08.73.11 Algorithms, Logic and Complexity

Final exam **December 11th, 2007**

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All problems have the same value. ***You only have to solve 5 problems out of 6. The five best problems count.*** *All written material and a calculator are allowed.*

- Note that an answer without justification is worthless. Therefore justify all answers and remember that it is not necessary to write up definitions that are in the book.
- You can answer in either English or Icelandic.

1. Let $\{ L_1, L_2, L_3, ... \}$ be an infinite set of regular languages (i.e. each L_i is a regular language), where L_i is not a subset of L_j for $i \neq j$. Consider the language L formed with the infinite union

$$L = (L_1 \cup L_2 \cup L_3 \cup ...).$$

a) Give an example of such a set of languages for which L is regular.

b) Give an example of such a set of languages for where L is **not** regular.

2. Use the pumping lemma for context-free languages to show that the following language is not context-free

$$A = \{ \mathtt{a}^n \mathtt{b}^m \mathtt{c}^{nm} \mid n, m \geq 0 \}.$$

3. For each of the following statements say whether it is true or false. Give a short proof (or a counterexample) in each case.

a) Every subset of a regular language is regular.

b) If $A \subseteq B$ then $A \leq_m B$.

c) If $A \leq_p B$ and $B \leq_p A$ then both A and B are NP-complete.

d) If every NP-hard language is also PSPACE-hard then NP = PSPACE.

4. A Turing machine is said to be *reversal-bounded* on input w if its head changes direction only a finite number of times during the computation on w. Consider the language

$$RB_{\mathrm{TM}} = \{ \langle M, w \rangle \mid \text{Turing machine } M \text{ is reversal-bounded on } w \}$$

a) Show that the language RB_{TM} is Turing-recognizable.

b) Prove that RB_{TM} is undecidable (use a reduction from A_{TM}).

5. We call a Turing machine M *egotistic* if the only string that it accepts is its own description, i.e. $L(M) = \{\langle M \rangle\}$. Prove that there are infinitely many egotistic Turing machines.

6. Using the distributive law, a 3CNF Boolean formula can be transformed to an equivalent Boolean formula in 2DNF (Disjunctive Normal Form). Since the Satisfiability problem for 2DNF formulas is in P, it looks like we can design a polynomial time algorithm to solve a 3SAT instance. For example, the 3SAT instance of

$$(x_1 \vee x_2 \vee x_3) \wedge (x_4 \vee x_5 \vee x_6)$$

is equivalent to

$$(x_1 \wedge x_4) \vee (x_1 \wedge x_5) \vee (x_1 \wedge x_6) \vee (x_2 \wedge x_4) \vee (x_2 \wedge x_5) \vee (x_2 \wedge x_6) \vee (x_3 \wedge x_4) \vee (x_3 \wedge x_5) \vee (x_3 \wedge x_6).$$

Thus, the 3SAT instance can be solved in polynomial time and 3SAT is in P. Is that correct? Justify your answer.