[Assignment View]

Eðlisfræði 2, vor 2007

25. Current, Resistance, and Electromagnetic Force

Assignment is due at 2:00am on Wednesday, February 14, 2007

Credit for problems submitted late will decrease to 0% after the deadline has passed.
The wrong answer penalty is 2% per part. Multiple choice questions are penalized as described in the online help.
The unopened hint bonus is 2% per part.
You are allowed 4 attempts per answer.

Current as Charge Flow

Electricity and Water Analogy

Learning Goal: To understand the analogy between water pressure, water flow, voltage, and current

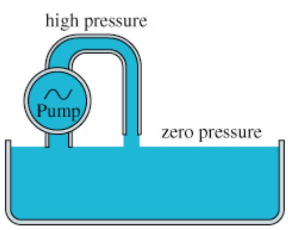
As suggested by the fact that we call both *currents,* the flow of charged particles through an electrical circuit is analogous in some ways to the flow of water through a pipe.

When water flows from a small pipe to a large pipe, the flow (measured, for instance, in gallons per minute) is the same in both pipes, because the amount of water entering one pipe must equal the amount leaving the other. If not, water would accumulate in the pipes. For the same reason, the total electric current I is constant for circuit elements in series.

Water pressure is analogous to total electric potential (voltage), and a pump is analogous to a battery. Water flowing through pipes loses pressure, just as current flowing through a resistor falls to lower voltage. A pump uses mechanical work to raise the water's pressure and thus its potential energy; in a battery, chemical reactions cause charges to flow against the average local electric field, from low to high voltage, increasing their potential energy.

Part A

Consider the following water circuit: water is continually pumped to high pressure by a pump, and then funnelled into a pipe that has lower pressure at its far end (else the water would not flow through the pipe) and back to the pump. Two such circuits are identical, except for one difference: the pipes in one circuit have a larger diameter than the pipes in the other circuit. Through which circuit is the flow of water greater?

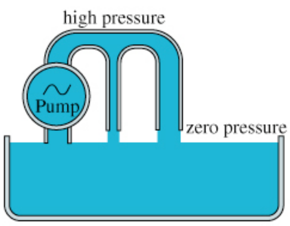


ANSWER: ***Answer not displayed***

Part B

Now consider a variant on the circuit. The water is pumped to high pressure, but the water then faces a fork in the pipe. Two pipes lead back to the pump: large pipe L and small pipe S. Since the water can flow through either pipe, the pipes are said to be *in parallel*:

The overall flow of water that enters the system before the fork is equal to _____



Hint B.1 Water conservation

Hint not displayed

ANSWER: *Answer not displayed*

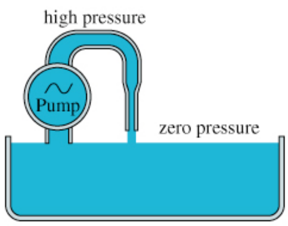
Part C

Part not displayed

Part D

Consider a new circuit: water is pumped to high pressure and fed into only one pipe. The pipe has two distinct segments of different diameters; the second half of the pipe has a smaller diameter than the first half:

Which of the following statements about the flow and change in pressure through each segment is true?

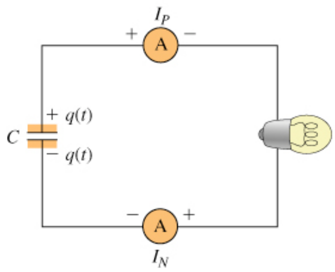


ANSWER: *Answer not displayed*

Capacitor Supplies Current to Bulb

A large capacitor is charged with q on one electrode and $-q$ on the other. At time $t=0$, the capacitor is connected in series to two ammeters and a bulb. The ammeter connected to the positive side of the capacitor reads I_P, and the ammeter connected to the negative side of the capacitor reads I_N.

Both ammeters will read positive if current flows in a clockwise sense through the circuit (from the + to the - terminal of the meter).



Part A

Immediately after time $t=0$, what happens to the charge on the capacitor plates?

a. Electrons flow through the circuit from the positive to the negative side of the capacitor.
b. Electrons flow through the circuit from the negative to the positive side of the capacitor.
c. The positive and negative charges attract each other, so they stay in the capacitor.
d. Current flows clockwise through the circuit.
e. Current flows counterclockwise through the circuit.

