Magnetic Field near a Moving Charge

A particle with positive charge $q$ is moving with speed $v$ along the $z$ axis toward positive $z$. At the time of this problem it is located at the origin, $x = y = z = 0$. Your task is to find the magnetic field at various locations in the three-dimensional space around the moving charge.

Part A
Which of the following expressions gives the magnetic field at the point $r'$ due to the moving charge?

A. $\frac{\mu_0 q v \times \hat{r}}{4\pi r^2}$
B. $\frac{\mu_0 q v \times \hat{r}}{4\pi r^3}$
C. $\frac{\mu_0 q v \times \hat{v}}{4\pi r}$
D. $\frac{\mu_0 q v \times \hat{v}}{4\pi r^2}$

ANSWER: Answer not displayed

Part B
Find the magnetic field at the point $r_1 = z_1 \hat{z}$.

Part B.1 Find the magnetic field direction

Express your answer in terms of $\hat{x}$, $\hat{y}$, and $\hat{z}$, and use $\hat{x}$, $\hat{y}$, and $\hat{z}$ for the three unit vectors.

ANSWER: $\vec{B}(r_1) = $ Answer not displayed

Part C
Find the magnetic field at the point $r_2 = y_1 \hat{y}$.

Express your answer in terms of $\hat{x}$, $\hat{y}$, and $\hat{z}$, and use $\hat{x}$, $\hat{y}$, and $\hat{z}$ for the three unit vectors.

ANSWER: $\vec{B}(r_2) = $ Answer not displayed

Part D
Find the magnetic field at the point $r_3 = x_1 \hat{x} + z_1 \hat{z}$.

Part D.1 Evaluate the cross product

Part not displayed

Part D.2 Find the distance from the charge

Part not displayed

Express your answer in terms of $\mu_0 q v \times \hat{x}$, $\hat{y}$, and $\hat{z}$, and use $\hat{x}$, $\hat{y}$, and $\hat{z}$ for the three unit vectors.
Part E
The field found in this problem for a moving charge is the same as the field from a current element of length \( dl \) carrying current \( i \) provided that the quantity \( qv \) is replaced by which quantity?

**Hint E.1**
Making a correlation

**Answer:**
Answer not displayed

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**Force between Moving Charges**

Two point charges, with charges \( q_1 \) and \( q_2 \), are each moving with speed \( v \) toward the origin. At the instant shown \( q_1 \) is at position \((0, 0)\) and \( q_2 \) is at \((d, 0)\). (Note that the signs of the charges are not given because they are not needed to determine the magnitude of the forces between the charges.)

**Part A**
What is the magnitude of the electric force between the two charges?

**Hint A.1**
Which law to use

Apply Coulomb's law:

\[
F = \frac{q_1 q_2}{4\pi\epsilon_0 r^2}
\]

where \( r \) is the distance between the two charges.

**Part A.2**
Find the value of \( r^2 \)

What is the value of \( r^2 \) for the given situation?

**Answer:**
\[ r^2 = 2d^2 \]

**Express \( F \) in terms of \( q_1, q_2, d, \) and \( \epsilon_0 \).**

**Answer:**
\[ F = \frac{q_1 q_2}{4\pi\epsilon_0 2d^2} \]

---

**Part B**
What is the magnitude of the magnetic force on \( q_2 \) due to the magnetic field caused by \( q_1 \)?

**Hint B.1**
How to approach the problem

First, find the magnetic field generated by charge \( q_1 \) at the position of charge \( q_2 \). Then evaluate the magnetic force on \( q_2 \) due to the field of \( q_1 \).

**Part B.2**
Magnitude of the magnetic field

The Biot-Savart law, which gives the magnetic field produced by a moving charge, can be written

\[
\vec{B} = \frac{\mu_0 |i| \times \vec{r}}{4\pi r^3},
\]

where \( \mu_0 \) is the permeability of free space and \( \vec{r} \) is the vector from the charge to the point where the magnetic field is produced. Note we have \( \vec{r} \) in the numerator, not \( \vec{r} \), necessitating an extra power of \( \vec{r} \) in the denominator.

Using this equation find the expression for the magnitude of the magnetic field experienced by charge \( q_2 \) due to charge \( q_1 \).

**Part B.2.a**
Determine the cross product

What is the magnitude of \( \vec{r} \times \vec{r} \)?

