TEMPERATURE

A Temperature Distribution in Glaciers

Why study temperature?

- Deformation rate of ice decreases by a factor of 5 by cooling from -10°C to -25°C.
- Temperature profiles provide information of past variations of surface temperature.
- Want to know whether the glacier is at or near melting point at the base. That has implications for erosion, sliding, and unstable advance.

What controls $T$:

- surface $T$,
- geothermal heat,
- friction,
- water freezing (melt, rain, ...)

It is important to keep in mind that the solution of the temperature distribution for the whole glacier depends on the flow (advection), which again depends on the temperature, so this is a coupled problem.

If the ice in the glacier is at a temperature below the pressure melting point throughout it is characterized as a cold glacier. It is still called a cold glacier if it reaches the pressure melting point only at the bed. If there is a basal layer at the pressure melting point it is called polythermal. And, finally, if all the ice is at the pressure melting point, except in a surface layer during the winter, it is called temperate.

Glaciers in Iceland are all temperate. Most of the glaciers in Svalbard are polythermal. The large ice-sheets, Greenland and Antarctica, are cold glaciers.

1 Thermal parameters

Table 1 lists the main parameters needed for calculations of temperature.

<table>
<thead>
<tr>
<th>Thermal parameters of pure ice</th>
<th>0°C</th>
<th>-50°C</th>
</tr>
</thead>
<tbody>
<tr>
<td>Specific heat capacity, $c$</td>
<td>J kg$^{-1}$K$^{-1}$</td>
<td>2097</td>
</tr>
<tr>
<td>Latent heat of fusion, $L_f$</td>
<td>kJ kg$^{-1}$</td>
<td>333.5</td>
</tr>
<tr>
<td>Thermal conductivity, $K$</td>
<td>W m$^{-1}$ K$^{-1}$</td>
<td>2.10</td>
</tr>
<tr>
<td>Thermal diffusivity, $k$</td>
<td>10$^{-6}$ m$^2$ s$^{-1}$</td>
<td>1.09</td>
</tr>
</tbody>
</table>

The specific heat capacity $c$ and thermal conductivity $K$ vary with temperature according to

\begin{equation}
c = 152.5 + 7.122T,
\end{equation}

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\[ K = 9.828 \exp(5.7 \cdot 10^{-3}T). \]  

(2)

Thermal conductivity \( K \) also depends on density \( \rho \) (Paterson, 1994, p. 205). Figure 1 shows an upper limit

\[ K = \frac{2K_i \rho}{3\rho_i - \rho}, \]

and lower limit

\[ K = 2.1 \cdot 10^{-2} + 4.2 \cdot 10^{-4}\rho + 2.2 \cdot 10^{-9}\rho^3, \]

formulation of the variation as a function of density. Here \( \rho_i \) refers to the value for pure ice.

![Figure 1: Variation of thermal conductivity as a function of density according to an upper limit (solid line) and lower limit (dashed line) formulation.](image)

Thermal diffusivity for any density and temperature can be calculated from

\[ k = \frac{K}{\rho c}. \]  

(3)

2 Periodic surface temperature variation

According to Fourier’s law of heat conduction, the heat flux is proportional to the temperature gradient, that is

\[ Q = -K \frac{\partial T}{\partial z}. \]  

(4)

The amount of heat energy in volume \( V \) is,

\[ Q = \iiint_V \rho c T \, dv. \]
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The change in heat per unit time \( \frac{dQ}{dt} = -\rho c \frac{\partial T}{\partial t} \) is equal to the difference in heat flux through the area (in 1-D, over length scale \( \partial z \)),

\[ -\rho c \frac{\partial T}{\partial t} = \frac{\partial Q}{\partial z} = -K \frac{\partial^2 T}{\partial z^2}. \tag{5} \]

which leads to the change in temperature with time, for a constant \( K \),

\[ \frac{\partial T}{\partial t} = k \frac{\partial^2 T}{\partial z^2}. \tag{6} \]

To solve this equation boundary conditions are needed. Here we consider a periodic surface temperature variation, which can approximate daily, yearly, and possibly longer time scale variations:

\[ T(0, t) = T_S \sin(\omega t), \tag{7} \]

where \( T_S \) is the amplitude of the surface temperature variation, and \( \omega \) is the frequency, which is related to the periodicity,

\[ P = \frac{2\pi}{\omega}. \]

The solution to Equation 6 (also have the condition that \( T \to 0 \) as \( z \to \infty \)) is

\[ T(z, t) = T_S \exp \left( -z \sqrt{\frac{\omega}{2k}} \right) \sin \left( \omega t - z \sqrt{\frac{\omega}{2k}} \right). \tag{8} \]

In Figure 2 the solution is examined by showing first only the periodic part, then the exponential part, and then finally the combined properties of the temperature distribution.

