Theoretical Foundations for an Optimal Design of a Sequence of Hydro-Power Projects

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Summary: Optimal sizing of projects, and the determination of local optimal hydroelectric power plant parameters is an integral part in the design of individual hydropower stations. Such a design should be based on an economic trade-off between cost and benefit criteria. Actual optimization can for instance be performed by standard optimization methods or methods based on evolutionary computational techniques. The global optimization of design parameters in a sequence of hydro power projects, however, calls for similar criteria to do the corresponding trade-off between parameters of different projects in a sequence, based on timing and sizing of individual projects.

Previous papers have either focused on the local optimization problem or the overall sequencing and sizing problem. This paper attempts to address theoretically the comparison criteria between different projects by modeling the hydro power sequence as a whole. We investigate the interaction between different aspects of design such as capacity (firm energy or installed capacity), construction and operation costs, optimal timing and optimal sizing of projects and subsequently the interrelation between design parameters, such as dam height, tunnel width, different degrees of diversion etc. The results extend the previous results by encompassing multiple projects and should facilitate better practical global optimization of hydro-power sequences/systems.

Keywords: Optimal design, sequencing, sizing, hydro-plants

I. INTRODUCTION

The design of hydroelectric energy systems involves optimizing a complex range of factors. While it may be difficult to optimize all these simultaneously, to avoid sub optimization, an iterative scheme between them individually may be an appropriate choice. These factors include:

1. Optimal timing and capacity decisions. Determining the optimal project and system capacity of projects has been addressed in [1] and optimal project timing in [2]. It was shown that optimal timing and capacity will depend not only on the sequence chosen for previous projects but also on the capital cost of the next project.

2. Optimal sequencing decisions. Generally the project sequencing problem is a mixed zero/one nonlinear optimization problem involving uncertainty [3]. Optimal sequencing can for instance be determined by dynamic programming or heuristic techniques, as discussed in [4].

3. Optimal sizing of projects. With a fixed sequence of projects, the optimal utilization of resources at each project site will depend on the economic parameters of individual projects in the sequence. These include project capital and operations cost and discount rate. It has been shown that with a simple notion of fixed capacity definition for a project, the optimal sizing can be determined by the long range marginal cost (LRMC) [5] or alternatively, by evolutionary computational techniques [6].

In this paper we assume a definition of capacity that not only depends on plant parameters but also on the optimal timing of the next project and its economic factors such as capital costs, annual cost, interest rates etc.

The paper is arranged as follows. In Section II the hydro system design and sizing framework is described from the point of view of plant parameters and timing and optimal sizing of projects in a sequence.

In Section III we first review the elementary single project design model from [6] and extend it to include market price and gradually increasing demand. Finally, a recursive model describing the expansion process and accounting for independent and interdependent projects parameters is set forth, thereby extending the results previously presented [5]. Finally, Section IV presents conclusions and discussions of results.
II. THE HYDRO EXPANSION PROCESS FRAMEWORK

It is well known that hydroelectric plants are generally unique, due to local geo-topographical conditions. Separate plants may however depend on each other, since a design at each site may depend on how neighboring sites are utilized. These key characteristics, uniqueness and interdependence form a basis for the hydro expansion process described here. An interesting and important question is how strong and significant these interdependencies are.

A common core element in the hydro expansion process is the concept of design parameters. These are physical and measurable variables that describe a certain plant/project design. Examples of such parameters are dam heights and dimensions, tunnel widths, volumes and lengths etc. Some of these parameters may depend on others, while others form an independent basis. For instance, dam volume will depend on dam dimensions while the chosen tunnel width could generally be independent of the dam design.

