The far-infrared-active collective modes of short period arrays of quantum dots and antidots

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In this paper we investigate the far-infrared absorption of short period arrays of quantum dots and antidots and its underlying collective modes. For weak perpendicular magnetic fields when the magnetic length approaches the periodic length of the lattice the bandstructure manifests itself in the absorption. In high magnetic fields both systems show the characteristic two-peak structure, connected to the center-of-mass motion of quantum dots, and the edge and bulk modes of antidots. In the intermediate region the magnetoplasmon splits into Bernstein modes. The interaction between the electrons is treated in the Hartree approximation on equal footing in the ground state and the absorption calculation, both for the intra- and interdot case.

I. INTRODUCTION

Arrays of quantum dots and particularly antidots have been found to have fascinating dynamical properties. Transport[1] and microwave photoconductivity measurements[2] on antidot arrays have established commensurability effects between the magnetic length $l = (\hbar c/(eB))^{1/2}$ and the lattice period L. These experiments on large period arrays manifest resonance structures that have been explained by classical orbit models with inherent chaos. Measurements of the far-infrared (FIR) absorption have mainly revealed the expected two-peak structure explained by the center-of-mass motion of parabolically confined electrons in dots,[3] and anticrossing bulk and edge modes in antidots,[4] together with the Bernstein splitting[5] at intermediate magnetic fields, made visible by deviations from the parabolic potential shape.[6]

Classical models have been successful describing the FIR absorption of large period arrays of antidots,[7] but most quantum mechanical models have either oversimplified the confinement potential,[8] or neglected its periodicity[9,10], to adequately model a short period array. In arrays of quantum dots the interaction between electrons in different dots has typically been approximated to a lower order than the intradot interaction,[11] again making the models inappropriate to describe a short period lattice of dots that can be strongly coupled electrically or even having an overlapping electron density.

In this publication we present a model of the FIR absorption of a short period system, that can equally well be applied to an array of dots or antidots. The electron-electron Coulomb interaction is treated self-consistently within the Hartree approximation both in the ground state and the absorption calculation. We use the model to explore the FIR-active collective modes of both systems in the quantum regime.

II. MODEL

The square array of quantum dots or antidots is described by the static external potential

$$V_{per}(\mathbf{r}) = V\left\{\sin\left(\frac{gx}{2}\right)\sin\left(\frac{gy}{2}\right)\right\}^2 \quad , \tag{2.1}$$

where g is the length of the fundamental inverse lattice vectors, $\mathbf{g}_1 = 2\pi\hat{\mathbf{x}}/L$, and $\mathbf{g}_2 = 2\pi\hat{\mathbf{y}}/L$. The Bravais lattice defined by V_{per} has the periodic length L and the inverse lattice is spanned by $\mathbf{G} = G_1\mathbf{g}_1 + G_2\mathbf{g}_2$, with $G_1, G_2 \in Z$. The ground-state properties of the interacting 2DEG in a perpendicular homogeneous magnetic field $\mathbf{B} = B\hat{\mathbf{z}}$ and the periodic potential (2.1) are calculated within the Hartree approximation for the Coulomb interacting electrons at a finite temperature.[12] The commensurability condition between the magnetic length l and the period L requires the magnetic field to assume the values $B = pq\Phi_0/L^2$, with $pq \in \mathbf{Z}$, and $\Phi_0 = hc/e$ the unit magnetic flux quantum.[13,12] Arbitrary rational values can, in principle, be obtained by resizing the unit cell in the Bravais lattice. The spin degree of freedom and the spin degeneracy are negelected here.

The far-infrared absorption of the system is calculated within the time-dependent Hartree approximation perturbing the 2DEG with an incident electric field

$$\mathbf{E}_{ext}(\mathbf{r},t) = -i\mathcal{E}_0\frac{\mathbf{k}+\mathbf{G}}{|\mathbf{k}+\mathbf{G}|}\exp\{i(\mathbf{k}+\mathbf{G})\cdot\mathbf{r} - i\omega t\}. \tag{2.2}$$

The power absorption $P(\mathbf{k},\omega)$ is found as the Joul heating of the self-consistent time-dependent electric field composed of the external field (2.2) and an induced field.[14].

