Basics of spherical mirrors followed by increasing complex problems

Understanding Spherical Mirrors

**Learning Goal:** To be able to calculate locations and heights of images formed by spherical mirrors, as well as to understand the qualities of such an image.

In this problem, you will learn to use the spherical mirror equation:

\[
\frac{1}{s} + \frac{1}{s'} = \frac{1}{f}
\]

This equation relates three quantities important to the formation of images with a spherical mirror: The object distance \( s \) is the distance from the mirror to the object, along the axis of the mirror. The image distance \( s' \) is the distance from the mirror to the image, along the axis of the mirror. The focal length \( f \) is an intrinsic property of the mirror. It is equal to half the radius of curvature \( R \) (i.e., \( f = R/2 \)).

The equation given above allows you to calculate the locations of images and objects. Frequently, you will also be interested in the size of an image or object. The ratio of the size of an image to the size of the object is called the magnification; it is given by

\[
m = \frac{y'}{y} = \frac{s'}{s},
\]

where \( y' \) is the height of the image, and \( y \) is the height of the object. The second equality allows you to find the size of the image (or object) with the information provided by the spherical mirror equation.

All of the quantities in the above equations can take both positive and negative values. Positive distances correspond to real images or objects, while negative distances correspond to virtual images or objects. Positive heights correspond to upright images or objects, while negative heights correspond to inverted images or objects. The following table summarizes this information:

<table>
<thead>
<tr>
<th>Positive</th>
<th>Negative</th>
</tr>
</thead>
<tbody>
<tr>
<td>( s ) Real</td>
<td>Virtual</td>
</tr>
<tr>
<td>( s' ) Real</td>
<td>Virtual</td>
</tr>
<tr>
<td>( y' ) Upright</td>
<td>Inverted</td>
</tr>
<tr>
<td>( y ) Upright</td>
<td>Inverted</td>
</tr>
</tbody>
</table>

Focal length can also take positive and negative values. Positive \( f \) corresponds to a concave mirror, while negative \( f \) corresponds to a convex mirror. While it is possible for \( s \) or \( y \) to be negative, this can happen only in situations with multiple mirrors or mirrors and lenses. In this problem, you will consider only a single mirror, so \( s \) and \( y \) will be positive.

You will begin with a relatively standard calculation. Consider a concave spherical mirror with a radius of curvature equal to 60.0 centimeters. An object 6.00 centimeters tall is placed along the axis of the mirror, 45.0 centimeters from the mirror. You are to find the location and height of the image.

**Part A**
What is the focal length \( f \) of this mirror?

Express your answer in centimeters to three significant figures.

**ANSWER:** \( f = \text{Answer not displayed} \) cm

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**Part B**

Now use the spherical mirror equation to find the image distance \( s' \).

Express your answer in centimeters, as a fraction or to three significant figures.

**ANSWER:** \( s' = \text{Answer not displayed} \) cm

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**Part C**

Find the magnification \( m \), using \( s \) and \( s' \).

Express your answer numerically, as a fraction or to three significant figures.

**ANSWER:** \( m = \text{Answer not displayed} \)

---

**Part D**

Finally, use the magnification to find the height of the image \( y' \).

Express your answer in centimeters, as a fraction or to three significant figures.

**ANSWER:** \( y' = \text{Answer not displayed} \) cm

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**Part E**

Look at the signs of your answers to determine which of the following describes the image formed by this mirror:

**ANSWER:** \( \text{Answer not displayed} \)

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Next, you will manipulate the spherical mirror equation to explore the qualities (real/virtual and upright/inverted) of some images.

**Part F**

Solve the spherical mirror equation for \( s' \).

Express your answer in terms of \( f \) and \( s \).

**ANSWER:** \( s' = \text{Answer not displayed} \)

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**Part G**

Part not displayed

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**Part H**

Part not displayed

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**Part I**

Part not displayed

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As a spherical mirror becomes flatter, the radius of curvature \( R \) gets larger. Notice that as \( R \) goes to infinity, so does \( f \), because \( f = R/2 \). Thus, as \( R \) gets larger, \( 1/f \) gets smaller. In the limit where you allow \( R \) to go to infinity, \( 1/f \) becomes zero. Therefore, if you could construct a mirror with an infinitely large radius of curvature, it would obey the equation \( 1/a + 1/s = 0 \).
Part J
What is the value of \( s \) obtained from this new equation?
Express your answer in terms of \( s \).
\[ s' = \text{Answer not displayed} \]

Spherical Mirror 1
You wish to create an image that is 10 meters from an object. This image is to be inverted and half the height of the object. You wish to accomplish this using one spherical mirror.