Consider first a cascade of hydro projects in the same valley (river system). While, obviously, these individual plants are strongly linked hydrologically, design parameters may contribute to further interdependence. For instance, tail water elevation at one plant may depend on the headwater elevation at the next plant downstream. In a single river system, therefore there is undeniably a strong coupling between plant designs. Next consider plants/projects in separate riverbeds. In this case the hydrological linkage may be weaker i.e. mostly confined to cross-correlation of inflow time series to the plants (especially if the inflow is derived from the same hydrological/meteorological system). At first glance, plant parameters (as defined above) seem in this case to be independent between plants. However the economic choice of such parameters will always depend on other plants in time and space since a rational choice must consider the trade-off in benefits a certain parameter value will give against the cost of a particular design or parameter choice.

It is the objective of this paper to analyze the rational economic choice of plant design parameter and how this choice is affected by the above interdependencies.

To analyze the optimal design and of hydroelectric plants we must approach both sides of the coin, i.e. the cost and benefit of a particular design parameter.

Consider first the project cost which may be assumed to depend on the plant design parameters. This may apply to the investment cost as well as the operations cost. The total cost, whether represented as the levelized annual equivalent cost and annual operations cost or as the total discounted project cost NPV (net present value) is therefore a function of these parameters.

Next consider the plant and system benefits, what they are and if and how they depend on plant parameters. Clearly the energy output of a project is in some sense a measure of project benefits if there are customers to purchase the product (electrical energy). As discussed in [1] the marketable output can be defined as project or system capacity and will depend on external conditions of the market.

The concept of project/system capacity may in principle be multi-dimensional but will in any case depend on a number of plant parameters. An obvious example of benefits is the firm energy production capability (to be called FEPC normally in GWh/year). Figure 1 shows an example of how this benefit could depend on the parameter of reservoir volume. Such relationships have been very common in Icelandic engineering reports on new hydro-power projects.

The FEPC is the plant’s possibility to deliver firm energy, assuming a given time-distribution of water flow and power demand of the market within the year that the plant serves. Another benefit would be the income or price times quantity from actual energy produced by the plant. Other examples include the available instantaneous power or installed capacity (MW), the produced reactive power, reactive power capacity, etc.

Therefore, in conclusion, both project costs and quantifiable benefits will generally depend on the design parameters of individual plants.

III. MODELS FOR OPTIMAL DESIGN OF PROJECTS

A. An elementary optimal design model for a single project

As in [6], consider a single hydroelectric project with \( n \) real valued design parameters represented by a vector \( \mathbf{u} = [u_1, u_2, \ldots, u_n] \). The vector, \( \mathbf{u} \), determines the project’s design, representing, for instance, tunnel width, reservoir size, and system capacity and will depend on external conditions of the market.

Figure 1. An example diagram from the Icelandic hydroelectric power system of design curves linking FEPC with reservoir size (Thorisvatn) for different configurations [7]
First, assume that the project cost is a continuous real value function of the design or $C(u) = C(u_1, u_2, \ldots, u_n)$. Secondly, assume a single benefit measure, $x$, such as the project energy output, which also is a function of the design parameters, or $x(u) = g(u_1, u_2, \ldots, u_n)$. To determine the optimal project design, define the following optimization problem:

$$C'(u) = \min_{u_1, u_2, \ldots, u_n} C(u_1, u_2, \ldots, u_n)$$

subject to:

$$x = g(u)$$

The problem is readily solved using Lagrange multiplier theory by forming the Lagrangian:

$$\mathcal{L}(u, \lambda) = C(u) - \lambda [g(u) - x]$$

The necessary optimality conditions are:

$$\frac{\partial \mathcal{L}}{\partial u_j} = 0 \quad \forall j$$

which leads to:

$$\frac{\partial C}{\partial u_j} = \lambda \quad \forall j$$

This means that the incremental cost to benefit ratio should be the same with respect to all design parameters, the ratio being the Lagrange multiplier, $\lambda$.

We can extend the above results to assume, instead, a market unit price of the output, $p$. Then the total profit function, income minus cost, will be:

$$\Pi(u) = p \cdot x - \alpha C(u)$$

where $\alpha$ is the annuity factor relating total discounted project investment and operations cost to annual cost. The necessary conditions for maximizing this profit are:

$$\frac{\partial \Pi(u)}{\partial u_i} = p \cdot \frac{\partial g(u)}{\partial u_i} - \alpha \frac{\partial C(u)}{\partial u_i} = 0$$

which leads to similar results as in (5) or:

$$\frac{\partial C}{\partial u_i} = p \quad \forall j$$

Here (8) shows similarly that the incremental cost to benefit ratio for all parameters should equal the market price, $p$.

