

08.71.14 Stærðfræðimynstur í tölvunarfræði (English exam)

Makeup exam

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Time: 13³⁰ – 16³⁰

All problems have the same value. You only have to solve 5 problems out of 6. The five best solutions count. All written material and a calculator allowed.

- Please note that an answer without justification is worth nothing. Justify therefore all answers and remember that it is not necessary to write up definitions that are in the textbook.

1. Show quantified predicates for the following sentences. Use only the predicates $F(x, y)$: "x and y are relatives", $Y(x, y)$: "x is younger than y", and $N(x)$: "x is a student".

- a) All students have some relative who is also a student.
- b) There is a student who is younger than all of his relatives.
- c) Jón has no relatives that are older than him.

2. Prove the following equation by induction:

$$\sum_{i=n}^{2n} \frac{i}{3} = \frac{n^2 + n}{2}$$

- 3.**
- a) How many different directed graphs can be made with the nodes v_1, \dots, v_n ? Note that edges can not be duplicated and the graph does not have to be connected.
 - b) At a pizza place you can get 7 different toppings. How many different two-pizza orders can you make, if any subset of the toppings can be put on each pizza?
 - c) The textbook states that the number of different relations on an n -item set is 2^{n^2} . How many different *symmetric* relations are there on an n -item set?

- 4.**
- a) Can relations be **both** symmetric and asymmetric? How about **neither** symmetric nor asymmetric? Justify your answers.
 - b) What kind of relations are both asymmetric and antisymmetric? Describe their properties in general terms.

5. Assume that there are 8 teams in the North group and 7 teams in the South group. Is it possible to set up a schedule of games such that each team plays exactly 5 other teams in their own group and 2 teams in the other group? Present the problem as a graph theoretic problem and explain your answer in terms of graph properties.

- 6.** You are given the language defined by the regular expression $(0^*10^*10^*)^*$.
- a)* Show a grammar for the language.
 - b)* Show a finite automata for the language.