10 Interest Rates and Monetary Policy

10.1 Introduction

Most central banks in the major industrialized economies implement policy by intervening in the money market to achieve a target level for a short-term interest rate.[1] In the United States, the Federal Reserve sets a target level for the federal funds rate and then influences the supply of bank reserves to maintain the funds rate at the targeted value. The funds rate serves as the operational target for policy, and because it can be closely controlled, it can for most purposes be treated as the instrument of policy. Nevertheless, the theoretical models examined in chapters 2–4 generally treated monetary policy as if it were implemented through the use of a money-supply operating procedure or by some sort of policy rule oriented toward the control of a monetary aggregate. Treating the nominal money supply as the instrument of monetary policy is the approach taken in most undergraduate textbooks in which financial-market equilibrium is summarized by an LM curve.[2] This perspective emphasizes the role of money-demand disturbances in affecting the link between the money supply, interest rates, and real economic activity.

If the central bank can directly control short-term interest rates, the predictability (or unpredictability) of money demand becomes less relevant. Instead, the linkages between the very short-term interest rate the central bank controls and the broad range of market interest rates that affect investment and consumption spending, as well as the link between interest rates and exchange rates, become of critical importance.

In this chapter, we examine the implications of using a nominal interest rate as the operational instrument of monetary policy. The actual policy instrument of central banks is usually the supply of reserve assets, but these can be adjusted to maintain close control of a short-term rate such as the interbank overnight rate. Under the assumption that components of aggregate spending are more closely linked to movements in long-term interest rates, monetary policy actions affecting short-term interest rates are linked to the aggregate economy through the term structure of interest rates. The term structure and the relationship between long-term rates and expected inflation become important in policy design. By treating an interest rate as the variable under the control of the central bank, one obtains a framework that more closely matches the way in which most policy makers view policy.

1. This is certainly true in the United States. As discussed in Bernanke and Mishkin (1992), Kasman (1993), and Morton and Wood (1993), the cental banks in the major OCED countries also use short-term market interest rates as their instrument of policy.

2. See, for example, Abel and Bernanke (1995), Hall and Taylor (1997), and Mankiw (1997).

10.2 Interest-Rate Rules and the Price Level

In this section, we explore the implications for the price level of policies that focus on the interest rate. As the models of chapters 2 and 3 showed, the steady-state real rate of interest is determined by the marginal product of capital, and so, in the long run, monetary policy has no effect on real rates of return.[3] Monetary policy can affect nominal rates, both in the short run and in the long run, but the Fisher relationship links the real rate, expected inflation, and the nominal rate of interest. Targets for nominal interest rates and inflation cannot be independently chosen, and controlling the nominal interest rate has important implications for the behavior of the aggregate price level.

10.2.1 Price-Level Determinacy

Section 5.3.1 made use of the following model, expressed in terms of the price level:

$$y_t = y^c + a(p_t - \mathrm{E}_{t-1}p_t) + e_t \tag{10.1}$$

$$y_t = \alpha_0 - \alpha_1 r_t + u_t \tag{10.2}$$

$$m_t - p_t = y_t - ci_t + v_t \tag{10.3}$$

$$i_t = r_t + (\mathrm{E}_t p_{t+1} - p_t), \tag{10.4}$$

where y, m, and p are the natural logs of output, the money stock, and the price level, and r and i are the real and nominal rates of interest. In chapter 5, the specification of the model was completed by treating the nominal money supply as the policy instrument; the only exception occurred in section 5.4. We want to change this treatment, returning to the approach of section 5.4, by assuming that the nominal interest rate is the policy instrument. Given real output, the price level, and the nominal interest rate, the nominal money supply will be determined endogenously by the money-demand equation (10.3). Although central banks may closely control the nominal rate i, it is the expected real rate of interest r that influences consumption and investment decisions and therefore aggregate demand.[4] This distinction has important implications for the feasibility of an interest targeting rule.

3. If money is superneutral, this will certainly be the case; in models in which money is not superneutral, the simulations in chapters 2 and 3 indicated that the effect of inflation on equilibrium real returns was small.

