Comparison of energy balance and degree-day models of summer ablation on the Langjökull ice cap, SW-Iceland

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Abstract — Physical and empirical degree-day models are presented which simulate melting of the Langjökull ice cap in SW-Iceland, during the ablation seasons 2001 to 2005. The models were derived and evaluated using micro-meteorological observations on the glacier and tested with mass balance observations. The observed daily melt rates were successfully simulated by energy balance calculations based on meteorological observations on the glacier. Air temperatures in the low-albedo surroundings of the glacier represent daily variations in the incoming radiation flux better than the damped boundary layer temperature above the melting glacier. Comparison of the empirical degree-day and physical energy balance models indicates that for a regional temperature change less than 3 °C, the empirical degree-day model may provide realistic predictions of changes in glacier ablation.

INTRODUCTION

In order to describe the relationship between climate and the mass balance of glaciers, various physical and empirical models have been presented that are based on measurements obtained either on or away from glaciers. The physical models provide direct estimates of the energy balance components but they are often impractical due to lack of observations of weather parameters and surface characteristics on the glaciers (e.g. Lang, 1968; Van de Wal, 1996). Empirical models describe statistical relations between melting and weather parameters. They are often based on temperature alone that is easily observed and has often been recorded over decades. The temperature observations are either away from the glacier (e.g. Jóhannesson et al., 1995), or within the glacier boundary layer (e.g. Braithwaite, 1995a).

The present paper is a contribution to the discussion on melt models, presenting both physical budget calculations of energy and empirical degree-day models describing melting. The experimental site is Hagafellsjökull, an outlet glacier of the Langjökull ice cap in SW-Iceland (Figure 1). Radiation components were measured directly in situ, and turbulent fluxes were calculated using wind, air temperature and humidity measured in the boundary layer. We examine whether temperature records within the glacier boundary layer or at ice free areas away from the glacier signify better changes in glacial melting. Based on physical models of the energy balance we test the performance of degree-day melt models and their applicability for predicting changes in melt rates in response to changes in regional temperatures. The degree-day models are also tested against mass balance observations at stakes that have not been used for model calibration.

LOCATION AND OBSERVATIONS

Langjökull is the second largest ice cap in Iceland (925 km² in area), located in Southwest Iceland at ~64.7° N and ~20.4° W (Figure 1). Elevations on the glacier surface range from 450 to 1450 m above sea...
level, and average around 900 m. The average altitude of the equilibrium line is about 1100 m at the southern outlets and 1200 m at the northern ones. Two major rivers drain from the ice cap, Hvítá in Borgarfjörður and Hvítá in Árnessýsla, although a significant part of the glacial meltwater drains directly into groundwater reservoirs (Sigurðsson, 1990).

Measurements of summer and winter balance have been conducted annually at 23 locations on Langjökull since 1996, supplemented by glaciometeorological observations that were initiated in 2001. Each year in April-May, cores for measuring snow density have been drilled through the winter layer. The summer balance has been measured in
September-October using stakes and wires drilled into the glacier in April or May (Björnsson et al., 1998; 2003).

Air temperature has been recorded since 2001 at two automatic weather stations (AWSs) situated in the surroundings of the ice cap (28 and 11 km away from the glacier): S300 at 300 m a. s. l. on Söðulhólar, and S475 at 475 m a. s. l. north of the mountain Skjaldbreiður (Figure 1). The surroundings of these sites comprise lava, sand, and glacial lakes.

Our glacio-meteorological study was carried out on the south-facing outlet Hagafellsjökull (Figure 1). Mass balance measurements were done at 8 points evenly distributed along a profile ranging from 500 to 1450 m a. s. l. One AWS (at G500) was located in the ablation area at 500 m a. s. l. close to the glacier terminus, and another (G1100) close to the equilibrium line, at 1100 m a. s. l. The AWSs were operated from April to October during each year of observations, fully covering the ablation season.

The AWSs measured each 10 minutes the incoming ($Q_i$) and reflected ($Q_o$) solar radiation, incoming ($I_i$) and outgoing ($I_o$) long-wave radiation, wind speed ($u$), wind direction ($WD$), air temperature ($T$), and relative humidity ($r$) at 2 m above the surface (Figure 1 and Table 1). The surface elevation changes due to melting ($d$) were measured with a 30 minutes interval by a sonic echo sounder. The meteorological instruments were mounted on a mast that followed the melting surface, but the sonic echo sounder on a mast drilled several metres into the glacier (Figure 1). Artificial ventilation of the temperature and humidity sensors (Vaisala HMP35 in Table 1) was not required due to the enduring wind blowing on the glacier. This effective natural ventilation has been confirmed by several experiments both on the Vatnajökull glacier (Figure 1) and during calibration of instruments off the glacier.