III. RESULTS

The calculations have been performed for GaAs parameters, $m^* = 0.067m$, and $\kappa = 12.4$ at the temperature $T = 1$ K. The arrays have the periodic length $L = 100$ nm and the modulation $V = \pm 5$ meV. The power absorption is achieved by an energy dissipation described

by a damping parameter $\hbar\eta = \hbar\omega_c/20$. This is probably narrower linewidth than in present experiments in the FIR-regime, and for the parameters used here it is comparable to the thermal broadening for $pq = 2$. Numerical results have been verified with respect to accuracy and convergence.

In a strong magnetic field and a weak modulation of the periodic potential the resulting Landau levels would show a slight dispersion. Each one of them would have pq subbands, reflecting the number of unit magnetic flux quanta flowing through the unit cell of the Bravais lattice. Here we shall investigate the case of an intermediate strength of the magnetic field, when the confining potential still strongly affects the energy bands. This situation is depicted in Fig. 1 with $pq = 1$, corresponding to $\hbar\omega_c = 0.7$ meV, for both quantum dots and antidots. The energy bands are shown in the first Brillouin zone mapped out by the dimensionless quantities $\Theta_i = k_i L$, with $i = 1, 2$. $N_s = 0.5$ here, meaning that, on average, only half an electron occupies each unit cell. This is possible here, even in the case of a dot array, since there is always a small overlap of the electron density of neighboring dots and we are using Bloch-type basis states.[12,15] Our model is thus not appropriate to describe electrons in a dot array in the tight-binding limit. Later on we discuss the case of a higher electron density, but the value $N_s = 0.5$ leads to relatively well localized quantum dots as can be verified by the electron density or the gap in the dot spectrum between the lowest (half filled) band and the second (empty) band on the right panel of Fig. 1. In both spectra the chemical potential μ lies in the lowest energy band.

The power absorption P is seen in Fig. 2 for dots and antidots as a function of energy $E = \hbar\omega$ and magnetic flux in the unit cell, pq, for $\mathbf{k}L = (0.2, 0)$. For high flux the familiar two-peak structure is seen, but in the case of lower magnetic flux pq the absorption is more complicated. This phenomenon is particular to the short period lattice and can be explained as follows: An examination of the ground state electron density for the dot array reveals that for all the flux values referred to in Fig. 2 the electrons are well localized in the dots. The two-peak structure of the absorption can be expected for parabolically confined electrons in quantum dots where the generalized Kohn's theorem is satisfied for long wavelength incident radiation. Furthermore the two-peak structure has been shown to be quite stable against perturbation of the confinement.[16,17] As the strength of the magnetic field is increased the magnetic length becomes smaller and the electron density is more localized within each dot. This is also reflected by the fact that $\mu - (\hbar\omega_c/2)$ decreases with increasing magnetic field. The electrons see to more extent only the parabolic bottom part of the confinement. For the lowest magnetic flux, $pq = 1$, the FIR absorption of the 2DEG attests that for the given density of electrons their behavior is strongly influenced by the periodic properties of the array potential. The dots are strongly coupled. The absorption curves for the $pq = 1$ cases are extended only as high in energy as the number of Landau bands included in the calulation allows with respect to accuracy, (6 bands are used here).

In an antidot array the electrons are not localized in the same sense as in a dot array but a similar argument about the shape of the potential the electron see applies. In fact, Fig. 3 shows how features of the absorption for the antidot array in the case of $pq = 1$ can be directly related to the bandstructure in the ground state. In the left side of the figure we see the band structure projected on one plane parallel to the Θ_2-axis in the first Brillouin zone. The energy scale is changed such that the chemical potential μ coincides with the lower edge of the left panel, the zero-plane. Clearly, the lowest peak in the right panel of the figure is caused by intraband transitions in the lowest band. The two upper peaks in the absorption reflect a high- and a low-energy van Hove singularity of the second band. The peaks are shifted in energy to different extent due to many-body effects. What happens when the lowest band is increasingly populated can be seen in Fig. 4. As expected, the intraband peak vanishes when the band is full, the small remainder still existing for $N_s = 1.0$ is due to the finite temperature $T = 1$ K.

