x| - c \}$. Show that $COMP_c$ is Turing-recognizable.

6. The language *CLIQUE* is defined as $\{ \langle G, k \rangle \mid G$ is an undirected graph with a k-clique $\}$. Let us now define *MAXCLIQUE* = $\{ \langle G, k \rangle \mid G$ is an undirected graph whose largest clique is a k-clique$\}$.

a) Show that *MAXCLIQUE* is NP-hard.

b) Is *MAXCLIQUE* in *NP*? Give a convincing argument for your answer.