The instruments were calibrated in Reykjavík in April and the beginning of September each year. The AWSs were visited regularly to ensure that they were functioning properly and to lower the sonic echo sounder. There are gaps in the data at both AWSs from 21 May to 29 May 2001 because the memory capacity was exceeded. The sonic echo sounders failed to record the melting rate continuously after 26 June 2001 at G1100 and after 11 August 2001 at G500, and the whole summer 2004 at both stations, but the cumulative melting during those periods was measured (Figure 2).

Air pressure was not measured on the glacier but estimated at an elevation $h$ from synoptic observations at meteorological stations of the Icelandic Meteorological Office, according to the empirical formula:

$$P(h) = P(h_0) \left(1 - \frac{0.0065(h - h_0)}{T(h_0) + T_0}\right)^{5.25}$$

where $P(h_0)$ and $T(h_0)$ are the air pressure and air temperature observed at a station at elevation $h_0$, and $T_0 = 273.15$ K (e. g. Wallace and Hobbs, 1977). This relationship has been applied successfully at various locations on and around Vatnajökull. Vapour pressure was calculated as $e = r \cdot e_s/100$, where $r$ is the measured relative humidity and

$$e_s = 611.213 \exp \left(17.5043 \frac{T}{T + 241.2}\right)$$

Table 1: Instruments and accuracy. Parameters are defined in the main text. – Nákvæmni mæltækja.

<table>
<thead>
<tr>
<th>Observation</th>
<th>Equipment</th>
<th>Accuracy</th>
<th>Operating range</th>
</tr>
</thead>
<tbody>
<tr>
<td>$T, r$</td>
<td>Vaisala HMP35</td>
<td>0.2°C, 2%</td>
<td>–</td>
</tr>
<tr>
<td>$WD, u$</td>
<td>R.M. Young</td>
<td>5°, 0.1 m s$^{-1}$</td>
<td>–</td>
</tr>
<tr>
<td>$Q, I$</td>
<td>Kipp &amp; Zonen CNR1</td>
<td>3%, 3%</td>
<td>0.3–2.8 µm, 5–50 µm</td>
</tr>
<tr>
<td>$d$</td>
<td>SR50</td>
<td>max(1 cm, 0.4%)</td>
<td>–</td>
</tr>
</tbody>
</table>
is a formulation of the saturation vapour pressure in Pa of temperature $T$ (e.g. Buck, 1981; Murray, 1967).

**METHODS**

**Melt energy derived from the sonic echo sounder data**

The observed melt rate derived from daily records of the sonic echo sounder ($a_s$ in m d$^{-1}$ w. eq.) was used to estimate the average daily energy supplied for ablation (in W m$^{-2}$), described as

$$M_m = L_l \cdot \rho \cdot f \cdot a_s \quad (3)$$

where $\rho = 10^3$ kg m$^{-3}$ is the density of water, $L_l = 3.3 \cdot 10^5$ J kg$^{-1}$ is the specific latent heat of melting and $f = 1/86400$ d s$^{-1}$.

**Physical energy budget model**

The energy budget on the melting surface of the glacier can be written as

$$M_c = R + H_d + H_l \quad (4)$$

where $R = Q_i - Q_o + I_i - I_o = Q_i(1 - \alpha) + I_i - I_o$ is the net radiation depending on the incoming solar radiation, surface albedo ($\alpha$) and the long-wave radiation balance, while $H_d$ and $H_l$ represent the vertical turbulent fluxes of sensible and latent heat, respectively. The heat supplied by rain is assumed to be negligible, as well as the sub-surface heat flux which is appropriate under melt conditions. The water equivalent (in m d$^{-1}$) of the daily energy budget (in W m$^{-2}$) is calculated as

$$a_s = \begin{cases} \frac{M_c}{L_l \cdot \rho \cdot f} & M_c \geq 0 \\ 0 & \text{otherwise} \end{cases} \quad (5)$$

hereafter referred to as EBM. The effect of sub-surface heat transport and refreezing is omitted.

Radiation components were measured directly and the turbulent energy exchange calculated from hourly mean values of the wind, temperature and relative humidity measured in the boundary layer of the glacier. The Monin-Obukhov model can be adapted for the single-level measurements as (e.g. Munro, 1989):
Here, \( \beta \) is the von Kármán constant, \( c_p \) is the specific heat capacity of air under constant pressure, \( L_v = 2.5 \times 10^6 \) J kg\(^{-1}\) K\(^{-1}\) is the specific latent heat of evaporation. The density of the air is included as \( \rho_1 = \rho_0(P/P_0) \), in which \( \rho_0 = 1.29 \) kg m\(^{-3}\), \( P_0 = 1.013 \times 10^5 \) Pa, and \( P \) is the air pressure in Pa, estimated with Eq. 1. The Monin-Obukhov length (e. g. Munro, 1989; Björnsen, 1972) is expressed for one-level measurements and when \( z >> z_0 \) as