Hint A.1 What is meant by current flow?

Hint not displayed

List the letters corresponding to the correct statements in alphabetical order. Do not use commas. For instance, if you think that only statements a and c are correct, write ac.

Part B

At any given instant after $t=0$, what is the relationship between the current flowing through the two ammeters, I_P and I_N, and the current through the bulb, I_B?

ANSWER:

- $I_P > I_B > I_N$
- $I_P = I_B > I_N$
- $I_P > I_B = I_N$
- $I_P = I_B = I_N$

This is a fundamental result that reflects conservation of charge. In a circuit where elements are arranged in series, the voltage changes as current flows through the circuit, but the current is constant. Otherwise, charge would accumulate in the circuit.

In a circuit where elements are arranged in parallel, the opposite is true; all parallel branches have the same voltage, although the current may be different in different branches. This result is formalized in Kirchoff's junction law -- the algebraic sum of currents entering any junction must be zero. (In this law, a current leaving a junction is considered negative).

Part C

What is the sign of the quantity dq/dt?

ANSWER: Positive Negative

Part D

Light bulbs are often assumed to obey Ohm's law, but this is not strictly true, because their resistance increases as the filament heats up at higher voltages. A typical flashlight bulb at full brilliance draws a current of approximately 0.5 A while attached to a 3 V source. For this problem, assume that the changing resistance causes the current to be 0.5 A for any voltage between 2 V and 3 V.

Suppose this flashlight bulb is attached to a capacitor as shown in the circuit from the problem introduction. If the capacitor has a capacitance of 3 F (an unusually large but not unrealistic value) and is initially charged to 3 V, how long will it take for the voltage across the flashlight bulb to drop to 2 V (where the bulb will be orange and dim)? Call this time t_{bright}.

Hint D.1 How to approach this problem

Hint not displayed

Part D.2 Initial charge on capacitor

Part not displayed

Part D.3 Charge on capacitor at 2 V

Part not displayed

Hint D.4 Relationship between charge and current

Hint not displayed

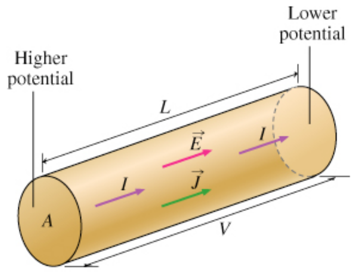
Express t_{bright} numerically, in seconds, to the nearest integer.

ANSWER: t_{bright} = **6** seconds

Ohm's Law and Resistance

Resistance from Microscopic Ohm's law

Your task is to calculate the resistance of a simple cylindrical resistor with wires connected to the ends, such as the carbon composition resistors that are used on electronic circuit boards. Imagine that the resistor is made by squirting material whose conductivity is σ into a cylindrical mold with length L and cross-sectional area A . Assume that this material satisfies Ohm's law. (It should if the resistor is operated within its power dissipation limits.)



Part A

What is the resistance R of this resistor?

Hint A.1 General approach

Hint not displayed

Part A.2 Microscopic Ohm's law

Part not displayed

Part A.3 Find the voltage from the electric field

Part not displayed

Part A.4 Find the current from the current density

Part not displayed

Hint A.5 Ohm's law for the resistor

Hint not displayed

Express the resistance in terms of variables given in the introduction. Do *not* use V or I in your answer.

ANSWER: R =***Answer not displayed***

Electrical Safety

Most of us have experienced an electrical shock one way or another in our lives. Most electrical shocks we receive are minor ones from wooly sweaters or from shoes. However, some shocks, especially from outlets or power mains, can be fatal. This question will show you how to estimate the current through a human body when subject to an electrical shock.

Imagine a situation in which a person accidentally touches an electrical socket with both hands. By modeling the arm and the chest to be a cylindrical tube with a total length $L = 2.0\ \mathrm{m}$, cross-sectional area $A = 10\ \mathrm{cm}^2$, and resistivity $\rho = 1.5\ \mathrm{ohm \cdot m}$, you can calculate the current in amperes through the person when a potential difference of $V = 110\ \mathrm{V}$ is applied across the two hands. Assume that the current flows only through the modeled cylindrical tube.

