

08.71.14 Stærðfræðimynstur í tölvunarfræði (English exam)

Final exam

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Time: 13³⁰ – 16³⁰

All problems have the same value. You only have to solve 5 problems out of 6. The five best solutions count. All written material and a calculator allowed.

- Please note that an answer without justification is worth nothing. Justify therefore all answers and remember that it is not necessary to write up definitions that are in the textbook.

1. Write the following sentences as quantified predicates **and** show their negation as a predicate and in words.

- a) For all integers n , if n^2 is even, then n is even.
- b) There is a man who answers all the questions correctly.
- c) Some students do not speak english.
- d) All of the teachers in the school teach exactly two classes.

2. Prove by induction the $2 \mid (n^2 + n)$ (i.e. 2 divides $(n^2 + n)$).

3. How many onto functions are there from a set with 7 elements to one with 3 elements? How about one-to-one functions? Give justification for each answer.

4. Determine whether the following relations are *i*) reflexive, *ii*) irreflexive, *iii*) symmetric, *iv*) antisymmetric, *v*) asymmetric, or *vi*) transitive. Give a justification for each instance.

- a) The relation R is over the real numbers and $(x, y) \in R$ if and only if $(x + y) \geq 0$.
- b) The relation R is over the natural numbers and $(x, y) \in R$ if and only if x and y have the same number of digits.
- c) The relation R is over $\{a, b, c\}$ and $R = \{ (a, b), (b, c), (c, a) \}$.

5. *a)* Construct a binary search tree from the words of the sentence "Þetta er léttasta prófið hjá mér í vetur". Enter the words into the tree in the order in which they appear in the sentence. Draw the tree after each insertion.

b) Show *i)* an inorder and *ii)* a preorder for the words in the tree from part *a)*.

6. *a)* Describe in words the strings in the regular set $(0 \cup 1)^* 00 1^*$.

b) Show a nondeterministic finite-state automaton that recognizes the set from part *a)*.

c) Show a grammar for the set from part *a)*.