\[
L = -A + \frac{1}{B}
\]

assuming \( A = \beta z/(\ln(z) - \ln(z_0)) \) and \( B = (g/T_0)(T(z)/u^2(z))(\ln(z) - \ln(z_0)) \), where \( g = 9.8 \) m s\(^{-2}\) is the acceleration of gravity. Published values for the empirical stability correction constant \( \beta \) are typically \( \sim 5 \) to 8 (e. g. Dyer, 1974; Högström, 1988, 1996), but variations within this range only cause a small uncertainty in the calculated sensible heat flux (Munro, 1989). This is supported by our energy balance calculations that show the same to be relevant for the latent heat flux. Here, \( \beta = 7 \) was chosen as a practical approximation for both \( H_d \) and \( H_l \).

We assume that the \( z_0 \)-values in Table 2, constant with time and only varying with the surface type, are appropriate for the presented study. The values are in a close agreement with more accurately estimated \( z_0 \)-values of Brock et al. (2006), derived by microtopographic and wind profile measurements at the Haut Glacier d’Arolla, Switzerland. Typically \( z_0 \) is in the range of 1–10 mm for glacier firn and ice (e. g. Björnsen, 1972; Moore, 1983; Morris, 1989; Greuell and Konzelmann, 1994; Braithwaite, 1995b; Hock and Holmgren, 1996; Brock et al., 2006), but values up to 7–10 cm have been reported for the rough lowermost ablation areas of Vatnajökull ice cap in Iceland (Smeets et al., 1999). Generally, the temporal variation of \( z_0 \) during the ablation season is unclear (e. g. Brock et al., 2006), but Denby and Smeets (2000) did not record any variations in \( z_0 \) for ice over several months on southern Vatnajökull.

**Table 2:** Applied values of surface roughness \((z_0)\). – *Hrjúfleikastuðull jökulyfirborðs.*

<table>
<thead>
<tr>
<th>( z_0 ) (mm)</th>
<th>( \ln(z_0) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>New snow</td>
<td>( \sim 0.1 )</td>
</tr>
<tr>
<td>Melting snow/firn</td>
<td>( \sim 2 )</td>
</tr>
<tr>
<td>Ice in ablation zone</td>
<td>( \sim 10 )</td>
</tr>
</tbody>
</table>

The selected \( z_0 \)-values gave in general a good fit between the derived values of \( M_m \) (Eq. 3) and \( M_c \) (Eq. 4). Varying \( z_0 \) from 1 to 14 mm in our calculations alters the total melting energy at most by 3% at G1100, and by 7% at G500 when changing \( z_0 \) from 1 to 7 cm. The high consistency between the derived \( M_c \) and \( M_m \) values (e. g. Figure 2b) indicates that the sonic echo sounder satisfactorily describes the cumulative daily melting rates despite the rather high uncertainty of the sonic echo sounder (Table 1). Up to 95% of the daily variation in \( M_m \) is described by \( M_c \), and the standard deviation of the difference between daily values of \( M_c \) and \( M_m \) is 33 and 20 W m\(^{-2}\) at
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G500 and G1100, respectively, which is well within the errors of ~35 W m⁻² of $M_a$ and 40 W m⁻² of $M_m$, calculated by using the instrument errors given in Table 1.

**Empirical ablation models**

Two degree-day models were used to relate the specific ablation rate ($a_s$ in m d⁻¹ w.eq.) at an elevation $h_G$ on the glacier to degree-days, either as observed on the glacier (DDM1, Eq. 9) or estimated at a base station outside the glacier (DDM2, Eq. 10):

$$a_s = \frac{d d f_1 \sum_{t_1}^{t_2} T_{G}^{+}}{t_1} \quad (9)$$

$$a_s = \frac{d d f_2 \sum_{t_1}^{t_2} (T_S - \gamma(h_G - h_S))^+}{t_1} \quad (10)$$

The sums are computed over the period from day $t_1$ to $t_2$ of the ablation season, $d d f_1$ and $d d f_2$ are degree-day factors that remain constant with time but are different for snow and ice/firn. The number of degree-days calculated for each day is the mean daily temperature above the melting point, 0 °C, $^+$ stands for degree-days over a threshold of 0 °C, $T_G$ is the observed temperature 2 m above the surface at elevation $h_G$ on the glacier, $T_S$ is the observed temperature at a weather station of elevation $h_S$ away from the glacier (here S475) and the constant $\gamma = 0.6 \times 10^{-2}$ °C m⁻¹ approximates the adiabatic lapse rate. The temperature $T_G$ is damped by energy exchange processes near the melting ice surface but $(T_S - \gamma(h_G - h_S))$ is typically representative for the atmosphere temperature at an elevation $h_G$ above the off-glacier weather station S474, and not influenced by a melting ice surface. Hence, $(T_S - \gamma(h_G - h_S))$ is not an estimate of the damped boundary layer temperature $T_G$, but rather an estimate of the temperature at height $h_G$ in the free atmosphere surrounding the glacier.

