08.71.14 Stærðfræðimynstur í tölvunarfræði
(English exam)

Final exam **December 20th, 2002**

Professor: Hjálmtýr Hafsteinsson **Time: $13^{30} - 16^{30}$**

All problems have the same value. ***You only have to solve 5 problems out of 6. The five best solutions count.*** *All written material and a calculator allowed.*

- Please note that an answer without justification is worth nothing. Justify therefore all answers and remember that it is not necessary to write up definitions that are in the textbook.

1. The proposition $\neg(\neg(p \to \neg q) \to (r \to (\neg s \to t))) \to u$ is only false for one particular assignment of truth values to the variables p, q, r, s, t, and u. Find this assigment without using a truth table (the truth table has 64 rows!). Instead convert the proposition to a form that makes it obvious which truth values of the variables make the proposition false.

2. The following predicates are given: $S(x, y)$: "Team x beat team y" and $P(x, y)$: "Team x has played team y". Use these predicates to build quantified logical expression for the following statements and their negation:

a) "All the teams have lost at least one game"

b) "Liverpool has wone exactly one game"

c) "There is a team that has beat all the other teams"

3. Remember (from chapter 4.4) that the probability of an event is the number of outcomes that satisfy the conditions / *(divided by)* the number of possible outcomes. You are supposed to draw four cards from a deck of 52 cards.

a) What is the probability that all four cards will be clubs?

b) What is the probability that none of the four cards will be a club?

c) What is the probability that all four cards will different suits (i.e. heart, spade, diamond, and club)?

4. *a)* If it is possible show a single relation that is neither reflexive, irreflexive, symmetric, nor antisymmetric. Justify that the relation you show has none of the above properties, **or** argue that it is impossible to make such a relation.

b) A function can be viewed as a special kind of relation. Can such "function"-relations also be *i)* reflexive, *ii)* symmetric or *iii)* transitive without violating the function property? Justify each part seperately.

5. In this problem we are working with general trees, not necessarily with a designated root.

a) If the number of nodes in the tree is 6 can the number sequence 1, 3, 3, 1, 1, 1 be a list of degrees of the six nodes? If that is the case show the tree, otherwise explain why this is not possible.

b) What is the smallest **and** largest number of nodes with the degree 1 in a *n*-node general tree? Draw those trees and justify you answer.

c) What do the trees look like that have a Hamilton **path**? Justify the conditions that such trees have to fulfill.

6. Show *i)* grammars **and** *ii)* finite-state machines for the following languages. The grammars do not have to be of any particular type (i.e. type 0, type 1, etc.), and the finite-state machines can be nondeterministic.

a) All bit-strings that do not **end with** "01".

b) All bit-strings that do not **start with** "01".