For any two vectors,
\[ \vec{a} \times \vec{b} = |a| |b| \sin \theta \vec{n} \]

where \( \theta \) is the angle between the vectors. Because, in this case, \( \theta \) is 45 degrees,

\[ |\vec{a} \times \vec{b}| = \frac{\sqrt{2}}{2} |b| r. \]

Substitute the appropriate value of \( r \) for this problem, to arrive at a surprisingly simple answer.

Express your answer in terms of \( \vec{a} \) and \( \vec{b} \).

**ANSWER:**

\[ |\vec{a} \times \vec{b}| = \frac{1}{r} \]

Note that \( r \) is cubed in the denominator of the Biot-Savart law.

Express the magnitude of the magnetic field of \( q_1 \) (at the location of \( q_2 \)) in terms of \( \vec{q}_1 \times \vec{r}_1 \) and \( \mu_0 \).

**ANSWER:**

\[ B = \frac{\mu_0 q_1 d \sin \theta}{(r^3 + \sqrt{2} r d)} \]

Part B.3 Find the direction of the magnetic field

Which of the following best describes the direction of the magnetic field from \( q_1 \) at \( q_2 \)? Remember, according to the Biot-Savart law, the field must be perpendicular to both \( \vec{q}_1 \) and \( \vec{r} \).

Ignore the effects of the sign of \( q_1 \).

**ANSWER:**

- Along the \( x \) axis
- Along the \( y \) axis
- Along the \( z \) axis into or out of the screen

Hint B.4 Computing the force

You can evaluate the force exerted on a moving charge by a magnetic field using the Lorentz force law:

\[ \vec{F} = q \vec{v} \times \vec{B} \]

where \( \vec{F} \) is the force on the moving charge, \( \vec{B} \) is the magnetic field, \( q \) is the charge of the moving charge, and \( \vec{v} \) is the velocity of the charge. Note that, as long as \( \vec{v} \) and \( \vec{B} \) are perpendicular, \( \sin \theta = 1 \).

Express the magnitude of the magnetic force in terms of \( q_1 \), \( \vec{q}_1 \times \vec{r}_1 \), and \( \mu_0 \).

**ANSWER:**

\[ F = \frac{q_1 \mu_0 |\vec{q}_1| d |\sin \theta|}{(r^3 + \sqrt{2} r d)} \]

Part C Assuming that the charges are moving nonrelativistically (\( v \ll c \)), what can you say about the relationship between the magnitudes of the magnetic and electrostatic forces?

**Hint C.1 How to approach the problem**

Determine which force has a greater magnitude by finding the ratio of the electric force to the magnetic force and then applying the approximation. Recall that \( \epsilon_0 \mu_0 = 1/c^2 \).

**ANSWER:**

- The magnitude of the magnetic force is greater than the magnitude of the electric force.
- The magnitude of the electric force is greater than the magnitude of the magnetic force.
- Both forces have the same magnitude.

This result holds quite generally: Magnetic forces between moving charges are much smaller than electric forces as long as the speeds of the charges are nonrelativistic.

Biot-Savart Law: B-Field from Current Segments

**Magnetic Field from Current Segments**

**Learning Goal:** To apply the Biot-Savart law to find the magnetic field produced on the \( z \) axis from current elements in the \( xy \) plane.

In this problem you are to find the magnetic field component along the \( z \) axis that results from various current elements in the \( xy \) plane (i.e., at \( z = 0 \)).

The field at a point due to a current-carrying wire is given by the Biot-Savart law,

\[ \vec{B}(\vec{r}) = \int \frac{\mu_0 |\vec{r} \times \vec{r}'|}{4\pi r^3} \]

where \( \vec{r} \) and \( \vec{r}' \) are the position vectors of the current carrying wire and the point of observation, respectively, and the integral is done over the current-carrying wire. Evaluating the vector integral will typically involve the following steps:

- Choose a convenient coordinate system--typically rectangular, say with coordinate axes \( x \), \( y \), and \( z \).
- Write \( \vec{r} \times \vec{r}' \) in terms of the coordinate variables and directions (\( x \), \( z \), etc.). To do this, you must find \( \vec{r} \times \vec{r}' \) and \( \vec{r}' \). Again, finding the cross product can be done either
  - geometrically (by finding the direction of the cross product vector first, then checking for cancellations from any other portion of the wire, and then finding the magnitude or relevant
component) or algebraically (by using $\hat{r} \times \hat{g} = \hat{\delta}$, etc.).

Evaluate the integral for the component(s) of interest.

In this problem, you will focus on the second of these steps and find the integrand for several different current elements. You may use either of the two methods suggested for doing this.