Part A
What is the focal length \( f \) of the mirror that would accomplish this?

Part A.1 Find the focal length
\[ \text{Part not displayed} \]

Part A.2 Find the magnification
\[ \text{Part not displayed} \]

Part A.3 Use the magnification to find \( s_o \) and \( s_i \)
\[ \text{Part not displayed} \]
Express your answer in meters, as a fraction or to three significant figures.
\[ f = \text{Answer not displayed} \]

Part B
\[ \text{Part not displayed} \]

Part C
\[ \text{Part not displayed} \]

Part D
\[ \text{Part not displayed} \]

Spherical Mirror 2
You wish to create an image that is 10 meters from an object. This image is to be upright and half the height of the object. You wish to accomplish this using one spherical mirror.

Part A
What is the focal length \( f \) of the mirror that would accomplish this?

Part A.1 Find the focal length
\[ \text{Part not displayed} \]

Part A.2 Find the magnification
\[ \text{Part not displayed} \]

Part A.3 Use the magnification to find \( s_o \) and \( s_i \)
\[ \text{Part not displayed} \]
Express your answer in meters, as a fraction or to three significant figures.
A guy named Joe, who is 1.6 meters tall, enters a room in which someone has placed a large convex mirror with radius of curvature $R$ equal to 30 meters. The mirror has been cut in half, so that the axis of the mirror is at ground level. As Joe enters the room, he is 5 meters in front of the mirror, but he is looking the other way, so he fails to see it. When he turns around, he is startled by his own image in the mirror.

**Image Size in a Mirror**

**Part A**
How far away does the image appear to Joe?

**Part A.1 Find the focal length**
What is the focal length $f$ of this mirror? Recall that $f = \frac{R}{2}$. Consider whether a convex mirror has positive or negative focal length.

Express your answer in meters, to three significant figures.

ANSWER: $f = -15 \text{ m}$

**Hint A.2 Finding the distance from Joe to his image**
Once you have the value for the image distance $s_i$, think about which side of the mirror the image must be formed on to have the sign that it has. Then, think about whether your should add or subtract the image distance from Joe's distance (object distance $s_o$) to obtain the distance from Joe to his image.

Express your answer in meters, to three significant figures or as a fraction.

ANSWER: $8.75 \text{ m}$

**Part B**
What is the image height $h_i$ that Joe sees in the mirror?

**Hint B.1 Finding the image height**
Remember that there are two definitions of magnification: $m = \frac{h_i}{h_o}$ and $m = \frac{s_i}{s_o}$. Using these together, you can find the height of Joe's image.

Express your answer in meters, to three significant figures or as a fraction.
Joe is so startled by his image that he falls forward.

**Part C**

Now what is the length (i.e., the distance from head to toe) of Joe's image?

**Hint C.1 Finding the image's length**

Where is the image of Joe's head? Where is the image of Joe's feet? The distance between these two images is how long Joe's image appears.

**Express your answer in meters, to three significant figures or as a fraction.**

**ANSWER:** $0.9783$ m

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**Understanding Lenses**

**Learning Goal:** To learn the quantitative use of the lens equation, as well as how to determine qualitative properties of solutions.

In working with lenses, there are three important quantities to consider: The object distance $s$ is the distance along the axis of the lens to the object. The image distance $s'$ is the distance along the axis of the lens to the image. The focal length $f$ is an intrinsic property of the lens. These three quantities are related through the equation

$$\frac{1}{s} + \frac{1}{s'} = \frac{1}{f}$$

Note that this equation is valid only for thin, spherical lenses. Unless otherwise specified, a lens problem always assumes that you are using thin, spherical lenses.

The equation above allows you to calculate the locations of images and objects. Frequently, you will also be interested in the size of the image or object, particularly if you are considering a magnifying glass or microscope. The ratio of the size of an image to the size of the object is called the magnification. It is given by

$$m = \frac{y'}{y} = \frac{s'}{s}$$

where $y'$ is the height of the image and $y$ is the height of the object. The second equality allows you to find the size of the image (or object) with the information provided by the thin lens equation.