Some of the properties of the solution are:

- The amplitude decreases as \( \exp \left( -z \sqrt{\frac{\omega}{2k}} \right) \). Thus the higher the frequency, the more rapid the attenuation.

- The velocity of propagation of temperature maxima and minima is,

\[ V = \sqrt{2k\omega}, \tag{9} \]

the higher the frequency, the higher the speed.

Figure 3 shows the variation in amplitude with depth for a seasonal (period of one year) variation, with an amplitude of \( T_S = 20 \, ^\circ C \), at different times through the year. Figure 4 shows the temperature at a fixed location on a time versus depth plot.

The temperature at 10-15 m depth can be considered the mean annual temperature, if no melt or refreezing (try calculating the temperature distribution with a period of 1 day).

Meltwater is important, since 1 g water that refreezes produces enough heat to raise the temperature of 160 g of snow by 1 K (\( L/c \) high!).

Percolating water may eliminate by refreezing the winter cold wave in the accumulation area. Below the ELA, in the ablation area, there is likely to be ice with temperature below the melting point. That is because the latent heat is lost (as surface run-off) and does not warm the glacier (except for a little bit of snow and surface ice that is lost); only a little by the slow process of conduction. Therefore the temperature in the accumulation area may be higher than in the ablation area, even if the air temperature is lower.
Figure 2: Variation of temperature as a function of depth. The first plot shows only the sin term, the second plot shows only the exp term, and the last plot shows the full solution. The depth is normalized with the skin depth, $z_0$ (see Problem).
Figure 3: Seasonal variation of temperature as a function of depth. The minimum surface temperature is in winter, then spring, fall, and highest temperature in summer.

Figure 4: Variation of temperature as a function of depth and time.
3 Temperate Glaciers

The water content of temperate glaciers is typically 0.1 - 2% by volume (refs).

If only pure ice and water,

\[ T = T_0 - \beta p, \]

where \( \beta = 7.42 \times 10^{-5} \text{ K (kPa)}^{-1} \), melting point depression with pressure, and \( T_0 = 273.16 \text{ K} = 0^\circ \text{C} \), the triple point (Figure 5).

![Phase diagram for water.](image)

Figure 5: Phase diagram for water.

Due to air-saturation, which lowers the equilibrium temperature, we should use \( \beta' = 9.8 \times 10^{-5} \text{ K (kPa)}^{-1} \), or \( 8.7 \times 10^{-4} \text{ K m}^{-1} \) for ice.

Impurities also depress the melting point. For small concentrations this term can be written as \( -An/W \), where \( W \) is the fractional water content by weight, \( n \) is the salt concentration in mol kg\(^{-1} \), and \( A = 1.86 \text{ K kg mol}^{-1} \) (Paterson, 1994). Thus the equilibrium temperature of the ice is,

\[ T = T_0 - \beta' p - \frac{An}{W}. \tag{10} \]

Impurities and air bubbles can greatly increase the effective specific heat capacity of ice near the melting point. If heat is added, some of it must be used to dilute the liquid. The effective specific heat capacity is then,

\[ c' = c + L_f \frac{dW}{dt}, \tag{11} \]

where \( c \) is the value for pure ice, \( L_f \) is latent heat of fusion, and \( W \) is related to \( T \) by Eq. 10.
4 Steady-State - No Horizontal Motion

The equation for the temperature variation with time in a glacier with vertical motion \( w \) is,

\[
\frac{\partial T}{\partial t} = k \frac{\partial^2 T}{\partial z^2} - w \frac{\partial T}{\partial z} \quad \text{advection}
\]  

(12)

If not in steady state, both \( w \) and \( T(z) \) will change with time.