### B. Optimal design of a single project to maximize profit assuming gradual project utilization

Next consider the same single project but with gradually increasing demand, where the marketable output of the project increases linearly with time by a quantity, $q$, as for instance in [4] and [5]. Then it can be shown that the discounted value of output is $X_{out} = \frac{q}{\alpha^2} (1 - \exp[-\alpha X_{out} / q])$ and the profit function (with discounted values in capital letters) replacing (6) will be:

$$\Pi(u) = p \cdot \frac{q}{\alpha^2} (1 - \exp[-\alpha x(u) / q]) - C(u)$$

By differentiating (9) we get the optimality condition:

$$\frac{\partial \Pi(u)}{\partial u_j} = \frac{p}{\alpha} \exp[-\alpha x(u)] \frac{\partial g(u)}{\partial u_j} - \frac{\partial C(u)}{\partial u_j} = 0$$

The analogous result to (5) and (8) will be:

$$\frac{\partial C}{\partial u_j} = \frac{p}{\alpha} \exp[-\alpha x(u)] = p \cdot d \quad \forall j$$

where $0 \leq d = \exp[-\alpha x(u)] \leq 1$ is the discount factor for the market price, $p$. The results in (11) indicate how the price should be reduced to account for the gradual utilization of the project’s output and the incremental cost to benefit ratio should be the same as this reduced price for all parameters.

### C. Optimal design of hydro-systems for multiple independent projects

Instead of a single project, consider a sequence of projects. This is the recursive expansion model described for instance in [4] and [5]. Assume a set of $N$ independent projects $\Omega = \{1, 2, 3, \ldots, N\}$ to be constructed in a fixed sequence. For project # $i$ the parameters are $u_i = [u_{i1}, u_{i2}, \ldots, u_{in}]$ where $n_i$ is the number of parameters for project # $i$. Expanding from [5], assume that both the total project cost and the benefit measure (such as the energy output) for each project is a function of the design parameters, or:

$$x_i(u_i) = g_i(u_{i1}, u_{i2}, \ldots, u_{in})$$

and

$$C_i(u_i) = C_i(u_{i1}, u_{i2}, \ldots, u_{in})$$

The total discounted cost (TDC) for all future projects, at a given stage, # $i$, in the sequence, is then given by:

$$P_i = C_i(u_i) + P_{i+1} \exp[-\alpha x(u_i)]$$

where $P_i$ is the TDC at stage # $i$. $q$ is the rate of linear demand increase and $\alpha$ is the continuous discount rate.

Similarly the objective function is as follows, i.e. it minimizes the TDC of the whole expansion sequence, at the construction time of the $i^{th}$ projects:

$$P^* = \min_{u_{i1}, u_{i2}, \ldots, u_{in}} \left\{ \sum_{i=1}^N C_i(u_i) \exp[-\alpha x(u_i)] \right\}$$

subject to:

$$x_i(u_i) = g_i(u_{i1}, u_{i2}, \ldots, u_{in})$$

and

$$C_i(u_i) = C_i(u_{i1}, u_{i2}, \ldots, u_{in})$$

The analogous result to (5) and (8) will be:

$$\frac{\partial C}{\partial u_j} = \frac{p_i}{\alpha} \exp[-\alpha x_i(u_i)] = p_i \cdot d \quad \forall j$$

where $0 \leq d = \exp[-\alpha x_i(u_i)] \leq 1$ is the discount factor for the market price, $p_i$. The results in (11) indicate how the price should be reduced to account for the gradual utilization of the project’s output and the incremental cost to benefit ratio should be the same as this reduced price for all parameters.
where $P_{S,i+1}$ indicates the TDC of the long range back-up supply.