The issue of how many electrons fit into a quantum dot with the chosen parameters and a particular magnetic field is resolved in Fig. 5 for the case of $pq = 3$. For, on the average, one or less electrons the spectrum displays features characteristic of the requirements of Kohn's theorem. For two electrons two peaks are present but not having the usual intensity relation expected for dots. In the case of three electrons the spectrum has assumed features characteristic of a modulated 2DEG, as can be verified independently by the strongly overlapping density of the 'dots'. Here one has to remember that we neglect both the spin degree of freedom and the exchange interaction.

The $E = 2\hbar\omega_c$-splitting of the upper dispersion mode is visible both for dots and antidots for $pq = 2$ reminding us that the Kohn theorem is broken both by the small external wavevector, $kL = 0.2$, and the confinement potential (2.1).[5,6,14]

The time-dependent density oscillations of the electron system representing the collective oscillations caused by the incident FIR radiation can be used to identify the absorption peaks. The induced density for an array of quantum dots is shown in Fig. 6 for 4 unit cells and $pq = 3$. The dot minima or the antidot maxima are localized in the center of each unit cell. The external electric field (2.2) is linearly polarized parallel to the x-axis with $kL = 0.2$ and the small asymmetric change in the induced density observed along the x-axis is caused by this small but nonvanishing wavevector. The ensuing oscillation modes are composed of this long-wavelength mode enforced by the external field and a short-wavelength mode caused by the self-consistent response of the system. For the two peaks present in the spectrum the collective oscillations are almost pure dipole center-of-mass modes, almost, since the confinement is not perfectly parabolic.

The antidot array allows a richer spectrum of collective modes as is presented in the last two figures. A mode that can typically be assigned to the lower peaks for $pq > 1$ is shown in the upper panel of Fig. 7. Here $pq = 3$, $E = 0.63$ meV, and we see the electron density oscillating around each antidot. The low-intensity peaks just above the $E = \hbar\omega_c$ line for $pq = 2, 3$ typically arise due to a collective motion as displayed in the lower panel of Fig. 7. Here we see the electron density circulating between and around the the antidots. The peaks with the highest energy and the largest oscillator strength correspond to almost linear waves travelling parallel to the x-axis as the incident electric field (2.2) but with a wavelength corresponding to the lattice length L of the array.

In Fig. 8 the induced density is presented for the antidot array with $pq = 1$. The peak with the highest energy, $E = 2.05$ meV corresponds to the mode seen in the bottom panel of the figure. This mode is quite similar to the modes observed for the peaks with the highest energy in a stronger magnetic field, $pq > 1$, except the linear wave is more affected by the antidots. The induced motion for the lower energies shown in the top and the middle panel is marked by the larger magnetic length l here compared to the modes seen in Fig. 7. Here a simpler dipole oscillation takes place in-between the antidots, specially in the case of the intraband mode, (the top panel in Fig. 8). Here $N_s = 0.50$ but the results for the induced density are almost the same for the values of $N_s = 0.25$ and 0.75.

IV. SUMMARY

In this paper we have investigated the FIR absorption of arrays of quantum dots and antidots in the limit of a short period applying the self-consistent Hartree-approximation. We have noticed how the placement of the system in the strong or weak coupling limit is governed by the electron density, the periodic length L, the magnetic field B, and the modulation strength V, all individually or hand in hand.

For the chosen period L and modulation V (2.2) we observe how the absorption is strongly influenced by the periodic bandstructure for low magnetic field, where $V >> \hbar\omega_c$. The system is here in the strong coupling limit, but the character of the system can still be changed by N_s.

On the other hand, the absorption assumes more and more the form predicted by the Kohn theorem as the strength of the magnetic field increases and the electrons get more localized in the dots or between the antidots. Even for $pq = 4$ the cyclotron energy $\hbar\omega_c$ is slightly smaller than the modulation V, but the strong screening ability of the 2DEG makes the modulation or dispersion of the subbands of one Landau level much smaller than the seperation of the Landau levels.