The $d d f$-parameters of DDM1 and DDM2 (Table 3) were scaled to fit the water equivalent of the daily energy supplied for melting (Eq. 5), using: a) a combined 2001–2005 energy budget calculations at the two AWSs (G500 and G1100), and b) the 2004 energy budget at three mass balance stakes at 700 to 1000 m a. s. l. (Figure 1), inferred by assuming the parameters $T_G$, $\alpha$, $Q_i$, and $I_i$ of Eqs. (4–7) to vary linearly with elevation between the observation sites G500 and G1100, and setting $I_o$ equal to 315 W m⁻² (for a melting surface) and $Q_o = Q_i \cdot \alpha$ with albedo ($\alpha$) estimated by combining information from the stake observed winter balance ($b_W$; for snow/ice transition), the observed daily albedo at the two AWS sites and three optical SPOT5 satellite images (5x5 m spatial resolution) acquired on August 12, 17 and 19, 2004.

When fitting the $d d f$-parameters, the energy balance data-sets were divided into periods with melting of snow and ice/firn, respectively. The timing for the exposure of ice/firm was estimated when an abrupt reduction in albedo took place as the summer surface of the previous year was exposed, as well as considering the melting needed to remove the measured winter accumulation ($b_W$). Albedo changes due to new snow that was deposited on an ice/firm surface and melted was also easily detected from the albedo profiles. The uncertainty of the $d d f$-parameters was estimated at G1100 and G500 (Table 3) as one standard deviation of the annual variance of $d d f$-parameters fitted separately to each year.

**RESULTS**

**Energy budget during the ablation seasons 2001 to 2005**

The ablation seasons under investigation started at the end of April/beginning of May and terminated in September/early October.

Net radiation was typically the main contributor to the total energy supplied for melting during the months of June through August but was equalled or surpassed by turbulent fluxes during occasional spells of high temperatures and strong winds (Figure 3a-b). Throughout the ablation seasons, the albedo and the global radiation were the main factors determining the net radiation (Figures 3c-d) as the long-wave net radiation was fairly constant; slightly negative, with radiation emitted from the melting glacier hovering around 315 W m⁻² (Figure 3c). Daily variations in the energy budget were, however, generally highly related to turbulent eddy fluxes (Table 4), especially during the
Figure 3. Energy budget at G500 during the summer 2001 (a), in comparison with weather parameters (b-c) and albedo (d), displayed as three-day moving averages. The error is estimated as ±20 W m⁻² for $M_c$, ±7 W m⁻² for $R$ and ±19 W m⁻² for $H_d + H_l$. – Veður- og orkuþættir í 500 m y. s.

Table 3: Degree-day factors of the models DDM1 and DDM2 (locations of stakes are in Figure 1). After melting through the winter snow layer, the surface at 1100 m (around the ELA) consists of firn, and ice at 500–1000 m. The errors are given as one standard deviation of estimated annual variance of the $ddf$-parameters. – Stuðlar í reynslubundnum líkönum.

<table>
<thead>
<tr>
<th>AWS/stake</th>
<th>Elevation (m a. s. l.)</th>
<th>$ddf_1$ mm °C⁻¹</th>
<th>$ddf_2$ mm °C⁻¹</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>snow</td>
<td>firn/ice</td>
</tr>
<tr>
<td>G1100: Stake &amp; AWS</td>
<td>1100</td>
<td>10.8 ± 0.9</td>
<td>11.5 ± 0.4</td>
</tr>
<tr>
<td>Stake</td>
<td>1000</td>
<td>10.9</td>
<td>13.5</td>
</tr>
<tr>
<td>Stake</td>
<td>900</td>
<td>11.1</td>
<td>12.3</td>
</tr>
<tr>
<td>Stake</td>
<td>700</td>
<td>10.0</td>
<td>11.3</td>
</tr>
<tr>
<td>G500: Stake &amp; AWS</td>
<td>500</td>
<td>8.0 ± 1.6</td>
<td>11.1 ± 1.2</td>
</tr>
</tbody>
</table>
latter part of the ablation season (Figure 4d). Our data include two years with high melting rates in September related to spells of strong wind speeds and high temperatures (Figure 3).

In May and early June, temperatures typically fluctuated close to 0 °C at G1100, but some ablation was caused by solar radiation penetrating into the glacier surface despite the high albedo. During the summers 2001–2005, the albedo declined rapidly at the lower station (G500), until reaching an approximately constant value of 0.07 (Figure 3d) between May 20 to June 11, depending on the year, when the ice surface was exposed.

