# Enhancement of the $g$-factor and spin-density wave state in a confined 2DEG in the quantum Hall regime.

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We investigate the spin splitting of the Landau bands (LB's) in a confined two-dimensional electron gas (2DEG) using the Hartree-Fock approximation (HFA) for the mutual Coulomb interaction of the electrons. The exchange term of the interaction causes a large splitting of the spin levels of a LB whenever the chemical potential lies between them. These oscillations of the splitting with the filling factor of the LB's are conveniently interpreted as an oscillating enhancement of the effective $g$-factor, $g^*$. The reduction of $g^*$ when a LB is becoming completely filled is accompanied by a spontaneous formation of a static spin-density wave state whose details depend on the system size and temperature.

71.70.Gm, 73.20.Dx, 75.30.Fv

## I. INTRODUCTION

The enhancement of the effective $g$-factor, $g^*$, of a 2DEG in the quantum Hall regime has been studied experimentally and theoretically by several researchers [1]. Ando and Uemura [2] presented a model of an unbounded 2DEG where the broadening of the Landau levels due to impurity scattering is treated in the self-consistent Born approximation (SCBA). The dielectric function is calculated with the inclusion of the lowest order exchange energy of the screened Coulomb interaction in the self-energy of the electrons. For a strong magnetic field the overlapping of Landau levels with different indices is neglected. In addition to the enhancement of $g^*$ the exchange interaction can lead to the spontaneous formation of spin-density [3] or charge-density waves [4,5] in two-dimensional electron systems.

The instability of the spin-density wave in wide parabolic quantum wells has been studied by Brey and Halperin using a modified HFA with a point-contact exchange interaction. They find a divergence of the electric susceptibility in the presence of a magnetic field of intermediate strength parallel to the quantum well and an infinitesimal fictious magnetic field perpendicular to the quantum well [6]. This spin-density wave state has a wavevector along the quantum well parallel to the intermediate magnetic field and occurs only when the quantum well is wide enough and the exchange interaction has a strength larger than a critical value. Kempa et. al. have investigated the spontaneous polarization of an array of quantum dots into a ferroelectric or antiferroelectric state [7].

Here we shall study the spin splitting of LB's, the oscillations of $g^*$, and the formation of spin-density wave state in a strictly two dimensional mesoscopic 2DEG system in a perpendicular magnetic field of intermediate strength. We thus observe a spontaneous spatial dependent polarization of the 2DEG within one mesoscopic system of the size of a quantum dot. We investigate the effects of the finite size of the system on the LB's in the absence of any impurity scattering of the electrons. The system size is chosen to be of the order of several magnetic lengths $l = \sqrt{\hbar c/(eB)}$. The LB's in the center of the system do then approach flat Landau levels indicating that an electron in the center does not feel the boundary. We are thus able to observe effects caused by the finite size of the system and the bulk character of some of the electron states together with the fine interplay of these two effects characteristic to mesoscopic systems. To accomplish these goals the mutual Coulomb interactions of the exactly two-dimensional electrons in several Lb's are included in the Hartree-Fock approximation at finite temperature. The effects of the exchange interaction on the screening of a simple Coulomb impurity by the 2DEG have been discussed elsewhere [8]

## II. MODEL

We consider $N_s$ strictly two-dimensional electrons to model qualitatively a real heterostructure where the 2DEG is confined to the lowest electrical subband. The 2DEG is confined to a disk of radius $R$ in the 2D-plane by a potential step

$$V_{\rm conf}(r) = U_0 \left[\exp\left(\frac{R-r}{4\Delta r}\right)+1\right]^{-1}, \tag{1}$$

where $\Delta r = 22$ Å. To ensure charge neutrality of the system a positive background charge $n_b$ resides on the disk

$$n_b(r) = \bar{n}_s \left[\exp\left(\frac{r-R}{\Delta r}\right)+1\right]^{-1}, \tag{2}$$

with the average electron density of the system given by $\bar{n}_s = N_s/(\pi R^2)$. In the Hartree-Fock approximation the state of each electron is described by a single-electron Schrödinger equation

$$\{H^0 + V_H(r) + V_{\rm conf}(r)\}\psi_\alpha(\vec{r}) - \int d^2r'\,\Delta(\vec{r},\vec{r}\,')\psi_\alpha(\vec{r}\,') = \epsilon_\alpha\psi_\alpha(\vec{r}) \tag{3}$$

for an electron moving in a Hartree potential

$$V_H(r) = \frac{e^2}{\kappa}\int d^2r'\,\frac{n_s(r')-n_b(r')}{|\vec{r}-\vec{r}\,'|} \tag{4}$$