The main response of the antidot array is at a wavelength different from the long wavelength, $kL = 0.2$, imposed on the system by the external incident field (2.2) but commensurate with L. In addition, the presence of the magnetic field can lead to a collective mode with polarization totally different from the one of the incident field. The similarities and differences of collective electron motion in the short period arrays of quantum antidots with the larger classical systems remain an interesting question that we have only slightly touched.

We have studied here the effects of the electron-electron interaction on the FIR absorption in strictly periodic arrays of dots or antidots. Recent research on disordered systems indicate that the electron interaction severly reduces the disorder effects on the absorption by a mode lock-in mechanism.[18]

The available CPU-size limits us to the values chosen for the magnetic field in the absorption calculation, we are thus unable, at present, to investigate the system at a very low magnetic field when the magnetic length becomes larger than the periodic length or to observe the system at finer steps in the magnetic field.

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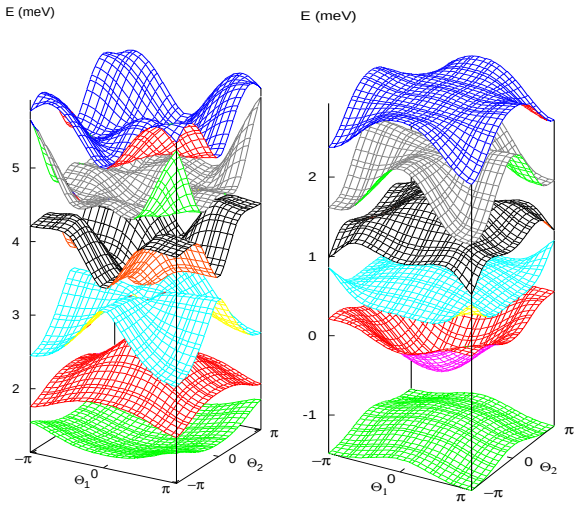

FIG. 1. The energy spectra in the first Brillouin zone for quantum antidots (left), and dots (right). $pq = 1$, $V = \pm 5$ meV, $N_s = 0.5$, $L = 100$ nm, and $T = 1$ K.

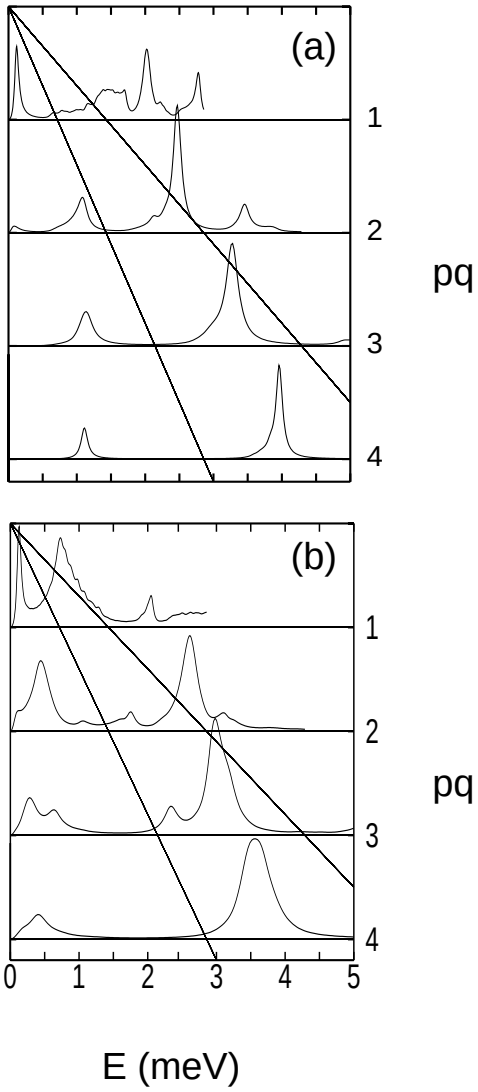

FIG. 2. The Power absorption as a function of $E = \hbar\omega$ and the number magnetic flux units pq in a unit cell for dots (a), and antidots (b). $\mathbf{k}L = (0.2, 0)$, $N_s = 0.5$, $V = \pm 5$ meV, $L = 100$ nm, and $T = 1$ K. The damping parameter is $\hbar\eta = 0.05\hbar\omega_c$ except in (a) for $pq = 4$ it is half this value. The straight lines representing $E = \hbar\omega_c$, and $E = 2\hbar\omega_c$ are shown as a guide to the eye.