4. Term structure considerations are postponed until section 10.3.

Suppose that the central bank conducts policy by pegging the nominal interest rate at some targeted value:

$$i_t = i^T. \tag{10.5}$$

Under an interest rate peg, the basic aggregate demand and supply system given by (10.1), (10.2), and (10.4) become

$$y_t = y^c + a(p_t - \mathrm{E}_{t-1}p_t) + e_t \tag{10.6}$$

$$y_t = \alpha_0 - \alpha_1 r_t + u_t \tag{10.7}$$

$$i^T = r_t + (\mathrm{E}_t p_{t+1} - p_t). \tag{10.8}$$

The money demand equation (10.3) is no longer relevant because the central bank must allow the nominal money stock to adjust to the level of money demand at the targeted interest rate and the equilibrium level of output.

Note that the price level only appears in the form of an expectation error (i.e., as $p_t - \mathrm{E}_{t-1}p_t$ in the aggregate-supply equation) or as an expected rate of change (i.e., as $\mathrm{E}_t p_{t+1} - p_t$ in the Fisher equation). This structure implies that the price *level* is indeterminate. That is, if the sequence $\{p^*_{t+i}\}_{i=0}^{\infty}$ is an equilibrium, so is any sequence $\{\hat{p}_{t+i}\}_{i=0}^{\infty}$ where $\hat{p}$ differs from p^* by any constant κ: $\hat{p}_t = p^*_t + \kappa$ for all t. Since κ is an arbitrary constant, $p^*_t - \mathrm{E}_{t-1}p^*_t = \hat{p}_t - \mathrm{E}_{t-1}\hat{p}_t$; hence, y_t is the same under either price sequence. From (10.7), the equilibrium real interest rate is equal to $(\alpha_0 - y_t + u_t)/\alpha_1$, so it too is the same. With expected inflation the same under either price sequence, the only restriction on the price path is that the expected rate of inflation be such that $i^T = (\alpha_0 - y_t + u_t)/\alpha_1 + \mathrm{E}_t p^*_{t+1} - p^*_t$.

The indeterminacy of the price level is perhaps even more apparent if (10.6)–(10.8) are rewritten explicitly in terms of the rate of inflation. By adding ap_{t-1} to and subtracting it from the supply function, the equilibrium conditions become

$$y_t = y^c + a(\pi_t - \mathrm{E}_{t-1}\pi_t) + e_t$$

$$y_t = \alpha_0 - \alpha_1 r_t + u_t$$

$$i^T = r_t + \mathrm{E}_t\pi_{t+1}.$$

These three equations can be solved for output, the real rate of interest, and the rate of inflation. Since the price level does not appear, it is formally indeterminate.[5]

5. Employing McCallum's minimum state solution method (McCallum 1983a), the equilibrium inflation rate is $\pi_t = i^T + (y^c - \alpha_0)/\alpha_1 + (u_t - e_t)/a$ when u and e are serially uncorrelated and the target nominal interest rate is expected to remain constant. In this case, $\mathrm{E}_t\pi_{t+1} = i^T + (y^c - \alpha_0)/\alpha_1$, so permanent changes in the target rate i^T do not affect the real interest rate: $r_t = i^T - \mathrm{E}_t\pi_{t+1} = -(y^c - \alpha_0)/\alpha_1$.

As stressed by McCallum (1986), the issue of indeterminacy differs from the problem of multiple equilibria. The latter involves situations in which multiple equilibrium price paths are consistent with a given path for the nominal supply of money. We saw one example of such a multiplicity of equilibria when studying models of hyperinflation in chapter 4. With indeterminacy, neither the price level nor the nominal supply of money is determined by the equilibrium conditions of the model. If the demand for real money balances is given by (10.3), then the price sequence p^* is associated with the sequence $m_t^* = p_t^* + y_t - ci_t^T + v_t$, while $\hat{p}$ is associated with $\hat{m}_t = \hat{p}_t + y_t - ci_t^T + v_t = m^* + \kappa$. The price sequences p^* and $\hat{p}$ will be associated with different paths for the nominal money stock.