Part A

What is the current flow through the body?

Hint A.1 Calculating the resistance

Hint not displayed

Part A.2 Obtaining the expression for current

Part not displayed

Express your answer numerically to two significant figures.

ANSWER: I = **0.037** A

The following are the effects of current on humans:

- 1 mA = 10^{-3} A or less: barely noticeable;
- 1 to 8 mA: strong surprise;
- 8 to 15 mA: unpleasant, victims able to detach from source;
- 15 to 75mA: painful, dangerous;
- 75 mA or more: fatal.

These values vary according to sex, age, and weight.

Resistance of a Heater

A 1500-W heater is designed to be plugged into a 120-V outlet.

Part A

What current will flow through the heating coil when the heater is plugged in?

Hint A.1 Setting it up

Hint not displayed

Part A.2 Power

Part not displayed

Part A.3 Finishing up

Part not displayed

Express your answer for the current numerically, to three significant figures.

ANSWER: I = **12.5** A

Note that watts/volts has the correct units: Since

$$\mathrm{watt} = \mathrm{joule/second}$$

and

$$\mathrm{volt} = \mathrm{joule/coulomb},$$

then

$$\mathrm{watt/volt} = \mathrm{coulomb/second} = \mathrm{ampere}.$$

Part B

What is R, the resistance of the heater?

Hint B.1 Which equation to use

Hint not displayed

Express your answer numerically, to three significant figures.

ANSWER: R = **9.6** ohms

Part C

How long does it take to raise the temperature of the air in a good-sized living room $(3.00 \mathrm{m} \times 5.00 \mathrm{m} \times 8.00 \mathrm{m})$ by $10.0^\circ \mathrm{C}$? Note that the heat capacity of air is 1006 $\mathrm{J/(kg \cdot ^\circ C)}$ and the density of air is $1.20 \ \mathrm{kg/m^3}$.

Part C.1 Mass of the air

Part not displayed

Part C.2 How many joules

Part not displayed

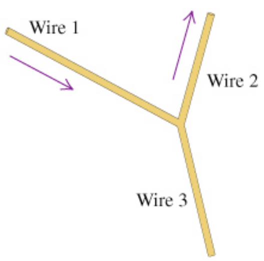
Express your answer numerically in minutes, to three significant figures.

ANSWER: t = **16.1** minutes

Actually, the heat capacity of the walls and other material in the room will generally exceed that of the air by several times, so an hour is a more reasonable time to heat the room by this much.

Current and Current Density at a Junction

Consider the juncion of three wires as shown in the diagram.



The magnitudes of the current density and the diameters for wires 1 and 2 are given in the table. The current directions are indicated by the arrows.

Wire	Current density ($\mathrm{A/mm^2}$)	Diameter (mm)
1	3.0	2.0
2	5.0	3.0

Part A

Find the current I_3 in wire 3.

Hint A.1 How to approach the problem

Find the total current through wires 1 and 2. Then use the Kirchhoff's junction rule to determine the current through wire 3.

Hint A.2 Kirchhoff's rule

Recall that Kirchhoff's junction rule is merely another way of stating the conservation of charge over time (charge flowing into a junction must equal charge flowing out). Otherwise, there would be an infinite buildup of charge over time. Therefore, the current entering a junction is equal to the current leaving a junction.

Hint A.3 Current density and current

Recall that current density is just the total current divided by the cross-sectional area of the wire. That is, $\vec{J} = \vec{I}/A$.

Hint A.4 Area of the wire

The wire cross-section is circular, so the area is just $A = \pi r^2$, where r is the radius of the wire.

Express your answer in amperes to two significant figures. Call current out of the junction positive and current into the junction negative.

ANSWER: I_3 = -26 A

Part B

Find the magnitude of the current density J_3 in wire 3. The diameter of wire 3 is 1.5 millimeters.

Hint B.1 Current density and current

Hint not displayed

Hint B.2 Area of the wire

Hint not displayed

Express your answer in amperes per square millimeter to two significant figures.

ANSWER: J_3 = 15 $\mathrm{A/mm^2}$.

How a Real Voltmeter Works

Unlike the idealized voltmeter, a real voltmeter has a resistance that is not infinitely large.