All of the quantities in the above equations can take both positive and negative values. Positive distances correspond to real images or objects, while negative distances correspond to virtual images or objects. Positive heights correspond to upright images or objects, while negative heights correspond to inverted images or objects. The following table summarizes these properties:
The focal length $f$ can also be positive or negative. A positive focal length corresponds to a converging lens, while a negative focal length corresponds to a diverging lens.

Consider an object with $s = 12 \text{ cm}$ that produces an image with $s' = 15 \text{ cm}$. Note that whenever you are working with a physical object, the object distance will be positive (in multiple optics setups, you will encounter "objects" that are actually images, but that is not a possibility in this problem). A positive image distance means that the image is formed on the side of the lens from which the light emerges.

**Part A**
Find the focal length of the lens that produces the image described in the problem introduction using the thin lens equation.
Express your answer in centimeters, as a fraction or to three significant figures.
**ANSWER:** $f = \text{Answer not displayed} \text{ cm}$

**Part B**
Part not displayed

**Part C**
Part not displayed

**Part D**
Part not displayed

Now consider a diverging lens with focal length $f = -15 \text{ cm}$, producing an upright image that is 5/9 as tall as the object.

**Part E**
Is the image real or virtual? Think about the magnification and how it relates to the sign of $s'$.
**ANSWER:** Answer not displayed

**Part F**
Part not displayed

**Part G**
Part not displayed

A lens placed at the origin with its axis pointing along the $x$ axis produces a real inverted image at $x = -24 \text{ cm}$ that is twice as tall as the object.

**Part H**
What is the image distance?
Express your answer in centimeters, as a fraction or to three significant figures.
**ANSWER:** $s' = \text{Answer not displayed} \text{ cm}$

**Part I**
Ray Tracing in Lenses and Mirrors

We usually locate the image formed by an object by looking for the intersection of just two of the rays coming from that object, and we usually choose these two rays to be as simple to draw as possible. In fact, all the rays coming from the object that pass through the lens must converge at the location of the image. In this problem you will be asked to identify some of these other rays, which are not so simple to draw.

**Part A**
The first figure shows an object to the left of a converging lens. The focal point of the lens is labeled F. Five rays are drawn from the tip of the object to the lens, but only one of them is traced through the lens correctly. Choose the ray that is traced correctly.

**Hint A.1 How to approach this problem**

**ANSWER: Answer not displayed**

**Part B**
The second figure shows an object to the right of a concave mirror. The focal point of the mirror is labeled F. Four rays are shown reflecting off the mirror, but only one of them is drawn correctly. Choose the ray that is traced correctly.

**Hint B.1 How to approach this problem**

**ANSWER: Answer not displayed**

**Laser and Lens**

A laser is mounted at distance $d$ to the left of, and at height $h$ above, the axis of a converging lens with positive focal length $f$. The laser is aimed so that the beam travels parallel to the axis of the lens. A screen is placed at distance $d_t$ to the right of the lens. The laser beam passes through the lens and makes a dot on the screen at height $h_s$, measured upward from the axis of the lens (a positive value means that the dot is above the axis, while a negative value means that it is below).
Part A
Find the height $h_s$ of the dot on the screen.

**Hint A.1 How to approach the problem**
*Hint not displayed*

**Hint A.2 Drawing the refracted ray**
*Hint not displayed*

**Part A.3 Find the slope of the refracted ray**
*Part not displayed*

Express your answer in terms of some or all of the variables $d_s, h_s, f_s,$ and $d_o$.

**ANSWER:** $h_s =$ *Answer not displayed*

Now consider a specific case. Let the laser be 50 centimeters to the left of the lens and at height 15 centimeters above the axis of the lens. The lens has focal length 30 centimeters, and the screen is 1 meter to the right of the lens.

**Part B**
What is the height $h_s$ of the dot on the screen?

Express your answer in centimeters, to two significant figures.