Intuitively, if all agents expect the price level to be 10% higher permanently, such an expectation is completely self-fulfilling. To peg the nominal rate of interest, the central bank simply lets the nominal money supply jump by 10%. This stands in contrast to the case in which the central bank controls the nominal quantity of money; a jump of 10% in the price level would reduce the real quantity of money, thereby disturbing the initial equilibrium. Under a rule such as (10.5) that has the policy maker pegging the nominal interest rate, the central bank lets the nominal quantity of money adjust as the price level does, leaving the real quantity unchanged.[6]

Price-level indeterminacy is often noted as a potential problem with pure interest-rate pegs; if private agents don't care about the absolute price level—and under pure interest-rate control, neither does the central bank—nothing pins down the price level. Simply pegging the nominal interest rate does not provide a *nominal anchor* to pin down the price level. However, this problem will not arise if the central bank's behavior does depend on a nominal quantity such as the nominal money supply.

For example, suppose the nominal money supply (or a narrow reserve aggregate) is the actual instrument used to affect control of the interest rate, and assume it is adjusted in response to interest-rate movements (Canzoneri, Henderson, and Rogoff 1983; McCallum 1986):

$$m_t = \mu_0 + m_{t-1} + \mu(i_t - i^T). \tag{10.9}$$

Under this policy rule, the monetary authority adjusts the nominal money-supply growth rate, $m_t - m_{t-1}$, in response to deviations of the nominal interest rate from its target value. If i_t fluctuates randomly around the target i^T, then the average rate of money growth will be μ_0. As $\mu \to \infty$, the variance of the nominal rate around the targeted value i^T will shrink to zero, but the price level can remain determinate.

6. See Patinkin (1965) for an early discussion of price-level determinacy and Schmidt-Grohe and Uribe (2000a) for a more recent discussion.

To verify these claims, we need to solve for the equilibrium price level (verifying that a determinate solution exists) and then show that the variance of the nominal interest rate around i^T can be made arbitrarily small by increasing μ. Employing the method of undetermined coefficients (see Sheffrin 1983, McCallum 1989, or Attfield, Demery, and Duck 1991), the model can be solved by first postulating candidate solutions for the price level and the nominal interest rate of the form

$$p_t = b_{10} + b_{11}m_{t-1} + b_{12}e_t + b_{13}u_t + b_{14}v_t \tag{10.10}$$

$$i_t = b_{20} + b_{21}m_{t-1} + b_{22}e_t + b_{23}u_t + b_{24}v_t, \tag{10.11}$$

where the b_{ij}'s are as yet unknown coefficients whose values we need to determine. Equations (10.10) and (10.11) express the equilibrium values of p_t and i_t as functions of the contemporaneous disturbances e_t, u_t, and v_t and the state variable m_{t-1}.

Using (10.1) and (10.2) to solve for r_t and substituting the result into (10.4) yields

$$i_t = \left(\frac{\alpha_0 - y^c}{\alpha_1}\right) - \frac{a}{\alpha_1}(p_t - \mathrm{E}_{t-1}p_t) + \frac{1}{\alpha_1}(u_t - e_t) + (\mathrm{E}_t p_{t+1} - p_t), \tag{10.12}$$

while substituting (10.6) into (10.3) and using the policy rule (10.9) yields

$$p_t = \mu_0 + m_{t-1} + (\mu + c)i_t - \mu i^T - y^c - a(p_t - \mathrm{E}_{t-1}p_t) - e_t - v_t. \tag{10.13}$$

This procedure gives two equations to solve for the equilibrium price level and the nominal interest rate, but they both involve expectational variables. These can be evaluated using the assumed solutions (10.10) and (10.11):

$$\mathrm{E}_{t-1}p_t = b_{10} + b_{11}m_{t-1}$$

and

$$\begin{aligned}\mathrm{E}_t p_{t+1} &= b_{10} + b_{11}m_t = b_{10} + b_{11}[\mu_0 + m_{t-1} + \mu(i_t - i^T)] \\ &= b_{10} + b_{11}\mu_0 + b_{11}m_{t-1} - b_{11}\mu i^T \\ &\quad + b_{11}\mu(b_{20} + b_{21}m_{t-1} + b_{22}e_t + b_{23}u_t + b_{24}v_t).\end{aligned}$$