and a nonlocal Fock potential with

$$\Delta(\vec{r},\vec{r}\,') = \frac{e^2}{\kappa}\sum_\beta f(\epsilon_\beta-\mu)\frac{\psi^*_\beta(\vec{r}\,')\psi_\beta(\vec{r})}{|\vec{r}-\vec{r}\,'|}, \tag{5}$$

where $f(\epsilon_\beta - \mu)$ is the Fermi distribution at the finite temperature $T$. $n_s(r)$ is the electron density

$$n_s(r) = \sum_\alpha |\psi_\alpha(\vec{r})|^2 f(\epsilon_\alpha - \mu), \tag{6}$$

with the chemical potential $\mu$. The label $\alpha$ represents the radial quantum number $n_r$, the angular quantum number $M$, and the spin quantum number $s = \pm\frac{1}{2}$. $H^0$ is the single particle Hamiltonian for one electron with spin in a constant perpendicular external magnetic field [9,10]. A Landau band index $n$ can be constructed from the quantum numbers $n_r$ and $M$ as $n = (|M| - M)/2 + n_r$. The Landau levels of $H^0$ with energy $E_{n,M,s} = \hbar\omega_c(n+\frac{1}{2}) + sg^*(\mu_B/\hbar)B$ are degenerate with respect to $M$ with the degeneracy $n_0 = (2\pi l^2)^{-1}$ per spin orientation. $\mu_B$ is the Bohr magneton ($e\hbar/2mc$). The filling factor of the Landau levels is defined by $\nu = \bar{n}_s/n_0 = 2\pi l^2 \bar{n}_s$. It assumes even integer values when an even number of spin Landau levels is filled. The cyclotron frequency is given by $\omega_c = eB/(mc)$. The Hartree-Fock energy spectrum $\epsilon_\alpha$ and the corresponding wave functions are now found by solving (3)-(6) iteratively in the basis [10–12] of $H^0$. The chemical potential $\mu$ is recalculated in each iteration in order to preserve the total number of electrons $N_s$. The number of basis functions used in the diagonalization is chosen such that a further increase of the subset results in an unchanged density $n_s(r)$.

## III. RESULTS

The calculations are carried out with GaAs parameters: $m^* = 0.067m_e$, $\kappa = 12.4$, and $g^* = -0.44$. The occupation of the LB's is varied by changing $N_s$ at a constant strength of the magnetic field $B = 3.0$ T. Since the radius of the system $R \geq 1000$ Å is much larger than the magnetic length $l \approx 148$ Å and the effective Bohr radius $a_0^* \approx 97.9$ Å we can use the average filling factor $\nu = 2\pi l^2 \bar{n}_s$ to describe the occupation of the lower bands in the interior of the system. The cyclotron energy $\hbar\omega_c \approx 5.2$ meV, so a sufficient height of the confining potential is $U_0 = 80$ meV in order to include several LB's in the calculation. For $B = 3.0$ T the bare spin splitting of the LB's $(g^*\mu_B/\hbar)B \approx 0.076$ meV is much smaller than their separation $\hbar\omega_c$ and corresponds to the thermal energy $k_BT$ at $T \approx 0.9$ K.

Fig. 1 shows the energy spectra of the electrons for 4 different values of $N_s$ at $T = 10$ K. For $N_s = 48$ the chemical potential $\mu$ is located between the two lowest almost spin degenerate LB's. When $N_s$ increases states of the LB with quantum numbers ($n = 1$, $s = +1/2$) become occupied and drop below $\mu$ until around $N_s = 62$ all the bulk states of that LB are occupied and are well separated from the empty LB with the same Landau level index $n$ but opposite spin $s = -1/2$. Further increase in $N_s$ brings the bulk states of this second LB all below $\mu$ drastically reducing the spin splitting. The root mean square $g$-factor, $g^*_{rms}$ for the lowest three LB's is seen in Fig. 2, where the splitting of the LB's has been interpreted as the normal Zeeman spin splitting $(g^*\mu_B/\hbar)B$ with an effective $g$-factor. $g^*_{rms}$ clearly takes on the bare value of 0.44 for even integer filling factor $\nu$, when an equal number of spin up and down states are occupied. On the other hand $g^*_{rms}$ peaks whenever $\nu$ assumes values in the neighborhood of odd integers, then $\mu$ lies between two spin LB's with the same $n$. The unequal number of spin up and down states then leads to different exchange energies for the two different spin directions [2]. The enhancement of the $g$-factor is well known from Shubnikov-de Haas experiments and the concurrent splitting of all the Landau levels has been established in optical measurements [13].