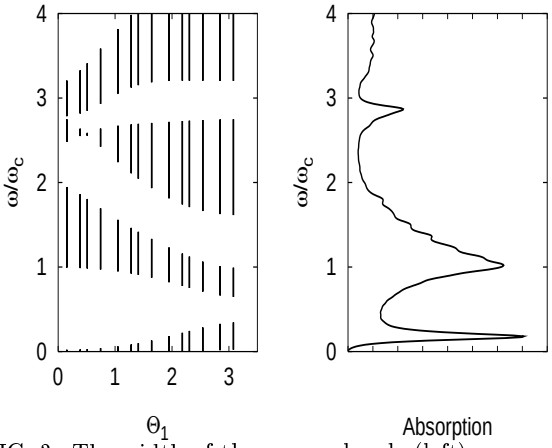

FIG. 3. The width of the energy bands (left) compared to the absorption (right) in the case of $\mathbf{k}L = (0.2, 0)$ $pq = 1$, $N_s = 0.5$, $V = 5$ meV, $L = 100$ nm, and $T = 1$ K. The zero of the energy bands coincides with the chemical potential μ.

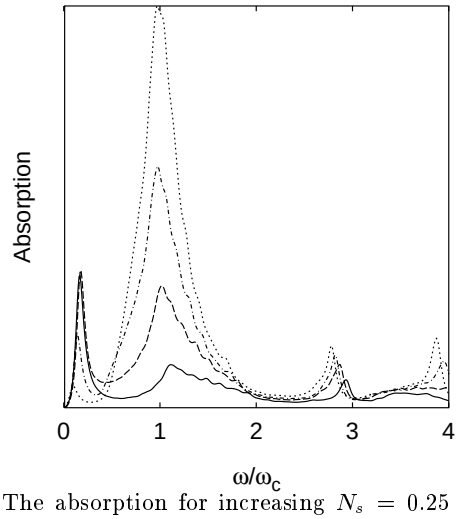

FIG. 4. The absorption for increasing $N_s = 0.25$ (solid), 0.50 (dashed), 0.75 (dashed-dotted), and 1.00 (dots) in the case of $\mathbf{k}L = (0.2, 0)$, $pq = 1$, $V = 5$ meV, $L = 100$ nm, and $T = 1$ K.

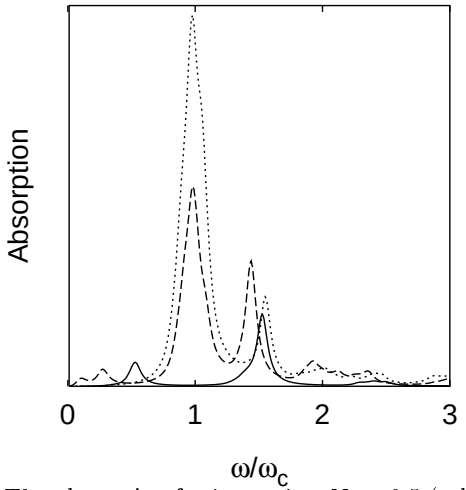

FIG. 5. The absorption for increasing $N_s = 0.5$ (solid), 2.0 (dashed), 3.0 (dots) in the case of $\mathbf{k}L = (0.2, 0)$, $pq = 3$, $V = -5$ meV, $L = 100$ nm, and $T = 1$ K.