Part A
The field at the point shown in the figure due to a single current element is given by

$$\vec{B}(\vec{r}) = \frac{\mu_0 I \, d\vec{l} \times \hat{r}}{4\pi r^2},$$

where $r = |\vec{r}|$ and $\vec{r} = \vec{r}/r$. In this expression, what is the variable $\vec{r}$ in terms of $\vec{r}_1$ and/or $\vec{r}_2$?

Hint A.1 Making sense of subscripts

**ANSWER:** \[ \vec{r}_1 - \vec{r}_2 \]

Part B
Find $B_z(0,0,0)$, the $z$ component of the magnetic field at the point $x = y = z = 0$ from the current $I$ flowing over a short distance $d\vec{l} = d\vec{r}$ located at the point $\vec{r}' = x_1, \hat{\hat{r}}$.

Hint B.1 Cross product

The key here is the cross product in the Biot-Savart law. What is the cross product when $\vec{r}$ and $d\vec{l}$ are parallel?

Express your answer in terms of $I$, $x_1$, $\mu_0$, $\gamma$, and $\beta$. Recall that a component is a scalar; do not enter any unit vectors.

**ANSWER:** $B_z(0,0,0) = 0$

Part C
Find $B_z(0,0,0)$, the $z$ component of the magnetic field at the point $x = y = z = 0$ from the current $I$ flowing over a short distance $d\vec{l} = d\vec{r}$ located at the point $\vec{r}' = x_1, \hat{\hat{r}}$.

Part C.1 Determine the displacement from the current element

What is $|\vec{r}_1|$ the distance (magnitude) from the current element to the point in question?

**ANSWER:** $|\vec{r}_1| = x_1$

Part C.2 Find the direction from the cross product
What is the unit vector that describes the direction of the magnetic field at the origin \( x = y = z = 0 \)?

Express your answer in terms of \( \hat{x} \), \( \hat{y} \), or \( \hat{z} \).

ANSWER: \( \hat{z} \)

Express your answer in terms of \( I \), \( \mu_0 \), \( \gamma \), and \( \beta \). Recall that a component is a scalar; do not enter any unit vectors.

ANSWER: \( B_z(0, 0, 0) = \frac{\mu_0 I \gamma}{4\pi} \)

Part D
Find \( B_z(0, 0, 0) \), the \( z \) component of the magnetic field at the point \( x = y = z = 0 \) from the current \( I \) flowing over a short distance \( d\ell = dx \) located at the point \( P_x = y; \hat{y} \).

Part D.1
Determine the displacement from the current element

Part D.2
Find the direction of the magnetic field vector

What is the unit vector that describes the direction of the magnetic field at the origin \( x = y = z = 0 \)?

Hint D.2.a
Evaluating cross products

Express your answer in terms of \( \hat{x} \), \( \hat{y} \), or \( \hat{z} \).

ANSWER: \( \hat{z} \)

Express your answer in terms of \( I \), \( \mu_0 \), \( \gamma \), and \( \beta \). Recall that a component is a scalar; do not enter any unit vectors.

ANSWER: \( B_z(0, 0, 0) = -\frac{\mu_0 I \gamma}{4\pi} \)

Part E
Find \( B_z(0, 0, z) \), the \( z \) component of the magnetic field at the point \( P \) located at \( x = y = 0, z = z_f \) from the current \( I \) flowing over a short distance \( d\ell = dx \) located at the point \( P_x = x; \hat{x} \).

Part E.1
Determine the displacement from the current element

Part E.2
Use the cross product to get the direction

Express your answer in terms of \( I \), \( \mu_0 \), \( \gamma \), and \( \beta \). Recall that a component is a scalar; do not enter any unit vectors.

ANSWER: \( B_z(0, 0, z_f) = 0 \)
Part F

Find \( B_z(0,0,z) \), the \( z \) component of the magnetic field at the point P located at \( x = y = 0, z = z_1 \) from the current \( I \) flowing over a short distance \( dl' = dl \hat{j} \) located at the point \( r'_z = x_1 \hat{z} \).

\[ \text{Part F.1 Determine the displacement from the current element} \]

What is \( |r'_z| \), the displacement (magnitude) from the current element to the point in question? The figure shows another perspective of the same situation to make this calculation easier.

\[ \text{ANSWER: } |r'_z| = \sqrt{x_1^2 + z_1^2} \]

\[ \text{Part F.2 Determine which unit vector to use} \]

Another way to write the Biot-Savart law is

\[ \vec{B}(\vec{r}) = \frac{\mu_0 I}{4\pi} \frac{\vec{r} \times \vec{r'}}{r'^3} \]

where you replace \( \vec{r} \) with \( \vec{r}/r' \). This eliminates the problem of finding \( \vec{r} \) and can make computation easier.

