FISH 105

Simple Backcalculation of a Cohort

The stock equation

The equation relating cohort size in one year to the next is:

$$N_{y+1} = e^{-(M+F_y)} N_y$$

which can also be written as:

$$N_y = N_{y+1} e^M e^{F_y} \tag{1}$$

Assume that by the end of the final year of a cohort all fish are caught with natural mortality occuring before fishing, then catch (at age) in the final year + natural mortality = stock size (at age) at the start of the year.

- Assume a yearclass was fished out and the catch in the final year was 57000 fish with $M = 0.2$ and natural mortality occured before fishing. What proportion of fish die from natural mortality? How many fish were there at the start of the year?
- Assume a yearclass was fished out and the catch in the final year was 57000 fish with natural mortality occurring before fishing at a rate of 10%. What was the cohort size at the start of the year?
- Given this matrix of catch data:

  ```
  age <- 13:3
  catch <- c(46,103,217,512,2054,7666, 12710,15119,13772,16286,2614)
  ```

  with $M = 0.2$

  Calculate the cohort size at the start of each year and the proportion of available the fish caught by the fishery — given the same assumptions on mortality as above.

  ```
  M <- 0.2

  N <- catch[1]*exp(M)
  for(i in 2:length(age)) {
    N <- c(N, (catch[i]+N[i-1])*exp(M))
  }

  p <- catch*exp(M)/N
  ```

- If only 50% of the cohort is caught in the final year calculate the cohort size at the start of each year and the proportion of available the fish caught by the fishery as before.

VPA

The catch equation

The catch equation is:

$$C_y = \frac{F_y}{Z_y}(1 - e^{-Z_y})N_y \qquad \text{or} \qquad C_y = \frac{F_y}{F_y + M}(1 - e^{-(F_y+M)})N_y$$

which can be inverted and written as:

$$N_y = \frac{C_y}{1 - e^{-(F_y+M)}} \frac{F_y + M}{F_y} \tag{2}$$

Backcalculations in R using the catch eqn

If these are catch at age data for the final year, assume F_a and M_a are known and calculate the stock size at age using the catch equation.

```
age <- 3:14
catch<-c(7.19,28.43,13.77,34.44,14.13,4.43,1.43,0.35,0.17,0.04,0.02,0)
Fa <- c(0.1,0.2,0.3,0.4,0.5,rep(0.6,7))
M <- rep(0.2, 12)
Z <- Fa+M
N <- catch/((Fa/Z)*(1-exp(-Z)))
```

Cohort Analysis

Approximate solution to the stock equation

This is an approximate solution to the stock equation which does not involve F but assumes all the catch is taken instantaneously half way through the year:

$$N_{a+1,y+1} = (N_{a,y}e^{-M/2} - C_{a,y})e^{-M/2} \tag{3}$$

Forward prediction

Using the catch at age data in:

www.hafro.is/∼lorna/codcatch

which is in millions.

Assume there are 100 million age 3 fish at the start of 1986 with M = 0.2.

```
N3 <- 100
M <- 0.2
```

The size of the age group at the middle of the year will be:

After 6 months of natural mortality:
`N3*exp(-M/2)`

After 6 months of natural mortality and all catch of the year:
`N3*exp(-M/2) - 20.642`

After applying natural mortality for the last 6 months of the year, the number at age 4 at the start of the following year (1987) is:
`N4 <- (N3*exp(-M/2) - 20.642)*exp(-M/2)`

Backcalculation

Reversing the previous equation:

$$N_{a,y} = (N_{a+1,y+1}e^{M/2} + C_y)e^{M/2} \tag{4}$$

Given an estimate of N for the final age group, the size of the cohort in the previous year can then be estimated using the catch at age data.