These expressions for expectations can be substituted into (10.12) and (10.13) to yield

$$\begin{aligned}i_t = {}& \frac{\alpha_0 - y^c}{\alpha_1} - \frac{a}{\alpha_1}(b_{12}e_t + b_{13}u_t + b_{14}v_t) + \frac{1}{\alpha_1}(u_t - e_t) + b_{11}\mu_0 - b_{11}\mu i^T \\ & + b_{11}\mu(b_{20} + b_{21}m_{t-1} + b_{22}e_t + b_{23}u_t + b_{24}v_t) - b_{12}e_t - b_{13}u_t - b_{14}v_t\end{aligned}$$

and

$$p_t = \mu_0 + m_{t-1} - \mu i^T - y^c + (\mu + c)(b_{20} + b_{21}m_{t-1} + b_{22}e_t + b_{23}u_t + b_{24}v_t)$$
$$- a(b_{12}e_t + b_{13}u_t + b_{14}v_t) - e_t - v_t.$$

Equating these to the trial solutions (10.11) and (10.10) implies the following values for the unknown b_{ij}s:[7]

$$b_{10} = \mu_0 - \mu i^T - y^c + (\mu + c)b_{20}$$

$$b_{11} = 1$$

$$b_{12} = -\frac{\alpha_1(1-\mu) + (\mu + c)}{\varphi}$$

$$b_{13} = \frac{\mu + c}{\varphi}$$

$$b_{14} = -\frac{\alpha_1(1-\mu)}{\varphi}$$

$$b_{20} = \frac{\alpha_0 - y^c}{\alpha_1(1-\mu)} + \frac{\mu_0 - \mu i^T}{1-\mu}$$

$$b_{21} = 0$$

$$b_{22} = \frac{\alpha_1 - 1}{\varphi}$$

$$b_{23} = \frac{1 + a}{\varphi}$$

$$b_{24} = \frac{\alpha_1 + a}{\varphi},$$

where $\varphi = \alpha_1(1-\mu)(1+a) + (\mu + c)(\alpha_1 + a)$.

Collecting these results, the equilibrium nominal interest rate is given by

$$i_t = b_{20} + \left(\frac{1}{\varphi}\right)[(\alpha_1 - 1)e_t + (1 + a)u_t + (\alpha_1 + a)v_t]. \tag{10.14}$$

7. There is a second solution with $b_{11} = 1/\mu$ and $b_{21} = (1-\mu)/[\mu(\mu + c)]$. McCallum (1986) shows that the solution in the text is the minimal state-variable solution. See also McCallum (1983a).

The nominal rate will fluctuate around i^T if $b_{20} = i^T$, which occurs when $i^T = r + \mu_0$, where $r = (\alpha_0 - y^c)/\alpha_1$ is the average real rate of interest. As a result, the target nominal rate i^T and the average money-growth rate μ_0 cannot be chosen independently. Since $\lim_{\mu\to\infty}|\varphi| = \infty$ and $\lim_{\mu\to\infty} b_{20} = i^T$, (10.14) implies that the variance of the nominal rate around i^T goes to zero as μ becomes arbitrarily large. At the same time,

$$\begin{aligned}\lim_{\mu\to\infty} b_{10} &= \mu_0 - y^c + \lim_{\mu\to\infty}[\mu(b_{20} - i^T) + cb_{20}]\\ &= \mu_0 - y^c - \left(\frac{\alpha_0 - y^c}{\alpha_1} + \mu_0 - i^T\right) + ci^T\\ &= \mu_0 + ci^T - y^c < \infty.\end{aligned}$$

It follows that

$$\lim_{\mu\to\infty} p_t = \mu_0 - y^c + ci^T + m_{t-1} - \left(\frac{1}{a}\right)\left[e_t - \left(\frac{1}{1-\alpha_1}\right)u_t - \left(\frac{\alpha_1}{1-\alpha_1}\right)v_t\right],$$

which remains well defined.

One property of (10.9) is that the nominal money stock is $I(1)$. That is, m_t is nonstationary and integrated of order 1. This property of m causes the price level to be nonstationary also.[8] One implication is that the error variance of price-level forecasts increases with the forecast horizon.