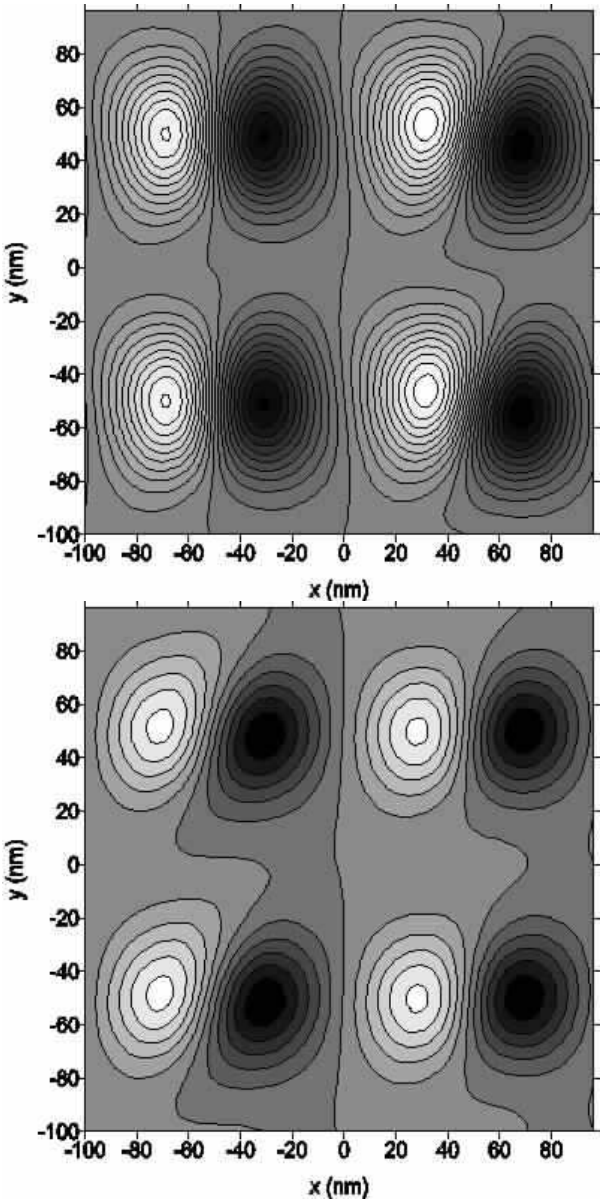

FIG. 6. The induced density for quantum dots for $E = 1.13$ meV (upper panel), and $E = 3.27$ meV (lower panel), in the case of $\mathbf{k}L = (0.2, 0)$, $pq = 3$, $N_s = 0.5$, $V = -5$ meV, $L = 100$ nm, and $T = 1$ K.