Part A

A voltmeter with resistance R_V is connected across the terminals of a battery of emf $\mathcal{E}$ and internal resistance r. Find the potential difference V_{meter} measured by the voltmeter.

Hint A.1 How to approach the problem

Hint not displayed

Hint A.2 How to find the potential between points a and b

Hint not displayed

Hint A.3 An expression for V_{ab}

Hint not displayed

Hint A.4 Using Kirchhoff's loop rule

Hint not displayed

ANSWER: $V_{\mathrm{meter}} = \mathcal{E} - ((\mathcal{E}/(R_V + r)) \cdot r)$

With a little algebraic manipulation, the answer can also be written as

$$\mathcal{E}\left(1 - \frac{r}{r + R_V}\right).$$

In this form it is easier to see why the voltmeter reading differs from the actual emf it is supposed to measure by only a small amount if $R_V \gg r$. It is a good idea to check that the answer gives the correct result in the limit that $R_V \to \infty$.

Part B

If $\mathcal{E}$ = $7.50\ \mathrm{V}$ and $r = 0.45\ \Omega$, find the minimum value of the voltmeter resistance R_V for which the voltmeter reading is within 1.0% of the emf of the battery.

Hint B.1 What is meant by "within 1.0%"

Examine the expression for the potential difference measured by the voltmeter:

$$\mathcal{E}\left(1 - \frac{r}{r+R_V}\right).$$

From this expression, you can see that the difference between the potential difference measured by the voltmeter and the actual emf of the battery is

$$\frac{r}{r+R_V}.$$

In our case, the specifications require that this expression be less than 1%.

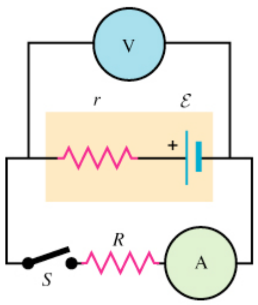
Express your answer numerically (in ohms) to at least three significant digits.

ANSWER: R_V = **44.6** Ω

Typical voltmeters have a range of possible resistances, some of which are much larger than the value you just obtained (on the order of megaohms). This allows reasonably accurate measurements of much larger resistances to be made.

Measuring the EMF and Internal Resistance of a Battery

When switch S in the figure is open, the voltmeter V of the battery reads 3.05 V. When the switch is closed, the voltmeter reading drops to 2.91 V, and the ammeter A reads 1.65 A. Assume that the two meters are ideal, so they do not affect the circuit.



Part A

Find the emf $\mathcal{E}$.

Express your answer in volts to three significant digits.

ANSWER: $\mathcal{E}$ = **3.05** V

Part B

Find the internal resistance R_{bat} of the battery.

Hint B.1 How to approach the problem

Hint not displayed

Express your answer in ohms to four significant digits.

ANSWER: R_{bat} = **8.485×10^{-2}** Ω

Part C

Find the circuit resistance R.

Part C.1 Find the voltage drop across the circuit resistor

Part not displayed

Express your answer in ohms to three significant digits.

ANSWER: R = **1.76** Ω

This is the kind of circuit you would use in real life to measure the emf and internal resistance of a battery. You need the second resistor R to increase the resistance in the circuit so that the current flowing through the ammeter is not too large. In fact, you would need to figure out roughly how big a resistance to use once you had determined the emf of the battery, depending on the range of the ammeter you were using.

Power in Resistive Circuits

Power in DC Circuits

A battery does work in moving charge around a circuit i.e. sustaining a current through the circuit. To illustrate this point, consider a resistor with a voltage V across it and a current I flowing through it.

Part A

Focus on a single charge, q, passing through the resistor. Find the work W done on the charge by the electric field in the resistor.

Hint A.1 How to approach this problem

Hint not displayed

Hint A.2 Formula for work

Hint not displayed

Part A.3 Find the force on the charge q

Part not displayed

Express the work W done on the charge in terms of q, V, and/or I.

ANSWER: W = *Answer not displayed*

Part B

Part not displayed

Power Delivered to a Resistor

In this problem you will derive two different formulas for the power delivered to a resistor.

Part A

What is the power P supplied to a resistor whose resistance is R when it is known that it has a voltage V across it?

Part A.1 Find an expression for power

Part not displayed

Part A.2 Find current in terms of voltage and resistance

Part not displayed

Express the power P in terms of R and V.