It is evident from our field work that the low albedo was due to sand and dirt blown over the surface that reached only to an elevation ∼100 m above the
Langjökull, energy balance and degree-day models

station according to the 2004 SPOT5 images. Albedo below 0.1 has also been observed at glacier outlets of the nearby Vatnajökull ice cap (e.g. Lister, 1959; Reijmer et al., 1999; Guðmundsson et al., 2005), but is in that case mainly related to exposed volcanic tephra. At G1100, albedo declined gradually through the ablation seasons from 0.8–0.9 to 0.5–0.6, but jumped rapidly down to 0.4 when the previous year’s summer surface was exposed around the middle of August 2001 and 2003 and the beginning of September 2004. Due to increased turbulent fluxes and reduced solar radiation, the relative contribution of net radiation decreased steadily throughout the summers in spite of the declining albedo. Frequent passage of low-pressure systems over Iceland results in September being windier than the summer months of June to August. High melt rates were produced by eddy fluxes during the warm September months of 2001 and 2002 (Figure 3a-b).

All the energy components supplied to the melting increased downglacier (Figure 5), the net long-wave radiation due to higher cloud cover and the net short-wave radiation as a result of lower albedo. Turbulent fluxes were maintained by down slope glacier winds and high air temperatures; wind directional constancy was 0.6 (in G500) and 0.5 (in G1100) where 0 means wind blowing equally from all directions and 1 wind solely down the steepest slope (e.g. Björnsson et al., 2005). The relative contribution of the net radiation components to melting was on average ~60–70% in the ablation area (Figure 5b).

The performance of degree-day models

Daily melting was calculated with the degree-models (DDM1 and DDM2) at stakes and the two weather stations on the Hagafellsjökull outlet 2001–2005 (Figure 1), using the dfdf-parameters in Table 3. Although not describing the physical melt processes, the degree-day models simulated annual and seasonal variations in the ablation at G500 and G1100 reasonably (Figure 6). The most successful degree-day predictions were obtained by applying temperature observations away from the glacier (Tₛ) using the constant wet adiabatic lapse rate (γ), rather than temperatures observed on the glacier itself (T₇). Lang (1968) concluded the same by investigating a small drainage basin of the Aletschgletscher glacier, Switzerland. This is shown in our data by correlations and residuals of both daily melting rates obtained with Eq. 5 (Table 4 and Figure 4c) and annual summer balance (Figure 6d-e). This
Table 4: Correlation of calculated daily values of total energy with during days of melting ($M_c \geq 0$) from 2001–2005, i.e. correlation of EBM with DDM1, DDM2, $T_G$ the temperature on the glacier, $T_S$ the temperature at S475, net radiation $R$ and eddy fluxes $H_d + H_l$. – Fylgni reynslubundinna likana og orkuþátta við heildar leysingarorku.

<table>
<thead>
<tr>
<th>Elevation</th>
<th>DDM1</th>
<th>DDM2</th>
<th>$T_G$</th>
<th>$T_S$</th>
<th>$R$</th>
<th>$H_d + H_l$</th>
</tr>
</thead>
<tbody>
<tr>
<td>G1100</td>
<td>1100</td>
<td>0.84</td>
<td>0.87</td>
<td>0.79</td>
<td>0.80</td>
<td>0.63</td>
</tr>
<tr>
<td>G500</td>
<td>500</td>
<td>0.79</td>
<td>0.89</td>
<td>0.67</td>
<td>0.84</td>
<td>0.78</td>
</tr>
</tbody>
</table>

Figure 6. (a): Daily ablation at G1100 from 2001 to 2005, calculated with EBM and DDM2. (b-c): Summer melting from stake measurements and calculated with EBM and DDM2; $M_c \geq 0$ is separated into contribution from $R$ and $H_d + H_l$. The sonic echo sounder was used to correct summer balance measured at stakes ($-b_S$) for snow that falls and melts within the ablation seasons, yielding the total summer melting. (d-e): Correlation ($r_c$) and standard deviation ($\sigma$) between summer balance ($b_S$) derived with the EBM, and DDM1 and DDM2, respectively. – Reynslubundin likón borin saman við orkuþátta í sjálfvirkum veðurstöðvum.
is above all evident during periods when melt was primarily varying with the incoming solar radiation (July in Figure 4), which suggests that temperatures above the low-albedo surface away from the glacier better signify the incoming radiation than the damped boundary layer temperatures over the melting glacier.

Good consistency is indicated between the degree-day models and the total energy supplied for melting when correlating all available daily values at both G1100 and G500 (Table 4 and Figure 6a). However, this consistency varied significantly within the ablation season and values from 0.9 and down to 0.2–0.4 were obtained when the correlation for the daily values was calculated separately for each month (Figure 4c). Thus, the lumped degree-day models are not fully reliable for estimating daily ablation. Typically, the best performance of the degree-day models was during periods when the total energy for melting correlated strongly with the turbulent eddy fluxes (Figure 4c-d). Exceptions to this appeared when eddy fluxes fluctuated due to variations in wind speeds rather than temperatures. Both DDM1 and DDM2 predicted high September melting in 2001 and 2002, however, more accurately on daily basis at G1100 than G500.