We have discovered quite an interesting phenomenon whenever $N_s$ is increased such that $\nu$ is changing from an odd integer to an even integer value, that is when the spin splitting of the LB's is closing down. This situation is displayed in Fig. 3 for the lowest LB. This apparent twisting of the LB's is repeated for a narrower range of $\nu$ or $N_s$ for $\mu$ in higher LB's. Fig. 4 showing the total density $n_s(r)$ and the electron density for each spin direction separately confirms that a *static spin-density wave state* forms spontaneously in the system concurrently with the twisting of the LB's.

An important difference of the present spin-density wave state in the 2D plane to the one parallel to $\vec{B}$ investigated by Brey and Halperin [6] is the fact that the wavelength of the present modulation varies strongly with $\nu$. This is caused by the strong dependence of the effective interaction, or the screening, in the 2D plane on $\nu$ [10,14–16]. One might expect the enhancement of $g^*$ and the spin-density wave state to interfer with the formation of incompressible regions of the 2DEG seen in the model of McEuen et. al. [16].

To exclude numerical deficiencies we have tested the stability of the spin-density structures by increasing the number of basis states included in the numerical calculation. The exact shape and formation of the spin-density waves state does depend on the size of the system emphasising that we are observing a confined spin-density wave state here [8]. In larger systems the spin-density wave occurs spontaneously together with a charge-density wave that dominates for stronger magnetic fields. Brey and Halperin can inhibit the formation of a charge-density wave state in their model by keeping the exchange interaction not too strong [6]. Calculations at a much higher temperature ($T = 100$ K) show no sign of any spin splitting since the thermal energy is then much larger than the Zeeman energy and of the same order as the separation of the LB's, $\hbar\omega_c$. The spin-density wave is better developed and exists for a larger region of $\nu$ for 10 K than 1 K. At the higher temperature the occupation of the different spin orientations is almost the same, such that small perturbations to the Landau levels can have large effects in the self-consistent evaluation of the LB's. The exchange interaction reduces the Coulomb repulsion between electrons of opposite spin orientation, and thus favors the spontaneous formation of a spin-density wave. The reduced dimensionality of the electron gas strengthens the electron-electron interaction and gives the spatial ordering of the spins an extra weight in the competition with the effects of the magnetic field that tend to order the spins all in the same direction. At much higher temperatures thermal excitations make states in neighboring LB's accessible for rearrangement of the 2DEG to neutralize any forces that might otherwise lead to splitting of LB's or spatial variations of the the spin or charge densities.

## IV. SUMMARY

In a mesoscopic system of a confined 2DEG we have been able to demonstrate both bulk effects and phenomena caused by the finite size of the system, in the absence of any impurity scattering of the electrons. The 2D system is large enough so that the LB's approach flat Landau levels for low values of the angular quantum number $M$. This can be interpreted as the formation of 2D bulk states inside the system. The ensuing singular density of states together with the exchange interaction causes the well known oscillations of the energy separation of the LB's with the same Landau level index $n$ but opposite spin orientations as a function of the filling factor $\nu$. Here we have seen that the enhancement of $g^*$ occurs not only in the LB where $\mu$ is located but in all the LB's included in the model. Similar behavior has been established in optical measurements of a 2DEG by Kukushkin [13]. In a large system of 2D electrons the state of homogeneous density becomes instable and a charge-density wave forms spontaneously [4,5,17]. In a small system the formation of such a wave is energetically unfavorable, but becomes possible with an increasing system size. Here we observe the spontaneous formation of concentric circular regions of different spin phases whenever the spin splitting of the LB's is closing down with an increasing $\nu$ at a low temperature. The shape of this spin-density wave depends on the size of the system, the temperature, and the filling factor $\nu$, such that the wavelength decreases as $\nu$ approaches an even integer. With an increasing size of the system the spin-density wave acquires properties of a charge-density wave. In the extreme quantum limit (when $\nu \to 0$) no such phase separation is observed in the Hartree-Fock approximation and only one spin state is occupied.

Whether this formation of a spin-density wave state in the HFA for finite systems is physical has to be investigated in higher order approximations. As the Hartree-Fock approximation does not describe the broadening of nearly degenerate Landau levels due to the Coulomb interaction correctly, it can be expected that this approximation overemphasizes the importance of the exchange interaction between the electrons, so one might speculate whether approximations of higher order would decrease the enhancement of the spin splitting or alter the occurance of the spin-density wave state.

In order to limit the approximations needed for the electron-electron interaction we have chosen the system to be strictly two-dimensional, this certainly precludes quantitative comparison with experimental results and the qualitative validity of the model in the limit of a 2DEG with a finite thickness has to be established. Models of 2DEG in quantum dots of finite thickness seem to indicate the qualitative applicability of such calculations as far as the electric subbands are well seperated [18].