You are asked for the \( z \) component of the magnetic field. \( dl' \) points in the \( \hat{y} \) direction. Which component \( (\hat{x}, \hat{y}, \hat{z}) \) must you cross with \( dl' \) to get a vector in the \( \hat{z} \) direction? Recall that

\[ \vec{A} \times (\vec{B} + \vec{C}) = \vec{A} \times \vec{B} + \vec{A} \times \vec{C}. \]

Express your answer in terms of \( \hat{x}, \hat{y}, \) or \( \hat{z} \) (ignoring the sign).

\[ \text{ANSWER: } \]  

\[ \text{Part F.3 Evaluate the cross product} \]

The \( z \) component of the magnetic field results from the cross product of \( dl' \) and the \( x \) component of \( r'_z \). These two vectors are orthogonal, so finding the cross product is relatively straightforward.

What is the value of \( dl' \times r'_z \)?

\[ \text{Give your answer in terms of } \hat{x}, \hat{y}, \hat{z}, \text{ and } \vec{r}. \]

\[ \text{ANSWER: } dl' \times r'_z = dl' \hat{x} \hat{z} \]

Substitute this expression into the formula for the magnetic field given in the last hint. Observe that it has \( 1/r^3 \) in the denominator since \( r' \) in the original equation was replaced with \( r' = r \).

Express your answer in terms of \( I, x_1, z_1, \mu_0, \gamma, \text{ and } dl'. \) Recall that a component is a scalar; do not enter any unit vectors.

\[ \text{ANSWER: } B_z(0,0,z_1) = \frac{\mu_0 I}{4\pi} \frac{1}{\sqrt{x_1^2 + z_1^2}} \]

Magnetic Field at the Center of a Wire Loop

A piece of wire is bent to form a circle with radius \( r \). It has a steady current \( I \) flowing through it in a counterclockwise direction as seen from the top (looking in the negative \( z \) direction).
Part A
What is \( B_z(0) \), the \( z \) component of \( \mathbf{B} \) at the center (i.e., \( x = y = z = 0 \)) of the loop?

Part A.1 Specify the integrand

Part A.2 Perform the integration

Express your answer in terms of \( I_1, \beta, \) and \( m \).

ANSWER: \( B_z(0) = \) Answer not displayed

Magnetic Field due to Semicircular Wires

A loop of wire is in the shape of two concentric semicircles as shown.

The inner circle has radius \( \alpha \); the outer circle has radius \( \beta \). A current \( I \) flows clockwise through the outer wire and counterclockwise through the inner wire.

Part A
What is the magnitude, \( B \), of the magnetic field at the center of the semicircles?

Hint A.1 What physical principle to use

Hint not displayed

Part A.2 Compute the field due to the inner semicircle

Part A.3 Direction of the field due to the inner semicircle

Part A.4 Compute the field due to the straight wire segments

Express \( B \) in terms of any or all of the following: \( I, \alpha, \beta, \) and \( m \).

ANSWER: \( B = \) Answer not displayed

Part B

Force between an Infinitely Long Wire and a Square Loop

A square loop of wire with side length \( a \) carries a current \( I \). The center of the loop is located a distance \( d \) from an infinite wire carrying a current \( I' \). The infinite wire and loop are in the same plane; two sides of the square loop are parallel to the wire and two are perpendicular as shown.
Part A
What is the magnitude, \( F \), of the net force on the loop?

Hint A.1  How to approach the problem
You need to find the total force as the sum of the forces on each straight segment of the wire loop. You'll save some work if you think ahead of time about which forces might cancel.

Part A.2  Determine the direction of force
Which of the following diagrams correctly indicates the direction of the force on each individual line segment?

Hint A.2.a  Direction of the magnetic field
In the region of the loop, the magnetic field points into the plane of the paper (by the right-hand rule).

Hint A.2.b  Formula for the force on a current-carrying conductor
The magnetic force on a straight wire segment of length \( l \) carrying a current \( I \) with a uniform magnetic field \( \vec{B} \) along its length, is
\[
F = lI \times \vec{B}.
\]
where \( l \) is a vector along the wire in the direction of the current.