The melting calculated with the EBM and degree-day models include snow that falls and melts during the summer. This melting is included in the sonic echo sounder observations, but not in the measured total summer balance at stakes ($b_S$). When using DDM1 and DDM2 to predict the observed summer balance at stakes (Figure 7), the $ddf$-parameters in Table 3 were applied up to 1100 m a. s. l., $ddf_{1snow}$ and $ddf_{2snow}$ at G1100 assumed to be valid within the accumulation area (above 1100 m a. s. l.) and the observed winter balance ($b_W$) was used to identify the transition from snow to ice/firn. The boundary layer temperature $T_G$ was assumed to vary linearly with elevation and calculated by interpolation of the temperatures observed at G500 and G1100. The annual variation of $b_S$ considering all stakes is better described with DDM2 than DDM1 (lower $\sigma$ values in Table 5), but the DDM1 model provides more accurate prediction on average in the accumulation area where too much summer balance is predicted with DDM2 (Figure 7 and Table 5).

This can be explained with snowfall within the ablation season that was frequently observed by the sonic echo sounder at G1100 (~15–20 cm w. eq. a$^{-1}$ during the years of observations) but hardly ever at the lower G500. By using sonic echo sounder and mass- and energy balance data, Guðmundsson et al. (2005) found the amount of snow falling and melting during the ablation season 2004 at the northeast Vatnajökull ice cap (Figure 1), to gradually increase up-glacier from ~15 cm w. eq. a$^{-1}$ at 1100 m a. s. l. up to 1 m w. eq. a$^{-1}$ at 1525 m a. s. l. Thus, the 36 cm w. eq. a$^{-1}$ higher ablation predicted with DDM2 than observed at the 1200–1300 m stake locations (Table 5) may be more realistic than the close fitting of the DDM1 model.

Table 5: Mean ($\mu$) and standard deviation ($\sigma$) of the predicted minus the stake observed summer balance ($b_S$) in Figure 7. The summer balance estimated with DDM1 and DDM2 includes snow that falls and melts within the summer that is not detected by the stake observations.

<table>
<thead>
<tr>
<th></th>
<th>DDM1 (cm w. eq. a$^{-1}$)</th>
<th>DDM2 (cm w. eq. a$^{-1}$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ablation area</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\mu$</td>
<td>-4</td>
<td>4</td>
</tr>
<tr>
<td>$\sigma$</td>
<td>60</td>
<td>41</td>
</tr>
<tr>
<td>Accumulation area</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\mu$</td>
<td>0</td>
<td>-36</td>
</tr>
<tr>
<td>$\sigma$</td>
<td>23</td>
<td>21</td>
</tr>
</tbody>
</table>

**DISCUSSION**

**Time and elevation dependency of the degree-day-parameters**

As a practical approximation, scaling parameters of temperature index models are generally assumed to be both time and elevation independent, only varying with the surface type (e.g. Jóhannesson et al., 1995). The model DDM2, using temperature away from the glacier, comes closer to be elevation independent than DDM1 (Table 3). Annual sensitivities...
(errors in Table 3) are identical for \(ddf_1\) and \(ddf_2\) at G1100 but substantially higher for \(ddf_1\) than \(ddf_2\) at the lower G500. One explanation for the increased \(ddf_{2_{\text{snow}}}\) downglacier could be the earlier timing of the snow/ice transition at lower elevations, resulting in higher incident solar radiation falling on a surface with already reduced albedo, and hence higher absorbed solar radiation, during the period of parameter optimisation. SPOT5 images show that the low albedo for ice at G500 (\(\alpha = 0.07\)), reflected as high \(ddf_{2_{\text{ice}}}\) (Table 3), reaches only to elevation \(\sim 100\) m above the station. The ice at 700–1000 m a. s. l. is much cleaner (\(\alpha \approx 0.35\)), resulting in stable \(ddf_{2_{\text{ice}}}\) (Table 3). The \(ddf_2\) values in Table 3 are close to being the same as previously found for northeastern Vatnajökull and northern Húfusjökull ice caps, Iceland (Figure 1), by using energy balance observations (Guðmundsson et al., 2003) and mass balance observations on stakes (Jóhannesson et al., 1995), respectively. The degree-day parameter was found to be slightly higher for ice on northern Hofsjökull than on southern Langjökull.