If in steady state, \( \partial T/\partial t = 0 \), and the solution is called the *Robin temperature profile*. The equation to solve is,

\[
k \frac{d^2 T}{dz^2} - w \frac{dT}{dz} = 0, \text{ where at } z = \begin{cases} 0, & Q = -K \frac{dT}{dz} |_{B} = c \\ h, & T_s = \text{const} \end{cases}
\]  

(13)

which gives after one integration,

\[
\frac{dT}{dz} = \left( \frac{dT}{dz} \right)_B \exp \left( \frac{1}{k} \int_0^z wdz \right).
\]  

(14)

To proceed we must know \( w \) as a function of \( z \). Use \( w = -\frac{bz}{h} \), assume \( u_s \cdot \tan \alpha \) is small. Then we get,

\[
T - T_s = \sqrt{\frac{\pi}{2}} l B \left[ \text{erf} \left( \frac{z}{l} \right) - \text{erf} \left( \frac{h}{l} \right) \right],
\]  

(15)

where \( l^2 \equiv 2kh/b \).

[FIGURE 10.4 in Paterson] If we now set \( \xi = z/h, \theta = \frac{K(T-T_s)}{Gh} \), and \( \gamma = \frac{bh}{k} \). If \( b = 0 \) we get a straight line where the slope is \( G \), the geothermal heat flux.

5 Horizontal Advection

Ice at depth in a glacier comes from higher up on the glacier, and thus, generally, from a region of lower temperature. Figure 6 shows a schematic of horizontal advection. Change in air temperature

![Figure 6: Horizontal advection of ice.](image-url)
with unit increase of elevation is $\lambda \sim -0.6$ to $-1.4$ K per 100 m.

Snow/ice at $P_1$ at $T = T_1$, buried to a depth $b$ and moved to $P'_1$ below $P_2$ where $T = T_2$. The temperature gradient between $P_2P'_1$ is $(T_2 - T_1)/b$, and the temperature difference between $P_1$ and $P_2$ is $T_1 - T_2 = u\alpha\lambda$, approximately. It follows that (put $-u\alpha\lambda$ in for $T_2 - T_1$),

$$\left( \frac{dT}{dz} \right)_S = -\frac{u\alpha\lambda}{b}. \quad (16)$$

Figure 7 shows the temperature profile at GISP2, Greenland.

![Temperature profile at GISP2, Greenland.](image)

**Figure 7:** Temperature profile at GISP2, Greenland.

## B Heat Sources

Some of the major heat sources are:
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- Ice deformation, \( f_1 = 2\dot{\varepsilon}_\tau \)
- Firn compaction, \( \nabla(\rho \mathbf{v}) \)
- Refreezing
- Heat of sliding, \( Q_f = u_b \tau_b \)
- Geothermal heat, \( \sim 46 - \sim 77 \text{ mW m}^{-2} \)

When modeling it is important to realize the interplay between mass balance, surface temperature and geothermal heat flux.

Inversions are hard, noise, diffusion process.

Temperature Problems

1. Derive the velocity of propagation, or phase velocity. Phase \( \Psi = \omega \cdot t - z \sqrt{\frac{\omega}{2k}} \), want constant phase, \( d\Psi = 0 \). (Hint: Take the derivative, and remember that \( t \) and \( z \) are variables, and that \( dz/dt = V \)).

2. a) Find the skin depth \( z_0 \), i.e. \( z \) where \( T = T_s e^{-1} \) from Equation 8 (use only the exponential term, i.e. \( T = T_s \exp \left( -z \sqrt{\frac{\omega}{2k}} \right) \)).
   b) Calculate \( z_0 \) for: i) Period of one day, \( P = 1 \text{ day} \), ii) \( P = 1 \text{ year} \), iii) \( P = 10000 \text{ year} \). (Use \( k = 10^{-2} \text{ cm}^2 \text{ s}^{-1} \)).
   c) How far does the wave travel in one period (for the periods above)?
   d) Does the bed of the Greenland ice sheet know that the ice age is over?
   e) How would advection affect this? Assume a simple linear vertical velocity, where the velocity at the surface is \( w(z = S) = b \), and at the bed is \( w(z = B) = 0 \), and use the mean velocity \( \overline{w} = b/2 \), where \( b = 10 \text{ cm yr}^{-1} \).
   f) How long should it take by diffusion alone (diffusion time scale). Do dimensional analysis on \( \frac{\partial T}{\partial t} = k \frac{\partial^2 T}{\partial z^2} \).

3. Show how we can say that freezing of 1 g of water can raise the temperature of \( \sim 160 \text{ g of ice} \) by 1 K. The numbers needed are in Table 1.

4. Explain why some glaciers are temperate in accumulation zone and frozen in ablation zone

5. Why would pure water in a temperate glacier freeze?