ANSWER: [a b c d]

Part A.3  Determine the magnitude of force
Which of the following diagrams correctly indicates the relative magnitudes of the forces on the parallel wire segments?

Part A.3.a  Find the magnetic field due to the wire
What is the magnitude, \( B \), of the wire's magnetic field as a function of perpendicular distance from the wire, \( r \).

Hint A.3.a.i  Ampère's law

Express the magnetic field magnitude in terms of \( J \), \( r \), and \( \mu_0 \)

ANSWER: \[
B = \frac{(\mu_0 I_1)}{(2\pi r)}
\]

ANSWER: [a b c d]
Part A.4  Find the force on the section of the loop closest to the wire
What is the magnitude of the force $F_1$ on the section of the loop closest to the wire, that is, a distance $d - a/2$ from it?

Hint A.4.a  Formula for the force on a current-carrying conductor  
$|\vec{F}| = \frac{|\mu_0 I_1 I_2 a|}{(2\pi \cdot (d - a/2))}$

Part A.4.b  Find the magnetic field due to the wire

Express your answer in terms of $\mu_0 I_1$, $I_2$, $a$, and $d$.

ANSWER: $B = \frac{|\mu_0 I_1|}{(2\pi \cdot r)}$

Part A.5  Find the magnetic field due to the wire
What is the magnitude, $B_0$, of the wire's magnetic field as a function of perpendicular distance from the wire, $r$.

Hint A.5.a  Ampère's law

Express the magnetic field magnitude in terms of $I_2$, $r$, and $\mu_0$

ANSWER: $B = \frac{|\mu_0 I_2|}{(2\pi \cdot r)}$

Express the force in terms of $I_2$, $I_1$, $a$, and $\mu_0$

ANSWER: $|\vec{F}| = \frac{|\mu_0 I_1 I_2 a|}{(2\pi \cdot (d + a/2))} - \frac{|\mu_0 I_1 I_2 a|}{(2\pi \cdot (d - a/2))}$

Part B
The magnetic moment $\mu_0 I_2$ of a current loop is defined as the vector whose magnitude equals the area of the loop times the magnitude of the current flowing in it ($\mu_0 I_2 = I_2 \cdot a$), and whose direction is perpendicular to the plane in which the current flows. Find the magnitude, $F$, of the force on the loop from Part A in terms of the magnitude of its magnetic moment.

Express $\vec{F}$ in terms of $\mu_0 I_2$, $a$, and $d$.

ANSWER: $F = \frac{|\mu_0 I_2 a|}{(2\pi \cdot (d + a/2))} - \frac{|\mu_0 I_2 a|}{(2\pi \cdot (d - a/2))}$

The direction of the net force would be reversed if the direction of the current in either the wire or the loop were reversed. The general result is that "like currents" (i.e., currents in the same direction) attract each other (or, more correctly, cause the wires to attract each other), whereas oppositely directed currents repel. Here, since the like currents were closer to each other than the unlike ones, the net force was attractive. The corresponding situation for an electric dipole is shown in the figure below.

Ampère's Law and Examples

Ampère's Law Explained

Learning Goal: To understand Ampère's law and its application.

Ampère's law is often written $\int \vec{B} \cdot d\vec{A} = \mu_0 I_{mag}$;

Part A
The integral on the left is

ANSWER: Answer not displayed

Part B
Part not displayed

Part C
The circle on the integral means that $\mathbf{\hat{n}}(r)$ must be integrated

**ANSWER:** Answer not displayed

---

**Part D**

Which of the following choices of path allow you to use Ampère’s law to find $\mathbf{H}(r)$?

a. The path must pass through the point $r$.

b. The path must have enough symmetry so that $\mathbf{H}(r) \cdot d\mathbf{l}$ is constant along large parts of it.

c. The path must be a circle.

**ANSWER:** Answer not displayed

---

**Part E**

Ampère’s law can be used to find the magnetic field around a straight current-carrying wire.

*Is this statement true or false?*

**ANSWER:** Answer not displayed

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**Part F**

Ampère’s law can be used to find the magnetic field at the center of a square loop carrying a constant current.

*Is this statement true or false?*

**ANSWER:** Answer not displayed

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**Part G**

Ampère’s law can be used to find the magnetic field at the center of a circle formed by a current-carrying conductor.

*Is this statement true or false?*

**ANSWER:** Answer not displayed

---

**Part H**

Ampère’s law can be used to find the magnetic field inside a toroid. (A toroid is a doughnut shape wound uniformly with many turns of wire.)