**ANSWER:** $h_s =$ *Answer not displayed* cm

**Part C**
If the laser is moved to the left, farther from the lens, what will happen to the dot on the screen?

**ANSWER:** *Answer not displayed*

**Part D**
If the laser is moved up, farther above the axis of the lens, what will happen to the dot on the screen?

**ANSWER:** *Answer not displayed*

**Part E**
If the laser (in its original position, in Part B) is tilted down, so that the beam strikes the lens less than 15 centimeters above the axis (as shown in the figure), what will happen to the dot on the screen? Note that the equation you developed in Part A applies specifically to a laser beam that is parallel to the lens's axis before striking the lens.
Focusing with the Human Eye

Joe is hiking through the woods when he decides to stop and take in the view. He is particularly interested in three objects: a squirrel sitting on a rock next to him, a tree a few meters away, and a distant mountain. As Joe is taking in the view, he thinks back to what he learned in his physics class about how the human eye works.

Light enters the eye at the curved front surface of the cornea, passes through the lens, and then strikes the retina and fovea on the back of the eye. The cornea and lens together form a compound lens system. The large difference between the index of refraction of air and that of the aqueous humor behind the cornea is responsible for most of the bending of the light rays that enter the eye, but it is the lens that allows our eyes to focus. The ciliary muscles surrounding the lens can be expanded and contracted to change the curvature of the lens, which in turn changes the effective focal length of the cornea-lens system. This in turn changes the location of the image of any object in one's field of view. Images formed on the fovea appear in focus. Images formed between the lens and the fovea appear blurry, as do images formed behind the fovea. Therefore, to focus on some object, you adjust your ciliary muscles until the image of that object is located on the fovea.

Part A
Joe first focuses his attention (and his eyes) on the tree. The focal length of the cornea-lens system in his eye must be __________ the distance between the front and back of his eye.

ANSWER: Answer not displayed

Part B

Part C

Part D

A Convex-Convex Lens

A convex-convex lens has radii of curvature with magnitudes of \( |R_1| = 10\,\text{cm} \) and \( |R_2| = 15\,\text{cm} \). The lens is made of glass with index of refraction \( n_{\text{glass}} = 1.5 \). We will employ the convention that \( R_1 \) refers to the radius of curvature of the surface through which light will enter the lens, and \( R_2 \) refers to the radius of curvature of the surface from which light will exit the lens.
Part A
Is this lens converging or diverging?

ANSWER: Answer not displayed

Part B
What is the focal length \( f \) of this lens in air (index of refraction for air is \( n_{\text{air}} = 1 \))?  

**Hint B.1** The signs of the radii of curvature

**Hint not displayed**

**Hint B.2** The lensmaker's equation

**Hint not displayed**

Express your answer in centimeters to two significant figures or as a fraction.

ANSWER: \( f = \text{Answer not displayed} \) cm

Part C

**Part not displayed**

**Chromatic Aberration**

There are two major types of aberration (defective image formation) in lenses: spherical and chromatic. Spherical aberration comes from the fact that a spherical lens has the property of focusing parallel rays to a single point only approximately. This approximation is better the closer the rays are to the axis of the lens, but there will always be some improper focusing. Chromatic aberration comes from the fact that different wavelengths of light have different indices of refraction in a specific material, a property known as dispersion. This means that a lens has different focal lengths for different wavelengths of light. Consider a lens made to the following specifications: radii of curvature \( r_1 = 20.00 \) centimeters and \( r_2 = -20.00 \) centimeters, and refractive indices for red and blue light of \( n_{\text{red}} = 1.511 \) and \( n_{\text{blue}} = 1.539 \).

Part A
What is \( \Delta f \), the magnitude of the difference in the lens's focal length between red and blue light?

**Hint A.1** How to approach the problem

**Hint not displayed**

**Part A.2** Find the focal length for red light

**Part not displayed**

**Part A.3** Find the focal length for blue light

**Part not displayed**

Express your answer in centimeters, to two significant figures.

ANSWER: \( \Delta f = \text{Answer not displayed} \) cm

**Part B**

**Part not displayed**

**Part C**

**Part not displayed**
Introduction to multiple optics and two increasingly challenging problems

Understanding Multiple Optics

Learning Goal: To become familiar with using the image of one instrument as the object of the next and tracing rays through a system of multiple instruments.