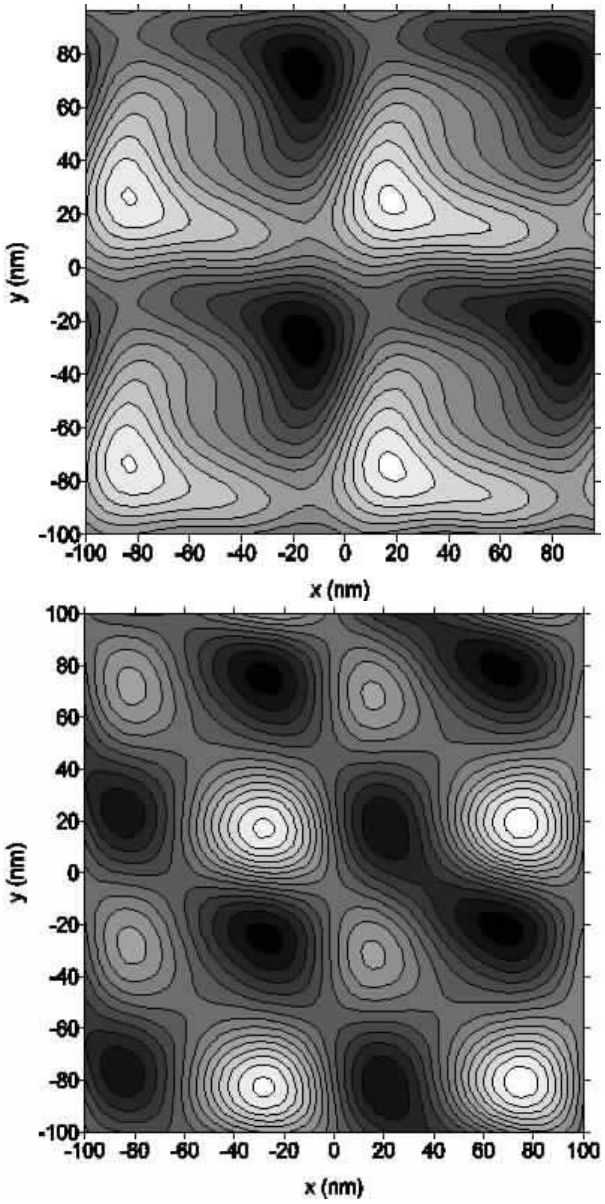

FIG. 7. The induced density for quantum antidots for $E = 0.63$ meV, $\mathbf{k}L = (0.2, 0)$, $pq = 3$ (upper panel), and $E = 1.74$ meV, $pq = 2$ (lower panel). $N_s = 0.5$, $V = 5$ meV, $L = 100$ nm, and $T = 1$ K.

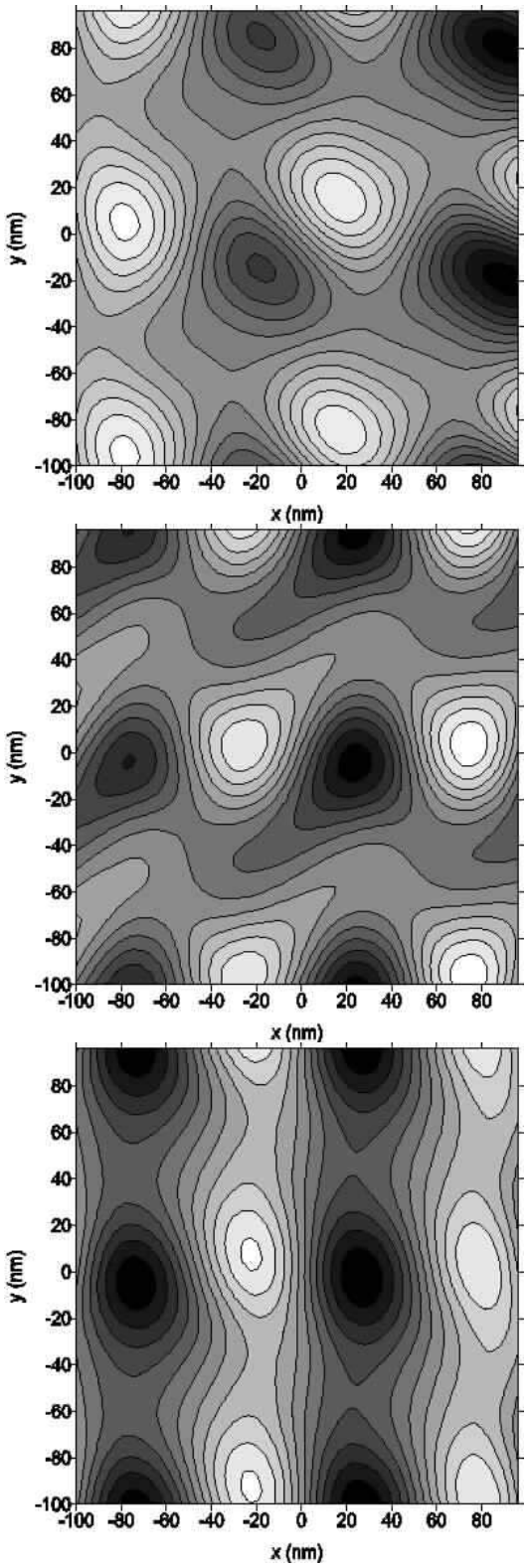

FIG. 8. The induced density of the 2DEG in an antidot array for $E = 0.126$ meV (top), $E = 0.729$ meV (middle), and $E = 2.05$ meV (bottom). $\mathbf{k}L = (0.2, 0)$, $pq = 1$, $N_s = 0.5$, $V = 5$ meV, $L = 100$ nm, and $T = 1$ K.