The present results give an indication what to expect in optical measurements of the properties of a 2DEG in lightly doped quantum wells and heterostructures, but calculations of the cyclotron resonance or the plasmonic structure of the model are necessary in order to compare with experimental data.

## ACKNOWLEDGMENTS

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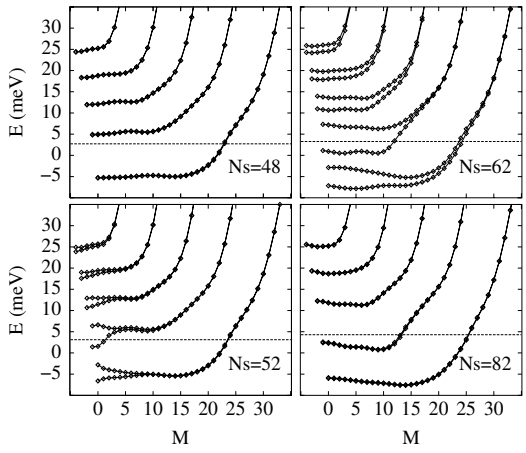


FIG. 1. Energy spectra and chemical potential $\mu$ (horizontal dashed line). The total number of electrons $N_s$ is indicated in each subfigure. $T = 10.0$ K. $R = 1000$ Å, $U_0 = 80$ meV, and $B = 3.0$ T. GaAs bulk parameters: $m^* = 0.067m_0$, $\kappa = 12.4$, $g^* = -0.44$.

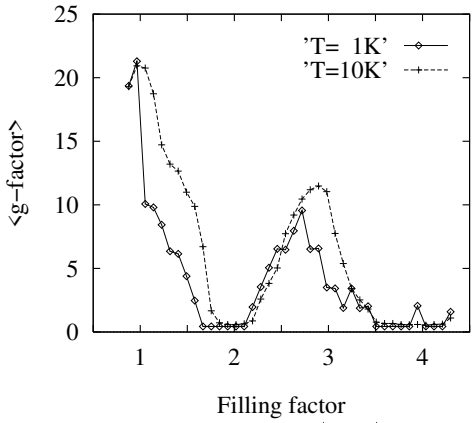


FIG. 2. $g^*_{rms}$ as a function of the average filling factor $\nu$ for $T = 1$ K (solid), and 10 K (dashed). Other parameters are as in Fig. 1.

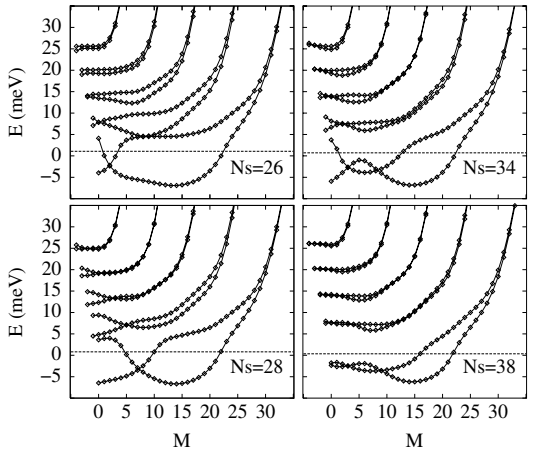


FIG. 3. Energy spectra and chemical potential $\mu$ (horizontal dashed line). The total number of electrons $N_s$ is indicated in each subfigure. Other parameters are as in Fig. 1.

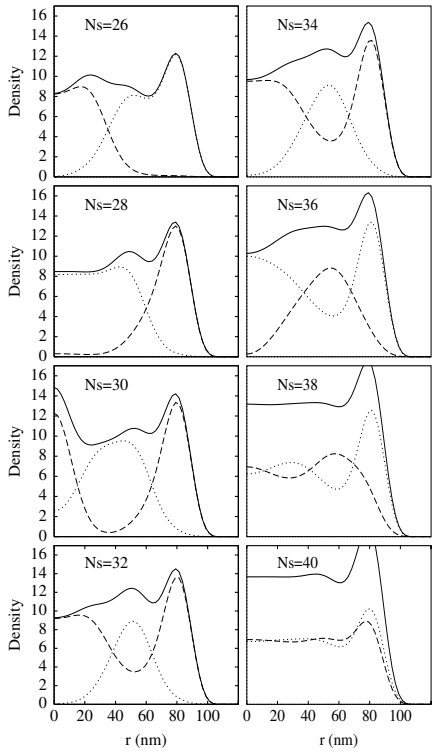


FIG. 4. The electron density $n_s(r) \times 10^4$ nm$^2$ for both spin directions (solid), spin up $s = +1/2$ (dashed), and spin down $s = -1/2$ (dotted) as functions of the radius $r$ of the system. Other parameters are as in Fig. 1.