Multiple optics refers to any system of more than one optical instrument through which light passes. Most devices related to optics, such as cameras, microscopes, and telescopes, contain multiple optics systems.

In multiple optics, the image of one optical instrument becomes the object of the next one. Thus, in multiple optics problems, you need to find the image created by the first optical instrument that the rays encounter. Then, you will use that image as the object of the next optical instrument, repeating this pattern until you have followed the rays all the way through the system. It is very important to be alert to the geometry and to signs when you find the object distance for one instrument from the location of the previous instrument’s image. Sometimes, the image is formed on the virtual side of the instrument, leading to a virtual object. This may sound strange, but in practice, its effect on your calculations is simply to make the object distance negative instead of positive.

Several optical instruments are placed along the x axis, with their axes aligned along the x axis. A plane mirror is located at \( x = 0 \). A converging lens with focal length 5.00 m is located at \( x = 12.5 \text{ m} \). An object is placed at \( x = 22.5 \text{ m} \).

In order to find the location of the final image of the object formed by this system, you will need to trace the rays through the system, instrument by instrument. You are strongly advised to draw a picture with the x axis and the location of the lens, mirror, and object marked. Then, as you proceed through the problem, you can mark where each image is located.

Part A
First, find the location of the image created by the lens by itself (as if no other instruments were present).

Hint A.1 The thin lens equation

Express your answer in meters, to three significant figures, or as a fraction.

\[ \text{ANSWER: } x = 2.50 \text{ m} \]

Part B
Next, find the location of the image created by the plane mirror (after the light has passed through the lens).

Hint B.1 The focal length of a plane mirror

Keep in mind that a plane mirror has a focal length of infinity (since a plane mirror has an infinite radius of curvature).

Part B.2 Find the object distance

You know from Part A that the image resulting from the passage of light though the lens is located at \( x = 2.50 \text{ m} \).

If the plane mirror is at \( x = 0 \), what is the object distance \( s \) for the plane mirror?

Express your answer in meters to three significant figures.

\[ \text{ANSWER: } s = 2.50 \text{ m} \]

Express your answer in meters, to three significant figures, or as a fraction.

\[ \text{ANSWER: } x = -2.50 \text{ m} \]

Proceed with caution! The light reflects off of the mirror and back through the lens a second time!

Part C
What is the location of the final image, as seen by an observer looking toward the mirror, through the lens? Keep in mind that the light must pass back through the lens, and thus you must do one more calculation with the thin lens equation.

**Part C.1 Object distance for the lens (second time)**
What is the object distance \( s \) for the lens the second time? Recall that the object is the image made by the mirror. You found its coordinate in the last part, so just subtract that from the coordinate of the lens to find the object distance for the lens the second time.

Express your answer in meters to three significant figures or as a fraction.

**ANSWER:** \( s = 15 \) \( \text{m} \)

Express your answer in meters, to three significant figures, or as a fraction.

**ANSWER:** \( x = 20.0 \) \( \text{m} \)

**Part D**
Is the final image formed by this system real or virtual?

**Hint D.1 Real versus virtual**
A real image is one that forms at a location that the light actually reaches (imagine a slide projector creating an image on a screen). A virtual image is one that forms at a location that the light does not actually reach. (Imagine the image created when you look in a mirror hanging on a wall. No light actually makes it past the wall; there only appears to be a copy of you on the other side of the wall!)

**Hint D.2 Real versus virtual with multiple optics**
Since the type of image is determined only by whether the light actually reaches the location of the image, whether the image is real or virtual depends only on the last optical instrument through which the light passes. Thus, the light may fail to reach an intermediate image (in this example, the one created by the mirror, which was a virtual image) but may still create a real final image, which it will do if the light reaches the location of the final image created by the system.

**ANSWER:** \( \checkmark \) real
\( \checkmark \) virtual

If you have more than one optical instrument, the total magnification is equal to the product of the individual magnifications. Keep in mind that the image of one device becomes the object of the next. If the first device creates an image that is ten times the size of the object, and the second creates an image that is twenty times the size of the first image (which was its object), then the final image will be two hundred times the size of the original object. This makes sense because the second device magnifies further what was already magnified by the first device. To find the magnitude of the magnification of the final image, you will need to consider each instrument and find the individual magnifications.