To investigate the seasonal sensitivity of the \(ddf\)-parameters during unchanged surface conditions, we used the observed weather parameters along with constant albedo values of 0.1 to 0.9 for the period of May through September at G1100 and G500, and optimised the \(ddf\)-parameters separately for each of the months and each albedo value (black lines in Figure 8). The lower solar radiation generally resulted in reduced degree-day factors during unchanged surface conditions, indicating that the degree-day factors are sensitive to seasonal changes in solar radiation; this is particularly evident at the higher station G1100. The high \(ddf\)-parameters optimised for May (black lines in Figure 8) can be explained by the relatively strong contribution of net radiation to melting during that month. The impact of the abrupt transition from snow to ice/firn depends on its timing during the summer. For example, a drop in the albedo at G1100 from 0.5 to 0.3 in June-July would increase \(ddf_1\) from \(\sim 17\) mm to 21 mm per °C, but the same decline in albedo during July-August would result in unchanged \(ddf_1\) (Figure 8a). The parameter \(ddf_2\) has a lower value, varies more slightly and is less sensitive to changes in the weather parameters and to the timing of the snow-ice transition than \(ddf_1\) (black lines in Figure 8); hence, \(ddf_2\) comes nearer to depending solely on conditions at the glacier surface. The typically strong contribution of net radiation in May affects \(ddf_2\) more than \(ddf_1\), but occurrence of the strong winds and relatively high temperatures in September affected \(ddf_1\), but not \(ddf_2\) (thick gray lines in Figure 8a,c).

A justification for assuming time-independent degree-day factors, varying only with surface conditions (snow or ice), is that the reduced solar radiation and increased heat fluxes as summer proceeds jointly counteract the lowering of albedo, which explains the stability with time in the monthly values for \(ddf_1\) and \(ddf_2\) at both stations obtained by using the observed albedo values (thick grey lines in Figure 8).

Our results indicate that the accuracy of degree-day models can be improved by accounting for the
solar zenith angle, which agrees with Guðmundsson et al. (2005) who gained better degree-day prediction of seasonal change in runoff at the northeastern part of Vatnajökull (Figure 1), by incorporating modelled clear sky irradiance into the degree-day scaling (as introduced by Hock, 1999).

The performance of degree-day models in response to regional temperature changes

Both the energy balance and the degree-day models (parameters from Table 3) were used to estimate melt rate changes in response to climate change. As a case study we considered a seasonally constant temperature change $\Delta T_S$ outside the ice cap (at S475). The calculation was restricted to a fixed period corresponding to the ablation season 2001 at the two AWSs sites. The eddy fluxes were calculated stepwise ($\Delta T_S = 1^\circ C$ ) for temperatures deviating -5 to $5^\circ C$ from the observed values of $T_S$ in 2001. The corresponding variations of $T_G$ (in $^\circ C$) were derived by using a piecewise-linear regression of the data in Figure 9.

The observed values of $Q_i$, $I_o$ (from a melting surface), $r$ (close to that of saturation), and the winter balance ($b_W$) for the year 2001 were selected as a reference. Changes in air temperature outside the glacier margin were expected to have an effect on the strength of the down slope glacier winds (e. g. Björnsson et al., 2005). For estimating this effect ($\Delta u$ in m s$^{-1}$), we approximate the relation between temperature and wind speed observations at G1100 and G500, and temperature at S475 during May-September 2001–2005 by piecewise linear regression (Björnsson et al., 2005) based on our glacio-meteorological data from 5 ablation seasons:

$$
\Delta u = \begin{cases} 
0.25 \Delta T_S & T_G \geq 0 \\
0 & \text{otherwise}
\end{cases}
$$

(11)
Figure 9. Relationship of air temperature $T_G$ at G1100 (a) and G500 (b) during the summers of 2001–2006, to temperature $T_S$ at S475, outside the glacier, presented in each instance as a one-hour mean. Piecewise linear regression between temperatures on and off the glacier are shown as imprinted grey lines. The scatter values in (b) were separated into southern regional winds (light grey) and northern down slope glacier winds (grey dark) when optimising Eq. 12. – Samband mælds hita yfir bráðnandi jökulyfirborði við hita mældan utan jökuls.

\[ \Delta u = \begin{cases} 
0.54 \Delta T_S & T_G \geq 0 \\
0 & \text{otherwise} 
\end{cases} \quad (12) \]

at G1100 and G500, respectively. Equations 11 and 12 apply for a melting glacier surface, approximated with $T_G \geq 0$. The observed incoming long wave radiation ($I_i$) of the ablation seasons 2001–2005 varies between $\sigma_S T_G^4$ and $\sigma_S T_S^4$ ($\sigma_S$ being the Stefan-Boltzmann constant), indicating a too shallow boundary layer to eliminate effects from the warm air above it. The boundary layer was expected to become thicker with increased $T_S$ and the changes of $I_i$ taken as mid-values between $\sigma_S T_G^4$ and $\sigma_S T_S^4$.

