# 08.71.14 Stærðfræðimynstur í tölvunarfræði

**Makeup exam** **August 18th, 2004**
**Teacher: Hjálmtýr Hafsteinsson** **Time: $9^{00} – 12^{00}$**

*All problems have the same value.* ***You only have to solve 5 problems out of 6. The five best solutions count****. All written material and a calculator allowed.*

- Please note that an answer without justification is worth nothing. Justify therefore all answers and remember that it is not necessary to write up definitions that are in the textbook.

**1.** Show the following sentences as quantified predicates. Also show their negation in as simple form as possible (i.e. you should "move the negation into the expression"). Use the predicates that fit best in each instance and describe them.

*a)* "No student is not good in logic."

*b)* "There is a prime number between each integer and twice that integer."

*c)* "There is an integer $n$, such that $n > 0$ and for all integers $m > n$, each polynomial equation $p(x) = 0$ of degree $m$ has no real number solutions."

**2.** Prove that for each integer $a > 7$ it holds that $a = 3i + 5j$, where $i$ and $j$ are integers that are $\geq 0$.

**3.** In the following questions show how you arrive at the answer. You do not have to complete all calculations, as long as the answer is clear.

*a)* What is the number of different 9 digit integers?

*b)* What is the number of 9 digit integers, where no two adjacent digits are identical?

*c)* What is the number of 9 digit integers that have at least one pair of adjacent digits that are identical?

*d)* How many 9 digit integers are palindromes, i.e. are the same read backwards and forwards?

**4.** Indicate whether the following relations are *i)* reflexive, *ii)* irreflexive, *iii)* symmetric, *iv)* antisymmetric, *v)* asymmetric), or *vi)* transitive. Justify each answer.

*a)* The relation $R_1$ on integers and $(x, y) \in R_1$ iff $x - y$ is an odd number.

*b)* The relation $R_2$ on the set of logic expressions and $(x, y) \in R_2$ iff the expression $x \rightarrow y$ is true.

*c)* The relation $R_3$ on the set $\{0, 1, 2\}$ and $R_3 = \{ (0, 1), (1, 1), (0, 2) \}$

**5.** Assume two *n*-node graphs *G* and *H*. *The degree sequence* of a graph is a listing of the degrees of the nodes in decending order, for example 3, 3, 3, 2, 1. If the graphs *G* and *H* have the same degree sequence are they then isomorphic? Show a counterexample, or prove that they must then be isomorphic.

**6.** Given the language *L*, that includes bit strings, where the number of 0-bits is not divisible by 3.

*a)* Show a finite automata that accepts *L*.

*b)* Show a regular grammar that generates *L*.