As McCallum (1986) demonstrates, a different equilibrium describing the stochastic behavior of the nominal interest rate and the price level is obtained if the money supply process takes the trend stationary form

$$m_t = \mu' + \mu_0 t + \mu(i_t - i^T) \tag{10.15}$$

even though (10.15) and (10.9) both imply that the average growth rate of money will equal μ_0 (see problem 2). With the money supply process (10.15), the equilibrium price level is trend stationary, and the forecast error variance does not increase without limit as the forecast horizon increases.

It is not surprising that (10.9) and (10.15) lead to different solutions for the price level. Under (10.9), the nominal money supply is a nontrend stationary process; random target misses have permanent effects on the future level of the money supply and therefore on the future price level. In contrast, (10.15) implies that the nominal

8. In contrast, the nominal interest rate is stationary since both the real rate of interest and the inflation rate (and therefore expected inflation) are stationary.

money supply is trend stationary. Deviations of money from the deterministic growth path $\mu' + \mu_0 t$ are temporary, so the price level is also trend stationary.

This discussion leads to two conclusions. First, monetary policy can be implemented to reduce fluctuations in the nominal interest rate without leading to price-level indeterminacy. The Canzoneri, Henderson, and Rogoff (1983) and McCallum (1986) papers showed that by adjusting the money supply aggressively in response to interest-rate movements, a central bank can reduce the variance of the nominal rate around its target level while leaving the price level determinate. However, the level at which the nominal rate can be set is determined by the growth rate of the nominal money supply. The choice of i^T determines μ_0 (or, equivalently, the choice of μ_0 determines the feasible value of i^T). Targets for the nominal interest rate and rate of inflation cannot be independently determined.

Second, the underlying behavior of the nominal money supply is not uniquely determined by the assumption that the nominal rate is to be fixed at i^T; this target can be achieved with different money supply processes. And the different processes for m will lead to different behavior of the price level. A complete description of policy, even under a nominal interest-rate targeting policy, requires a specification of the underlying money supply process.

Models of interest rate targeting are relevant for understanding actual policy. Barro (1989) analyzes interest-rate targeting under the assumption that the target for the nominal rate follows a random walk.[9] Based on U.S. data, Barro finds that the model's predictions for the time-series behavior of nominal rates, the money supply, and inflation are consistent with his interest-rate targeting specification. Rudebusch (1995) provides an empirical model of changes in the federal funds rate target set by the Federal Reserve's FOMC and demonstrates how this target-setting behavior helps account for the behavior of longer-term interest rates. Taylor (1993a) has proposed a simple rule for the nominal interest rate that mimics actual Federal Reserve behavior (see section 5.4), and Clarida, Galí, and Gertler (2000) have estimated variants of the Taylor rule for several major central banks.

10.2.2 Interest-Rate Policies in General Equilibrium

The analysis in the previous section employed a model that was not derived directly from the assumption of optimizing behavior on the part of the agents in the economy. One disadvantage of such models is that there is no natural welfare measure

9. Barro takes the actual policy instrument to be the nominal money supply, with the feedback rule for m determined by the desire to minimize a loss function that depends on the variance of the nominal rate around its target and the variance of one-step-ahead price-level forecast errors.

that can be used to evaluate alternative policies. Assuming that the central bank is concerned with output and inflation variability is probably reasonable, but to derive conclusions about optimal policies, one would like to be able to evaluate the welfare of the representative agent under alternative policies.

Among the more recent papers employing general equilibrium, representative-agent models to study interest-rate policies are those of Carlstrom and Fuerst (1995, 1997) and Woodford (1999b). Carlstrom and Fuerst address welfare issues associated with interest-rate policies. They employ a cash-in-advance (CIA) framework in which consumption must be financed from nominal money balances. As we saw in chapter 3, a positive nominal interest rate represents a distorting tax on consumption, affecting the household's choice between cash goods (i.e., consumption) and credit goods (i.e., investment and leisure). Introducing one-period price stickiness into their model, Carlstrom and Fuerst (1997) conclude that a constant nominal interest rate eliminates the distortion on capital accumulation, an interest-rate peg Pareto dominates a fixed money rule, and for any interest-rate peg, there exists a money growth process that replicates the real equilibrium in the flexible-price version of their model. That is, an appropriate movement in the nominal money growth rate can undo the effects of the one-period price stickiness.