Calculations of the energy fluxes, suggest that the melting rates would be accelerated both by eddy fluxes and net radiation, however, more by eddy fluxes at the lower station (Figures 10a-b and 11a-b). The net radiation would be affected more by increased long wave radiation than albedo despite earlier exposure of the summer surface.

Reasonable agreement was obtained between the complete model of physical energy balance (Eq. 5) and the degree-day models (Eqs. 9-10) at G1100 when $-5^\circ C \leq \Delta T_S \leq 5^\circ C$ (Figure 10c-d), and this also applied to G500 for $-5^\circ C \leq \Delta T_S \leq 2^\circ C$; keeping the wind speed unchanged from the 2001 reference values as well as wind speed changes proportional to temperature (Figure 11c-d). The empirical models diverged increasingly from the physical model when assuming glacier winds to change proportionally to $T_S$ (Figures 10d and 11d).
Figure 10. Weather parameters and energy components at G1100 corresponding to regional temperature changes ($\Delta T_s$) from $T_s$ of 2001 at S475. All parameters represent averages over a period equal to the observation season in 2001. (a-b): Energy budget using only the temperature changes and by assuming wind-speed to change according to Eq. 11, respectively. (c-d): EBM using the data from (a) and (b), respectively, compared to DDM1 and DDM2. – Orkubúskapar- og gráðudagalíkön notuð til að meta leysingu í 1100 m y. s. við breytt loftslag.

Figure 11. The variation in weather parameters and energy components at G500 corresponding to regional temperature changes at S475. Explanations of the subplots are given in the caption with Figure 10, except with wind speed changes according to Eq. 12. – Orkubúskapar- og gráðudagalíkön notuð til að meta leysingu í 500 m y. s. við breytt loftslag.
It should be noted that these synthetic calculations underestimate the absolute values of the temperature change on seasonal ablation. If the regional temperature rose, it would extend the ablation season, reduce $h_W$ (as long as winter precipitation did not also increase), moving the exposure of the low-albedo summer surfaces from earlier point in time in the season with the highest incident radiation falling on a surface with already reduced albedo. The ablation season would also be prolonged into the autumn when solar radiation has been reduced and melting is mainly driven by high temperatures and strong winds. The former effect is difficult to take into account with a degree-day model. Our data set include two years with a warm September month with high melting rates, which allow for testing of the degree-day models for warm and windy autumn months with low solar radiation. The degree-day models did satisfactorily predict the accumulative melting during those periods but were not reliable on a daily basis at the lower weather station.

CONCLUSIONS
The observed daily melt rates on the glacier were successfully simulated by energy balance calculations based on meteorological observations on the glacier. As a rule, net radiation was the main contribution to melting, although it was occasionally equalled by eddy fluxes. Sporadically, radiation contributed to melting even when eddy fluxes were negative. Net radiation typically peaked in the ablation area in May to June and in August around the ELA when the melting reached the previous year’s summer surfaces. Turbulent fluxes increased during the summer, reaching a maximum in August-September. Every energy component increased downglacier: radiation due to the lower albedo and turbulent fluxes owing to higher temperatures and the persistent down slope glacier wind.

Degree-day models successfully described seasonal variations in melting, but were less successful for simulating daily values. The most successful degree-day predictions were obtained by applying temperature observations away from the glacier and a constant adiabatic lapse rate with elevation, rather than temperatures observed on the glacier itself. Air temperatures in the low-albedo surroundings of the glacier represent daily variations in the global radiation flux better than the damped boundary layer temperatures above the melting glacier. Given no extreme changes in albedo the derived empirical degree-day models may provide reasonable predictions of increased ablation in response to a regional temperature change of less than 3 °C.

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ÁGRIP
Unnið hefur verið að því að kanna tengsl veðurfars og jökulleysingar á Langjökli á hverju sumri síðan 2001. Reknar hafa verið sjálfvirkar veðurstöðvar á Hagafellsjökli vestari (i um 500 m y. s. við jaðar jökulsins og í 1100 m y. s. nálægt snælínu í meðalári) til þess að meta orkustrauma sem berast að yfirborði jökulsins og valda leysingu. Orkuþættirnir eru sólgeislu, langbylgjugeislu, varmastraumur frá hlýju lofti og varmastraumur vegna þéttingar loftraka yfir jöklinum. Með þessum gögnum hafa verið sett fram og prófuð reiknilikön sem tengja jökulleysingu við orkustrauma og einstaka veðurfæði.

Nákvæm orkubúskaparlíkön krefjast viðamikilla mælinga á jökulum og því er reynt að finna einföld töfræðileg líkön sem lýsa leysingu í hlutfalli við melðan loftihita, svonefn gráðugalaðikön. Anmarkar reynslubundinna gráðugalaðikana hafa hins vegar verið lítið kannaðir. Þar er einkum áhyggjuefni hve tengsl

REFERENCES