*Is this statement true or false?*

**ANSWER:** Answer not displayed

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### Magnetic Field inside a Very Long Solenoid

**Learning Goal:** To apply Ampère’s law to find the magnetic field inside an infinite solenoid.

In this problem we will apply Ampère’s law, written

$$\oint \mathbf{B}(r) \cdot d\mathbf{l} = \mu_0 n I$$

to calculate the magnetic field inside a very long solenoid (only a relatively short segment of the solenoid is shown in the pictures). The solenoid has length $L$, diameter $D$, and $n$ turns per unit length with each carrying current $I$.

It is usual to assume that the component of the current along the $z$-axis is negligible. (This may be assured by winding two layers of closely spaced wires that spiral in opposite directions.)

From symmetry considerations it is possible to show that far from the ends of the solenoid, the magnetic field is axial.

---

**Part A**

Which figure shows the loop that must be used as the Ampèrean loop for finding $\mathbf{B}_m(r)$ for $r$ inside the solenoid?

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http://session.masteringphysics.com/myct/assignmentPrint?assignmentID=1064958  Page 10 of 16
Part A.1  
Choice of path for loop integral  
ANSWER:  
Part not displayed

Part B  
Part not displayed

Part C  
Part not displayed

Part D  
Part not displayed

Part E  
Find \( \mathbf{B}_z(r) \), the \( z \) component of the magnetic field inside the solenoid where Ampère's law applies.  
Express your answer in terms of \( r \), \( \mu_0 \), \( I \), and physical constants such as \( \mu_0 \).  
ANSWER:  
\[ \mathbf{B}_z(r) = \text{Answer not displayed} \]

Part F  
Part not displayed

Part G  
The magnetic field inside a solenoid can be found exactly using Ampère's law only if the solenoid is infinitely long. Otherwise, the Biot-Savart law must be used to find an exact answer. In practice, the field can be determined with very little error by using Ampère's law, as long as certain conditions hold that make the field similar to that in an infinitely long solenoid.  
Which of the following conditions must hold to allow you to use Ampère's law to find a good approximation?  
a. Consider only locations where the distance from the ends is many times \( D \).  
b. Consider any location inside the solenoid, as long as \( D \) is much larger than \( D \) for the solenoid.  
c. Consider only locations along the axis of the solenoid.  

Hint G.1  
Implications of symmetry  
Hint not displayed

Hint G.2  
Off-axis field dependence  
Hint not displayed

ANSWER:  
\[ \text{Answer not displayed} \]

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**Magnetic Field of a Current-Carrying Wire**

Find the magnetic field a distance \( r \) from the center of a long wire that has radius \( a \) and carries a uniform current per unit area \( j \) in the positive \( z \) direction.

Part A  
First find the magnetic field, \( \mathbf{B}_{\text{ext}}(\rho, \phi, z) \), outside the wire (i.e., when the distance \( r \) is greater than \( a \)).

Hint A.1  
Ampère's law with current density  
Hint not displayed

Part A.2  
Find the direction of the field  
Part not displayed
Part A.3 Find the left-hand side of Ampère's law

Part not displayed

Part A.4 Find the right-hand side of Ampère's law

Part not displayed

Express \( \mathbf{\vec{B}}_{\text{left}}(r) \) in terms of the given parameters, the permeability constant \( \mu_0 \), the variables \( \alpha, \beta, \gamma \) (the magnitude of \( \mathbf{\vec{j}}, \mathbf{\vec{r}}, \mathbf{\vec{\phi}}, \mathbf{\vec{z}} \)), and the corresponding unit vectors \( \mathbf{\hat{r}}, \mathbf{\hat{\phi}}, \) and \( \mathbf{\hat{z}} \). You may not need all these in your answer.

ANSWER: \( \mathbf{\vec{B}}_{\text{left}}(r) = \text{Answer not displayed} \)

Part B

Now find the magnetic field \( \mathbf{\vec{B}}_{\text{right}}(r) \) inside the wire (i.e., when the distance \( r \) is less than \( \alpha \)).

Part B.1 Establish the relationship to Part A

Part not displayed

Part B.2 Integrate over the Ampèrean loop

Part not displayed

Express \( \mathbf{\vec{B}}_{\text{right}}(r) \) in terms of the given parameters, the permeability constant \( \mu_0 \), the distance \( r \) from the center of the wire, and the unit vectors \( \mathbf{\hat{r}}, \mathbf{\hat{\phi}}, \) and \( \mathbf{\hat{z}} \). You may not need all these in your answer.