ANSWER: P =*Answer not displayed*

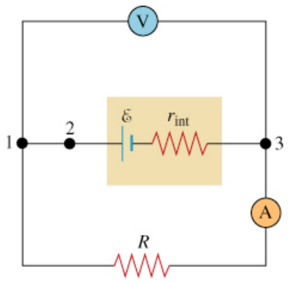
Part B

Part not displayed

Power in Resistive Electric Circuits

Learning Goal: To understand how to compute power dissipation in a resistive circuit.

The circuit in the diagram consists of a battery with EMF $\mathcal{E}$, a resistor with resistance R, an ammeter, and a voltmeter. The voltmeter and the ammeter (labeled V and A) can be considered ideal; that is, their resistances are infinity and zero, respectively. The current in the resistor is I, and the voltage across it is V. The internal resistance of the battery $r_{\rm int}$ is not zero.



Part A

What is the ammeter reading I?

Express your answer in terms of $\mathcal{E}$, R, and $r_{\rm int}$.

ANSWER: $I = \mathcal{E}/(R+r_{\rm int})$

Note that the resistances of the ammeter and voltmeter do not appear in the answer. That is because these two circuit elements are "ideal." The voltmeter has infinite resistance, so no current flows through it (imagine that there is a short circuit inside the voltmeter). The ammeter has zero resistance, so there is no voltage drop as current flows through it.

Part B

What is the voltmeter reading V?

Part B.1 Potential difference across the internal resistance

Part not displayed

Express your answer in terms of $\mathcal{E}$, R, and $r_{\rm int}$.

ANSWER: $V = (\mathcal{E}/(R+r_{\rm int}))\cdot R$

In the following parts, you will express the power dissipated in the resistor of resistance R using three different sets of variables.

Part C

What is the power P_R dissipated in the resistor?

Express your answer in terms of I and V.

ANSWER: $P_R = I\cdot V$

Part D

Again, what is the power P_R dissipated in the resistor?

This time, express your answer in terms of one or more of the following variables: I, $r_{\rm int}$, and R.

ANSWER: $P_R = I^2 \cdot R$

Part E

For the third time, what is the power P_R dissipated in the resistor?

Express your answer in terms of one or more of the following variables: $\mathcal{E}$, $r_{\rm int}$, and R.

ANSWER: $P_R = (\mathcal{E}/(R+r_{int}))^2 \cdot R$

Part F

What is the power $P_{\rm total}$ dissipated in the entire circuit?

Hint F.1 How to approach the problem

The total power dissapated in the circuit is the algebraic sum of the power dissipated in all the elements of the circuit. You have already found the power dissipated in the resistor. Now consider the other circuit elements.

Part F.2 Find the power dissipated in the battery

How much power $P_{\rm battery}$ is dissipated in the battery?

Hint F.2.a How can power be dissipated inside a battery?

Hint not displayed

Express your answer in terms of one or more of the following variables: $\mathcal{E}$, $r_{\rm int}$, and R.

ANSWER: $P_{\rm battery} = \left((\mathcal{E}/(R+r_{int}))^2 \cdot r_{int}\right)$

Part F.3 Find the power dissipated between points 1 and 2

What is the power dissipated in the part of the circuit between points 1 and 2?

ANSWER: ◉ 0 ○ I^2R ○ $I^2r_{\rm int}$ ○ $\frac{\mathcal{E}^2}{R}$ ○ $\frac{\mathcal{E}^2}{r_{\rm int}}$

Express your answer in terms of one or more of the following variables: $\mathcal{E}$, $r_{\rm int}$, and R.

ANSWER: $P_{\rm total} = \left((\mathcal{E}/(R+r_{int}))^2 \cdot R\right) + \left((\mathcal{E}/(R+r_{int}))^2 \cdot r_{int}\right)$

Part G

What is the total power $P_{\rm total}$ dissipated in the entire circuit, in terms of the EMF $\mathcal{E}$ of the battery and the current in the circuit?

Express your answer in terms of $\mathcal{E}$ and the ammeter current I.

ANSWER: $P_{\rm total} = I \cdot \mathcal{E}$

Summary	7 of 11 problems complete (62.24% avg. score) 34.23 of 35 points