

08.71.14 Stærðfræðimynstur í tölvunarfræði (English exam)

Makeup exam

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Time: 13³⁰ – 16³⁰

All problems have the same value. You only have to solve 5 problems out of 6. The five best solutions count. All written material and a calculator allowed.

- Please note that an answer without justification is worth nothing. Justify therefore all answers and remember that it is not necessary to write up definitions that are in the textbook.

1. Translate the following sentences to quantified predicates.

- a) There is exactly one student that lives on Laugavegur.
- b) No student, except Jón, is from Vestfirðir.
- c) At least two students are from Hafnarfjörður.
- d) No two students live in the same street.

2. Proof the following formula by induction:

$$(1 - 1/2) * (1 - 1/3) * \dots * (1 - 1/n) = 1/n$$

3. A 7 person group is to be formed from a larger group consisting of 8 men and 6 women.

- a) How many 7 person groups can be formed from these 14 individuals, where there are 4 women and 3 men?
- b) How many 7 person groups can be formed with at least one man?
- c) How many 7 person groups have at most 3 women?

4. You are given the relation $\{(2,3), (1,1), (2,1), (3,2), (3,3), (2,2)\}$ on the set $\{1, 2, 3, 4\}$.

- a) Determine whether the relation is *i*) reflexive, *ii*) irreflexive, *iii*) symmetric, *iv*) antisymmetric, *v*) asymmetric, or *vi*) transitive. Give a justification for each instance.
- b) How does the answer to part a) change if the relation is defined on the set $\{1, 2, 3\}$?
- c) How does the answer to part a) change if the item $(2,3)$ is not in the relation?

5. An edge in a graph is called a *cut edge* if the graph breaks up into two or more pieces when it is removed. Proof that an edge e is a cut edge if and only if it is not on any simple cycle in the graph.

6. Show finite-state automata that accept the following languages:

a) All bitstrings, where the third bit is 0, unless the fourth bit is 0.

b) The language described by the regular expression $01^*(1 \cup 00)^* \cup (01)^*$.