With a definition of the Long Range Marginal Cost (LRMC) by the following equation, (16) (See[5])

$$k_{i+1} = \alpha^2 \cdot \frac{P_{S,i+1}}{q}$$

equation (14) can be rearranged:

$$k_i = \alpha^2 C_i(u) + k_{i+1} \exp\left[-\frac{\alpha x_i(u)}{q}\right]$$ \hspace{1cm} (17)

since (14) is recursive, as shown in [5]. As the TDC depends on future project parameters, (14) can be written more accurately as:

$$P_i(u_i, u_{i+1}, \ldots, u_n) = C_i(u_i) + P_{i+1}(u_{i+1}, \ldots, u_n) \exp\left[-\alpha x_i(u_i)\right]$$ \hspace{1cm} (18)

Differentiating (14) and (18) we get the following optimality condition analogous to (5) and (8):

$$\frac{\partial P}{\partial u_{i,j}} = \frac{\partial C}{\partial u_{i,j}} - P_{i+1} \frac{\alpha}{q} \exp\left[-\alpha x_i(u_i)\right] \frac{\partial \alpha x_i(u_i)}{\partial u_{i,j}} = 0$$ \hspace{1cm} (19)

and obtain a necessary optimality condition:

$$\alpha \cdot \frac{\partial C}{\partial u_{i,i}} = k_{i+1}, d_i = K_{i+1} \hspace{1cm} \forall i, j$$ \hspace{1cm} (20)

where $k_{i+1}$ is the LRMC at stage $i+1$ in the expansion process and is given by:

$$k_{i+1} = \frac{\alpha^2 P_{S,i+1}}{q}$$ \hspace{1cm} (21)

and

$$d_i = \exp\left[-\frac{\alpha x_i(u_i)}{q}\right]$$ \hspace{1cm} (22)

is the discount factor. $K_{i+1}$ is then the “discounted “ LRMC.

Computational algorithms or details will not be discussed here. However, it should be noted that a simple algorithm to optimally design single projects by (5) and (8) would be to solve these equations given the price signals, for instance, by iterative search or by direct analytical methods.

Similarly, the algorithm to utilize (20) would, in summary, be to start with the last project in the fixed sequence and then move backwards. First calculate the LRMC (21) for the last projects and then determine the optimal design by (20). This can then be inserted into (17) and a new LRMC be calculated. By thus moving backward from stage $i+1$ to stage $i$ each time the optimal design of the entire sequence can be calculated stage by stage.

\section{D. Optimal design of hydro-systems for multiple independent projects}

Hydropower projects in separate valleys are generally independent, at least when it comes to the direct construction cost. However, they may be linked by a hydrological cross correlation. Therefore assume that certain project parameters of different projects are linked or related to each other. For instance, in a cascade of plants, the tail-water level of the plant above is the same as the head-water level for the plant above. Or the cost and output of each project depends on a common reservoir, associated with both projects.

Generally, we can assume that certain parameters are linked to each other with a given pattern through a common set of projects. Formally a set of subsets of all projects could be defined where a set of common parameter is associated with each such set.

However, to simplify the notation, consider an expansion example with only 2 projects, each with 2 parameters, one common to both projects and one specific to each project. Thus there is a total of 3 parameters and the parameter vectors can then be defined as $u_i = [u_i, v_i]$ and $u_k = [u_i, v_k]$. Thus the common parameter is $u$.

From (15) the TDC of the sequence with a linear demand function, can now be written:

$$P_i(u_i, u_{i+1}) = C_i(u_i) + C_i'(u_i) \exp\left[-\frac{\alpha x_i(u_i)}{q}\right]$$

$$+ P_{i+1}(u_{i+1}, u_{i+2}) \exp\left[-\frac{\alpha x_i(u_i)}{q}\right]$$ \hspace{1cm} (23)

or recursively.

$$P_1 = C_1(u_1, v_1) + P_2 \exp\left[-\frac{\alpha x_2(u_2, v_2)}{q}\right]$$

$$P_2 = C_2(u_2, v_2) + P_3 \exp\left[-\frac{\alpha x_2(u_2, v_2)}{q}\right]$$ \hspace{1cm} (24)

where $P_1 = \frac{q k_1}{\alpha^2}$ is the a constant TDC of the backup supply and $k_3$ its LRMC (21). Again, differentiating both equations of (24) gives necessary conditions for optimal design with respect to all parameters $u_i, v_i$ and $v_{i+1}$.

