08.71.14 Stærðfræðimynstur í tölvunarfræði
(English exam)

Final exam **December 15th, 2004**
Professor: Hjálmtýr Hafsteinsson **Time: 13^{30} – 16^{30}**

All problems have the same value. ***You only have to solve 5 problems out of 6. The five best solutions count.*** *All written material and a calculator allowed.*

- Please note that an answer without justification is worth nothing. Justify therefore all answers and remember that it is not necessary to write up definitions that are in the textbook.

1. Prove the following statements using the rules in section 1.2 í the textbook (page 24). In each step show which rule you are using. **Do not** use truth tables.

a) The proposition $((\neg p \vee q) \wedge (p \vee r)) \rightarrow (q \vee r)$ is a tautology.

b) The proposition $(p \wedge \neg q \wedge r) \vee (p \wedge \neg q \wedge \neg r)$ is logically equivalent to $\neg(p \rightarrow q)$.

2. *a)* Use induction to prove the equation $\sum_{i=0}^{n} \frac{i}{2^i} = 2 - \frac{n+2}{2^n}$ for all natural numbers n.

b) Let the integers $x_1, x_2, ..., x_n$ all be odd numbers. Prove that the multiplication of all of them, i.e. the integer $x_1 * x_2 * ... * x_n$, is an odd number.

3. In each instance show the **smallest relation** on the set { *a, b, c* } that have the properties. Justify in each instance that the relation is the smallest one that fulfills the conditons.

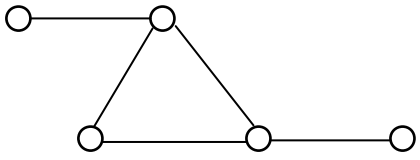
a) Transitive, not reflexive, not symmetric.

b) Reflexive, symmetric, not transitive.

c) Antisymmetric, symmetric, irreflexive.

d) Transitive, asymmetric, not irreflexive.

4. You are given the graph below:



In how many ways can the vertices be marked with the letters *a, b, c, d*, and *e* such that the graphs are isometric, but not the same graph? In other words, how many different labelled graphs are isomorphic with the graph above? Explain how you calculate the number.

5. *The complementary graph* $\bar{G}$ has the same vertices as the graph G, but an edge (u, v) is in $\bar{G}$ if and only if it is not in G. Show that if G is bipartite with more than 4 vertices, then $\bar{G}$ is not bipartite.

6. *a)* Show a finite-state automaton that accepts the language described with the regular expression:

$$(\ \mathbf{1}\ (\mathbf{0} \cup \mathbf{1}\ )\mathbf{0}^*\ (\mathbf{0} \cup \mathbf{1})\ )^*$$

b) A grammar is said to be *ambiguous* if you can construct two different derivation trees for the same string, which is in the language. You are given a grammar with the nonterminal S (which also is the start symbol), the terminals a and b, and the productions $S \to SS$, $S \to a$, and $S \to b$. Is this grammar ambiguous? Justify your answer.