To illustrate the basic issues in a very simple manner, consider the following five equilibrium conditions for a basic CIA economy with a positive nominal interest rate:

$$\frac{u_{c,t}}{1+i_t} = \beta \mathrm{E}_t R_t \left(\frac{u_{c,t+1}}{1+i_{t+1}} \right)$$

$$\frac{u_{l,t}}{u_{c,t}} = \frac{MPL_t}{1+i_t}$$

$$R_t = 1 + \mathrm{E}_t(MPK_{t+1})$$

$$m_t = \frac{M_t}{P_t} = c_t$$

$$1 + i_{t+1} = \mathrm{E}_t \left(\frac{R_t P_{t+1}}{P_t} \right),$$

where $u_{c,t}$ is the marginal utility of consumption at time t, β is the subjective rate of time preference, R_t is 1 plus the real rate of return, $u_{l,t}$ is the marginal utility of leisure at time t, i_t is the nominal interest rate, MPL_t (MPK_t) is the marginal product of labor (capital), P_t is the price level, and m_t is the level of *real* money balances. The first of these five equations can be derived from a basic CIA model by recalling that $u_{c,t} = (1 + i_t)\lambda_t$, where λ_t is the time-t marginal value of wealth. (This assumes that assets markets open before goods markets; see chapter 3.) Since $\lambda_t = \beta \mathrm{E}_t R_t \lambda_{t+1}$ (see 3.26), it follows that $u_{c,t}/(1 + i_t) = \lambda_t = \beta \mathrm{E}_t R_t u_{c,t+1}/(1 + i_{t+1})$. The second equation equates the marginal rate of substitution between leisure and wealth to the marginal product of labor, again using the result that $\lambda_t = u_{c,t}/(1 + i_t)$. The third equation is the definition of the real return on capital. The fourth equation is the binding CIA constraint that determines the demand for money as a function of the level of consumption. The final equation is simply the Fisher relationship linking nominal and real returns. The fourth and fifth equations of this system, as Woodford (1999c) emphasizes, are traditionally interpreted as determining the price level and the nominal interest rate for an exogenous nominal money supply process. The model could be completed by adding the production function and the economy-wide resource constraint.

Rebelo and Xie (1999) argue that this CIA economy will replicate the behavior of a nonmonetary real economy under any nominal interest-rate peg. To demonstrate the conditions under which their result holds, assume that the nominal interest rate is pegged at a value $\bar{\imath}$ for all t. Under an interest-rate peg, the first two equations of the basic CIA model become

$$\frac{u_{c,t}}{1+\bar{\imath}} = \beta \mathrm{E}_t R_t \left(\frac{u_{c,t}}{1+\bar{\imath}}\right) \Rightarrow u_{c,t} = \beta \mathrm{E}_t R_t u_{c,t}$$

and

$$\frac{u_{l,t}}{u_{c,t}} = \frac{MPL_t}{1+\bar{\imath}}.$$

The Euler condition is now identical to the form obtained in a real, nonmonetary economy, an economy not facing a CIA constraint.[10] The level at which the nominal interest rate is pegged only appears in the labor-market equilibrium condition. Thus, Rebelo and Xie conclude that if labor supply is inelastic, the equilibrium with an interest-rate peg is the same as the equilibrium in the corresponding nonmonetary real economy. Any equilibrium of the purely real economy can be achieved by a CIA model with a nominal interest-rate peg if labor supply is inelastic. If labor supply is elastic, however, the choice of $\bar{\imath}$ does have effects on the real equilibrium.

10. If output follows an exogenous process and all output is perishable, equilibrium requires that c_t equal output; the Euler condition then determines the real rate of return.

