08.71.14 Stærðfræðimynstur í tölvunarfræði
(English exam)

Final exam **December 17th, 2003**

Professor: Hjálmtýr Hafsteinsson **Time: $13^{30} – 16^{30}$**

All problems have the same value. ***You only have to solve 5 problems out of 6. The five best solutions count.*** *All written material and a calculator allowed.*

- Please note that an answer without justification is worth nothing. Justify therefore all answers and remember that it is not necessary to write up definitions that are in the textbook.

1. Remember that $\oplus$ is the logic operator *exclusive or* (i.e. XOR).

a) What is the truth value of the proposition $\mathbf{T} \oplus \mathbf{T} \oplus \mathbf{T}$, where **T** is the the truth value **true**?

b) What is the truth value of the proposition $\mathbf{T} \oplus \mathbf{T} \oplus \ldots \oplus \mathbf{T}$, where **T** occurs n times?

c) Prove a kind of DeMorgan law for $\oplus$:

$$\neg(p \oplus q) \equiv (p \oplus \neg q)$$

You are not to use truth tables, but derive this rule by using the equivalences in tables 5, 6, and 7 on page 24 in the textbook, and the following definition of the operation $\oplus$:

$$(p \oplus q) \equiv (p \vee q) \wedge \neg(p \wedge q)$$

2. Prove that if n is a prime number larger than 3, then n is either congruent to 1 or 5 modulo 6. In other words, we either have $n \equiv 1 \ (\mathbf{mod}\ 6)$ or $n \equiv 5 \ (\mathbf{mod}\ 6)$.

3. *a)* In how many ways can you get a *three of a kind* in poker, i.e. in a 5-card hand of poker there are 3 cards with the same value, but the other 2 have other values? For example the hand of 5-5-5-9-K.

b) In how many ways can you get a *straight*, where the 5 cards are in a sequence and the suit does not matter? For example 4-5-6-7-8.

c) In how many ways can you get an *around-the-corner straight*, where the cards are in a sequence, but the sequence can also warp around? For example Q-K-A-2-3.

d) In how many ways can you get a *no pair hand*, where no two cards have the same value? For example 3-7-8-10-K.

4. *a)* Let R and S be antisymmetric relations on the set A. Prove or disprove the following statements:

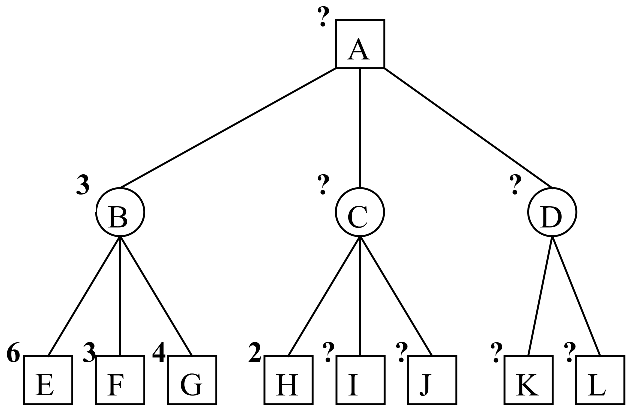
i) The relation $R \cup S$ is antisymmetric.
ii) The relation $R \cap S$ is antisymmetric.
iii) The relation $R - S$ is antisymmetric.

b) Let R be an asymmetric relation. Is the complementary relation $\overline{R}$ *i)* symmetric, or *ii)* asymmetric? Justify your answers.

5. In game trees the values of the nodes (i.e. game positions) can be any numbers, not just +1, -1, and 0 as in the game tree for *Nim*. As before the first player (box nodes) wants to maximize the value, and the second player (circle nodes) wants to minimize the value.

a) In the game tree below we are calculating the value of the game position A. We have already calculated B (from the values of E, F, and G), and we have also found the value of H. All the nodes left to calculate are marked with **?**. Explain why it is not necessary to finish the calculation of C after finding out this value of H.

b) Describe a general method for evaluating game trees based on the above idea.



6. *a)* Show a regular expression for bit strings that do not end in "00".

b) Show a grammar for the language described by **0(11)*(00)*1**.

c) Show a finite automaton that accepts bit strings where the number of 0-bits has the remainder 2 when you divide the number of 0-bits by 3 (i.e. the number of 0-bits is ≡ 2 **mod** 3), and the number of 1-bits is an even number.