ANSWER: \( \mathbf{\vec{B}}_{\text{right}}(r) = \text{Answer not displayed} \)

Magnetic Field inside a Toroid

A toroid is a solenoid bent into the shape of a doughnut. It looks similar to a toy Slinky® with ends joined to make a circle. Consider a toroid consisting of \( N \) turns of a single wire with current \( I \) flowing through it.

Consider the toroid to be lying in the \( r \theta \) plane of a cylindrical coordinate system, with the \( z \) axis along the axis of the toroid (pointing out of the screen). Let \( \theta \) represent the angular position around the toroid, and let \( r \) be the distance from the axis of the toroid.

For now, treat the toroid as ideal; that is, ignore the component of the current in the \( \mathbf{\hat{z}} \) direction.

Part A

The magnetic field inside the toroid varies as a function of which parameters?

Hint A.1 Consider rotational symmetry

You were asked to assume that the toroid is rotationally symmetric, so its magnetic field cannot depend on \( \theta \).

ANSWER: • \( r \) only
  • \( \theta \) only
  • both \( r \) and \( \theta \)

Part B

Inside the toroid, in which direction does the magnetic field point?

ANSWER: • \( \mathbf{\hat{\phi}} \) (outward from the center)
  • \( \mathbf{\hat{r}} \) (inward toward the center)
Part C

What is \( B (r) \), the magnitude of the magnetic field inside the toroid and at a distance \( r \) from the axis of the toroid?

Hint C.1  How to approach the problem

Hint not displayed

Part C.2  Evaluate the line integral

Part not displayed

Part C.3  Find \( \ell_{\text{int}} \)

Part not displayed

Express the magnetic field in terms of \( \mu_0 \) (the permeability of free space), \( N \), and \( r \).

ANSWER: \[ B (r) = \frac{\mu_0 I N}{2\pi r} \]

Part D

In an ideal toroid, current would flow only in the \( \hat{\phi} \) and \( \hat{z} \) directions. The magnetic field in the central plane, outside of the coils of such a toroid, is zero. For the toroid shown in the figures however, this field is not quite zero. This is because in this problem, there is a single wire that is wrapped around a doughnut shape. This wire must point somewhat in the \(-\hat{\phi}\) direction, and thus the current must actually have a component in the \(-\hat{z}\) direction.

Compute \( B_z \), the magnitude of the magnetic field in the center of the toroid, that is, on the \( z \) axis in the plane of the toroid. Assume that the toroid has an overall radius of \( R \) (the distance from the center of the toroid to the middle of the wire loops) and that \( R \) is large compared to the diameter \( d \) of the individual turns of the toroid coils.

Note that whether the field points upward or downward depends on the direction of the current, that is, on whether the coil is wound clockwise or counterclockwise.

Hint D.1  Simplifying the problem

For this question, it is useful to consider the Biot-Savart law:

\[ d\vec{B} = \frac{\mu_0 I \times \hat{r}}{4\pi r^2} \]

Since the question asks only for the \( z \) component of the magnetic field, we need only deal with those portions of \( d\vec{B} \) that are parallel to \( \hat{z} \) (keeping in mind that \( \hat{z} \times \hat{r} = \hat{\phi} \)). In other words, the fact that the wire is wrapped in \( N \) loops of radius \( d \) is irrelevant; only the component of the current flow in the \( \hat{\phi} \) direction is important.

With this simplification, the problem of finding the magnetic field at the center of the toroid becomes equivalent to finding the magnetic field at the center of a single circular loop of wire!

Express \( \vec{B} \) in terms of \( \mu_0 \), \( B \), \( I \), \( N \), and the local diameter \( d \) of the coils.

ANSWER: \[ B = \frac{\mu_0 I}{2R} \]

This is the same expression that you would derive for the magnetic field at the center of a circular loop of current-carrying wire. To see why this makes sense, imagine that the local diameter \( d \) of the coils gets so small that it is negligible in comparison to the radius of the toroid. The wire makes one complete turn around the axis of the toroid. So, to a point in the center, the toroid looks like a simple current loop.

Current Sheet

Consider an infinite sheet of parallel wires. The sheet lies in the xy plane. A current \( I \) runs in the -y direction through each wire. There are \( N/\alpha \) wires per unit length in the x direction.
Part A
Write an expression for \( \mathbf{B}(z) \), the magnetic field a distance \( z \) above the \( xy \) plane of the sheet.