Under an interest-rate peg, the price-level process must satisfy

$$\mathrm{E}_t\left(\frac{R_t P_t}{P_{t+1}}\right) = 1 + \bar{\imath},$$

while the nominal money supply must satisfy

$$M_t = P_t c_t.$$

These requirements do not, however, uniquely determine the nominal money-supply process. For example, suppose the utility of consumption is $\ln c_t$. Then $u_{c,t} = 1/c_t$, and the Euler condition under an interest-rate peg can be written as

$$\frac{1}{c_t} = \frac{P_t}{M_t} = \beta \mathrm{E}_t R_t \left(\frac{P_{t+1}}{M_{t+1}}\right).$$

Rearranging this equation yields

$$1 = \beta \mathrm{E}_t R_t \left(\frac{P_{t+1}}{P_t} \frac{M_t}{M_{t+1}}\right).$$

If this equation is linearized around the steady state, one obtains

$$r_t + \mathrm{E}_t \pi_{t+1} - \mathrm{E}_t \mu_{t+1} = \bar{\imath} - \mathrm{E}_t \mu_{t+1} = 0,$$

where $\mathrm{E}_t \mu_{t+1}$ is the expected growth rate of money. In this formulation, while real money balances are determined $(m_t = c_t)$, there are many nominal money supply processes consistent with equilibrium, as long as they all generate the same expected rate of nominal money growth.

As we saw in section 10.2.1, the price level is indeterminate under such an interest-rate pegging policy. However, assuming that P_t is predetermined due to price-level stickiness still allows the money-demand equation and the Fisher equation to determine P_{t+1} and m_t (and so the implied nominal supply of money) without affecting the real equilibrium determined by the Euler condition. In that sense, Carlstrom and Fuerst (1997) conclude that there exists a path for the nominal money supply in the face of price stickiness that leads to the same real equilibrium under an interest-rate peg as would occur with a flexible price level.

Carlstrom and Fuerst (1995) provide some simulation evidence to suggest that nominal interest-rate pegs dominate constant money growth rate policies. While this suggests that a constant nominal interest-rate peg is desirable within the context of their model, Carlstrom and Fuerst do not explicitly derive the optimal policy.

Instead, their argument is based on quite different grounds than the traditional Poole (1970) argument for an interest-rate-oriented policy. In Poole's analysis, stabilizing the interest rate served to insulate the real economy from purely financial disturbances. In contrast, Carlstrom and Fuerst appeal to standard tax-smoothing arguments to speculate, based on intertemporal tax considerations, that an interest-rate peg might be optimal.

The tax-smoothing argument for an interest-rate peg is suggestive, but it is unlikely to be robust in the face of financial market disturbances. For example, in an analysis of optimal policy defined as money growth rate control, Ireland (1996) introduces a stochastic velocity shock by assuming that the CIA constraint applies to only a time-varying fraction v_t of all consumption. In this case, the CIA constraint takes the form $P_t v_t c_t \leq Q_t$, where Q_t is the nominal quantity out of which cash goods must be purchased. It is straightforward to show that the Euler condition must be modified in this case to become

$$\frac{u_c(c_t)}{1+v_t i_t} = \beta \mathrm{E}_t R_t \left(\frac{u_c(c_{t+1})}{1+v_{t+1} i_{t+1}} \right).$$

If $v_t \equiv 1$, the case considered by Carlstrom and Fuerst is obtained. If v_t is random, eliminating the intertemporal distortion requires that $v_t i_t$ be pegged and that the nominal interest rate vary over time to offset the stochastic fluctuations in v_t. The introduction of a stochastic velocity disturbance suggests that an interest-rate peg would not be optimal.

10.2.3 Liquidity Traps

The Euler condition and the Fisher equation from a standard money-in-the-utility function (MIU) model can be combined and written as $u_{c,t}/(1+i_t) = \beta \mathrm{E}_t P_t u_{c,t+1}/P_{t+1}$ or, in the absence of uncertainty,

$$\frac{P_t(1+i_t)}{P_{t+1}} = \frac{u_{c,t}}{\beta u_{c,t+1}} \equiv z_t. \tag{10.16}$$

Following Woodford (2000), an interest-rate policy can be written as

$$1 + i_t = \phi(P_t, z_t).$$

The function $\phi(P_t, z_t)$ specifies the setting for the policy instrument (the nominal rate i_t) as a function of the current price level and the variable z_t, which captures the real factors that determine the marginal utility of consumption. Woodford labels policies of this form *Wicksellian* policies. Under such a policy, equilibrium, if it exists, is a