**Part E**
First, find the magnitude \( m_{\text{first}} \) of the magnification of the image created when light from the object passes through the lens the first time (as if the mirror were not present).

**Hint E.1 Magnification**

**Express your answer to three significant figures or as a fraction.**

**ANSWER:** \( m_{\text{first}} = 1 \)

**Part F**
Next, find the magnitude $m_{\text{mirror}}$ of the magnification of the plane mirror.

Express your answer to three significant figures or as a fraction.

ANSWER: $m_{\text{mirror}} = 1$

Of course, this is expected. Plane mirrors don't magnify your image!

Part G

Now find the magnitude $m_{\text{lens}}$ of the magnification of the image created when light from the object passes through the lens the second time (after reflecting off the mirror).

Express your answer to three significant figures or as a fraction.

ANSWER: $m_{\text{lens}} = 0.5000$

Part H

What is the magnitude of the magnification of the final image?

Express your answer to three significant figures or as a fraction.

ANSWER: $m = 0.5000$

Part I

Is the final image upright or inverted? Give the orientation relative to the original object.

Hint I.1 Magnification in multiple optics (orientation)

If you have more than one optical instrument, the total magnification is just the product of the individual magnifications. The image of one device becomes the object of the next. If, for example, the first device creates an image that is inverted (negative magnification) and the second creates one that is also inverted (negative magnification), then the final image will be upright (positive magnification). Mathematically, this makes sense because the product of two negative numbers is positive. Conceptually, this makes sense because if the first device inverts the image, and the second also inverts the image, then the first device flips the image upside down, and the second flips it back again, resulting in an image that is right-side up.

Part I.2 Find the orientation from the lens (first time)

Is the image created by the lens (as if the mirror were not present) inverted or upright?

ANSWER:  

- upright
- inverted

Part I.3 Find the orientation from the mirror

Is the image created by the mirror (of the image created by the lens) inverted or upright, compared to the image created by the lens?

ANSWER:  

- upright
- inverted

Part I.4 Find the orientation from the lens (second time)

Is the image created by the lens (of the image created by the mirror) inverted or upright, compared to the image created by the mirror?

ANSWER:  

- upright
- inverted
A Two-Lens System

A compound lens system consists of two converging lenses, one at \( x = -20.0 \text{cm} \) with focal length \( f_1 = +19.0 \text{cm} \), and the other at \( x = +20.0 \text{cm} \) with focal length \( f_2 = +8.00 \text{cm} \). An object 1.00 centimeter tall is placed at \( x = -50.0 \text{cm} \).

Part A
What is the location of the final image produced by the compound lens system? Give the \( x \) coordinate of the image.

Hint A.1 How to handle multiple optics

Part A.2 Find the object distance for the first lens

Part A.3 Find the image distance from the first lens

Part A.4 Find the object distance for the second lens

Part A.5 Find the image distance from the second lens

Express your answer in centimeters, to three significant figures or as a fraction.

ANSWER: \( x = \text{Answer not displayed} \text{ cm} \)

Part B

Part C

Part not displayed

Now remove the two lenses at \( x = +20.0 \text{cm} \) and \( x = -20.0 \text{cm} \) and replace them with a single lens of focal length \( f_1 \) at \( x = 6 \). We want to choose this new lens so that it produces an image at the same location as before.

Part D
What is the focal length of the new lens at the origin?

Part D.1 Find the object distance for the third lens

Part not displayed
Part D.2 Find the image distance for the third lens

Express your answer in centimeters, to three significant figures or as a fraction.

ANSWER: \( f_s = \text{Answer not displayed} \)

Part E

Multiple Optics with Plane Mirror - Virtual Objects

A plane mirror is located at the origin. A converging lens with focal length 5.00 m is located at \( x = 1.00 \) m. An object is placed at \( x = 31.0 \) m.

Part A

What is the location of the final image, as seen by an observer looking toward the mirror, through the lens?

Hint A.1 Multiple Optics

Part A.2 Image due to lens

Part A.3 Image formed by plane mirror

Hint A.4 Image formed by lens (after reflection)

Express your answer in meters, to three significant figures.

ANSWER: \( x = \text{Answer not displayed} \)

Part B

Part C

Part D

Summary

2 of 13 problems completed (15.14% avg. score)
9.86 of 10 points