This results in:

$$\frac{\partial P}{\partial u} = \frac{\partial C}{\partial u} + \frac{\partial P}{\partial u} \exp\left[-\frac{\alpha x_i(u_i, v_i)}{q}\right]$$

$$- P_2 \frac{\alpha}{q} \exp\left[-\frac{\alpha x_2(u_2, v_2)}{q}\right] \frac{\partial \alpha x_i(u_i, v_i)}{\partial u} = 0$$ \hspace{1cm} (25)

Rearranging results in:

$$\frac{\partial C}{\partial u} + \frac{\partial P}{\partial u} \exp\left[-\frac{\alpha x_i}{q}\right] \frac{\partial x_i}{\partial u} = P_2 \frac{\alpha}{q} \exp\left[-\frac{\alpha x_2}{q}\right]$$ \hspace{1cm} (26)

Similarly, by differentiating the latter equation of (24), we get:

$$\frac{\partial P}{\partial u} = 0$$ \hspace{1cm} (27)

By inserting (27) in (26) we get:
\[
\frac{\partial C_i}{\partial a} = P_i \cdot \frac{\alpha \cdot \exp(-\alpha \cdot \frac{x_i}{q})}{q}
\]  

(28)

which is the same result for interdependent parameters as for independent parameters as shown in (20).

These results can be interpreted so that the ratio incremental cost to incremental benefit should be equal to the discounted long range marginal cost, irrespective of whether we have independent or interdependent parameters in the design of a sequence (or a cascade) of hydroelectric projects.

IV. CONCLUSIONS AND DISCUSSION

In this paper we have expanded from previous papers on the subject of optimal design of hydroelectric systems in a number of ways.

First, an elementary analysis of a single project under different circumstances has been introduced and, at the same time, indicating the linking of these models to the more general recursive models of a whole expansion sequence.

Secondly, it has been shown that the design parameters introduced lead to similar results as the previous models, where the cost for individual projects was assumed to be a function of the project output [5].

Third, the concept of independence and interdependence of project parameters was introduced and it was shown that dependent parameters will lead to the same results as independent parameters in terms of linking adjacent projects in the sequence with a discount factor.

Generally, it is intuitively clear that the marginal cost of each kilowatt-hour in any project (plant) in the whole expansion sequence should be equal with respect to all parameters, otherwise it would be economical to substitute these incremental capacities for each other, considering the time value of money. It should be also equally intuitive that interdependence of projects does not basically change this main conclusion.

In a practical design process in hydroelectric systems, different assumptions may be prevalent when projects are designed, in particular, when a wide time frame is considered, perhaps ranging from years to decades. However this paper has analyzed the interaction of different projects and the marginal price signals that will dictate a proper balance in terms of marginal cost between individual projects.

Finally a note on the planning environment. In the old regulated environment these cost signals should have been present in the overall planning process. In the new deregulated environment, corresponding marginal price (marginal cost) signals should flow between independent generators and influence the design of their individual hydroelectric plants. It remains to be investigated how these signals can be integrated into a planning/expansion market where a regulator can for instance ask for bids to cover the expansion needs of a particular market/country with the associated long term price signals. It is well known that a major problem with the current deregulated regime is the difficulty of ensuring adequate generation expansion, since the current price signals in present day markets are predominantly short term signals in short term markets. It is hoped that the present discussion in this paper could contribute to the determination of the associated marginal costs behind and supporting these long term price signals which are, to a considerable extent, absent form the present day’s electricity markets.

V. REFERENCES