Assume there is 1 age 14 fish at the end of 1996 ie 1 age 15 at the start of 1997.

```
N15 <- 1e-6
M <- 0.2
```

The number at age 14 at the start of 1996 is then:
`N14 <- (N15*exp(M/2) + 0.005)*exp(M/2)`

Backcalculation of a year

Read in catch at age data and use the first column as row names.

```
cdat <- read.table("codcatch", header=T, as.is=T, check.names=F)
dimnames(cdat)[[1]] <- cdat[,1]
cdat <- cdat[,-1]
cdat <- as.matrix(cdat)
```

Start with some assumptions, including the size of the population in the final year:

```
M <- 0.2
N00 <- c(0, 120, 50, 30, 25, 10, 3, 1, 0.25, 0.1, 7e-2, 2e-2, 1e-4)
names(N00) <- paste(3:15)
```

Using the assumed number at age in the final year and the known catch at age calculate the number at age at the start of the previous year:

```
C99 <- cdat["1999",]
N1 <- N00[-1]
N0 <- (N1*exp(M/2)+C99)*exp(M/2)
```

Add 100 fish for the final age. And shift the age.

```
N99 <- c(N0,1e-4)
names(N99) <- paste(3:15)
```

Then for the previous year:

```
C98 <- cdat["1998",]
N1 <- N99[-1]
N0 <- (N1*exp(M/2)+C98)*exp(M/2)
N98 <- c(N0,1e-4)
names(N98) <- paste(3:15)
```

And the previous:

```
C97 <- cdat["1997",]
N1 <- N98[-1]
N0 <- (N1*exp(M/2)+C97)*exp(M/2)
N97 <- c(N0,1e-4)
names(N97) <- paste(3:15)
```

And so on ...

Do this for some more years.

These can be combined using eg:

```
Nmat <- rbind(N95, N96, N97, N98, N99, N00)
```

Calculate F

Equation (1) can be rewritten as:

$$F_{a,y} = \ln\left(\frac{N_{a,y}}{N_{a+1,y+1}}\right) - M \tag{5}$$

to calculate F_{ay} from the numbers at age table.

Using the calculated cohort stock size `N98` and `N97`, to calculate Fa for a single year.

```
N97a <- N97[-length(N97)]
N98a <- N98[-1]
Fa <- log(N97a/N98a) - M
```

To calculate F for a table of stock numbers at age:

```
A <- ncol(Nmat)
Y <- nrow(Nmat)
mat0 <- Nmat[1:(Y-1), 1:(A-1)]
mat1 <- Nmat[2:Y, 2:A]
Fmat <- log(mat0/mat1)-M
```

- Plot Fa by year eg using `matplot`.
- Select a range of ages (eg 5 to 12) over which to calculate the mean F ($\bar{F}$) – this should exclude the oldest age and the youngest ages and should be representative of effort. Using this age range calculate the average annual fishing mortality and plot the trend in $\bar{F}$.
- The selection pattern for a single year is $s_{ay} = F_{ay}/\bar{F}_{ay}$. Calculate the selection pattern for each year.
- Calculate the average selection pattern for a group of years — this will be limited if you have few years. Ideally this should represent current fishing but possibly not include the final year as it is poorly defined.
- What happens to the stock numbers and fishing mortality if you change the values in `N00`?

Backcalculation of a catch at age table

This exercise is more time consuming and can be returned to later.

A function (`cohortN`) is described in your class notes on page 22 which backcalculates the number at age using assumptions of the number in the last year and the last age. Use this function to calculate stock numbers at age for one or more of the tables of catch at age data:

www.hafro.is/∼lorna/codcatch
www.hafro.is/∼lorna/haddockcatch
www.hafro.is/∼lorna/saithecatch

- Calculate the numbers at age.
- Calculate F from the numbers at age table.
- Select a range of ages over which to calculate the mean F – this should exclude the oldest age and the youngest ages. Using this age range calculate the average annual fishing mortality and plot it.
- Calculate the average fishing mortality at age and plot it.
- Calculate $\bar{F}$ and s_{ay}.
- Calculate the average selection pattern for a group of years.

Cohort analysis starting with assumptions on F

Use the function in your lecture notes (and sent by email `cohortF`) do cohort analysis assuming F on the oldest (for each year) and F for the final year.