Use \( \mu_0 \) for the permeability of free space.

Hint A.1 How to approach the problem

Hint not displayed

Part A.2 Find \( I_{\text{aux}} \)

Part not displayed

Part A.3 Determine the direction of the magnetic field

Part not displayed

Hint A.4 Magnitude of the magnetic field

Hint not displayed

Part A.5 Evaluate \( \int \mathbf{B} \cdot d\mathbf{l} \)

Part not displayed

Express the magnetic field as a vector in terms of any or all of the following: \( \mathbf{B} \), \( \mathbf{I} \), \( \mathbf{N} \), \( \mathbf{A} \), \( \mu_0 \) and the unit vectors \( \hat{x} \), \( \hat{y} \), and/or \( \hat{z} \).

ANSWER: \( \mathbf{B}(z) = \text{Answer not displayed} \)

Magnetic Fields in Matter

Magnetic Materials

Part A
You are given a material which produces no initial magnetic field when in free space. When it is placed in a region of uniform magnetic field, the material produces an additional internal magnetic field parallel to the original field. However, this induced magnetic field disappears when the external field is removed.

What type of magnetism does this material exhibit?

ANSWER: 

- diamagnetism
- paramagnetism
- ferromagnetism

When a paramagnetic material is placed in a magnetic field, the field helps align the magnetic moments of the atoms. This produces a magnetic field in the material that is parallel to the applied field.

Part B
Once again, you are given an unknown material that initially generates no magnetic field. When this material is placed in a magnetic field, it produces a strong internal magnetic field, parallel to the external magnetic field. This field is found to remain even after the external magnetic field is removed.

Your material is which of the following?

ANSWER: 

- diamagnetic
- paramagnetic
- ferromagnetic

Very good! Materials that exhibit a magnetic field even after an external magnetic field is removed are called ferromagnetic materials. Iron and nickel are the most common ferromagnetic elements, but the strongest permanent magnets are made from alloys that contain rare earth elements as well.

Part C
What type of magnetism is characteristic of most materials?

ANSWER: 

- ferromagnetism
- paramagnetism
- diamagnetism
- no magnetism

Almost all materials exhibit diamagnetism to some degree, even materials that also exhibit paramagnetism or ferromagnetism. This is because a magnetic moment can be induced in most common atoms when the atom is placed in a magnetic field. This induced magnetic moment is in a direction opposite to the external magnetic field. The addition of all of these weak magnetic moments gives the material a very weak magnetic field overall. This field disappears when the external magnetic field is removed. The effect of diamagnetism is often masked in paramagnetic or ferromagnetic materials, whose constituent atoms or molecules have permanent magnetic moments and a strong tendency to align in the same direction as the external magnetic field.

Forces between a Charge and a Bar Magnet

Learning Goal: To understand the forces between a bar magnet and 1. a stationary charge, 2. a moving charge, and 3. a ferromagnetic object.

A bar magnet oriented along the \( y \) axis can rotate about an axis parallel to the \( z \) axis. Its north pole initially points along \( \hat{y} \).

Interaction of stationary charge and bar magnet

A positive charge is displaced some distance in the \( \hat{y} \) direction from the magnet.
Assume that no charges are induced on the magnet.

Part A
Assume that the length of the magnet is much smaller than the separation between it and the charge. As a result of magnetic interaction (i.e., ignore pure Coulomb forces) between the charge and the bar magnet, the magnet will experience which of the following?

ANSWER: Answer not displayed

Interaction of moving charge and bar magnet
Consider a second case in which the charge is again some distance in the direction from the magnet, but now it is moving toward the center of the bar magnet, that is, with its velocity along $\vec{v}$.

Part B
Due to its motion in the magnetic field of the bar magnet, the charge will experience a force in which direction?

Part B.1 Determine the magnetic field direction near a charge
Part not displayed

Part B.2 Determine the direction of force on a charge moving in a magnetic field
Part not displayed

ANSWER: Answer not displayed

Interaction of iron and bar magnet
Now the charge is replaced by an electrically neutral piece of initially unmagnetized soft iron (for example, a nail) that is not moving.

Part C
As a result of the magnetic interaction between the soft iron and the bar magnet, which of the following will occur?

Hint C.1 Magnetic induction
Hint not displayed

ANSWER: Answer not displayed

Summary
5 of 13 problems complete (28.67% avg. score)
17.3 of 22 points