FISH 508

Production models

Equilibrium production

$$P = rB\left(1 - \frac{B}{K}\right) \tag{1}$$

where P is the production, B the biomass, K the maximum biomass with no fishing and r the rate of population growth.

- If $r = 0.25$ and $K = 1000$, calculate P for a range of values of B.
- Plot P against B (B as the x axis and P on the yaxis).
- Compare with other values of R and K.

Equilibrium – forward projection

Given biomass B, production is $rB\left(1 - \frac{B}{K}\right)$, if the yield is Y then the stock biomass will be:

$$B + rB\left(1 - \frac{B}{K}\right) - Y$$

i.e. biomass + production - yield

The stock biomass increases if yield < production.

Development of a stock.

- In the code below, the stock is being overfished – with 50% more than the sustainable yield removed for every timestep.
- Calculate the stock trajectory for several years.
- Change the code to remove less than the sustainable yield – how do the curves compare?

```
B0 <- 1000
K <- 1000
r <- 1

B <- 0.75*B0      # remove 25%
nextB <- B
Bvec <- c()
Yvec <- c()

prevB <- nextB
SY <- r*prevB*(1-prevB/K)    # sustainable yield = production
Y <- 1.5*SY                  # yield > production
nextB <- prevB+SY-Y
Bvec <- c(Bvec, prevB)
Yvec <- c(Yvec, Y)
```

```
prevB <- nextB
SY <- r*prevB*(1-prevB/K)
Y <- 1.5*SY
nextB <- prevB+SY-Y
Bvec <- c(Bvec, prevB)
Yvec <- c(Yvec, Y)

#######################
# repeat as necessary
#######################

Blevels <- 0:1000
EYlevels <- r*Blevels*(1-Blevels/K)   # equilibrium yield
# observed yield
plot(Blevels, EYlevels, type="l", lty=2, ylim=c(0, max(c(EYlevels,Yvec))))
lines(Bvec, Yvec, type="b", col=2)
```

Dynamic biomass model

The development of a stock through time can be written as:

$$B_{y+1} = B_y + rB_y\left(1 - \frac{B_y}{K}\right) - Y_y$$

where Y_y is the annual catch.

Catch can be assumed to follow a rule e.g. proportion of stock size.

$$Y_y = pB_y$$

R code to model this could be like this:

```
B0 <- 150
K <- 1000
r <- 1

MSY <- r*K/4   # maximum sustainable yield
BMSY <- K/2    # biomass at maximum sustainable yield

p <- 0.2       # proportion of biomass caught

Bvec <- NULL
Yvec <- NULL
prevB <- B0

for(y in 1:100) {  # 100 years
  Y <- p*prevB     # remove with constant effort
  nextB <- prevB + r*prevB*(1 - prevB/K) - Y
  Bvec <- c(Bvec, prevB)
  Yvec <- c(Yvec, Y)
  prevB <- nextB
}
```

```
Blevels <- 0:1000
EYlevels <- r*Blevels*(1-Blevels/K)   # equilibrium yield
# observed yield
plot(Blevels, EYlevels, type="l", lty=2, ylim=c(0, max(c(EYlevels,Yvec))))
lines(Bvec, Yvec, type="b", col=2)
```

- Confirm graphically that the calculations for `MSY` and `BMSY` are correct.
- Calculate the yield and biomass using the code above. What happens? Add a line representing MSY.
- Plot B_{t+1} against B_t. Add lines representing BMSY and the equilibrium relationship between B_{t+1} and B_t. e.g.

```
plot(Bvec[-100], Bvec[-1], xlim=c(0, 1000), ylim=c(0,1000),
xlab="Bt", ylab="Bt+1")
abline(0,1, col=2)
abline(h=BMSY, col=2)
abline(v=BMSY, col=2)
```

- Instead of removing with constant effort, remove a constant value e.g 80 and repeat the calculations.
- What happens when you fish at MSY with B0 = 150?
- What happens when you fish at MSY with B0 = 800?
- Next, add an error term to the production equation in the function.

  To get a random value from the Normal distribution with mean 0 and standard deviation 'sd' use `rnorm(1, 0, sd)`. This code should be added to the existing function.

```
sigp <- 0.05

for(y in 1:100){
.
  errp <- exp(rnorm(1,0,sigp))
  nextB <- prevB + r*prevB*(1-prevB/K)*errp - Y
.
}
```

Statistical stock assessment – dynamic biomass model

As before, the development of a stock through time can be written as:

$$B_{y+1} = B_y + rB_y\left(1 - \frac{B_y}{K}\right) - Y_y$$

Typically, only the total catch Y_y and an abundance index I_y are known.

There are errors on the index data:

$$I_y = qB_y e^{\epsilon_y}$$

Using the R code from the previous section:

- What could the abundance indices I_y look like? (Abundance indices from a survey are derived from constant effort.) Include I_y in the loop in a similar way to Y_y with an error term.
- Plot Y_y against I_y.
- Vary the amount of error and compare the resulting indices.

Given data on the catch and survey indices for nephrops:

```
# Annual catch data
Y<-c(2398,2520,2603,2672,2459,2385,
     2564,2712,2240,1866,1692,2157,
     2230,2381,2238,1027,1633,1228,1411)
# Annual abundance index
I<-c(45.5,51.8,51.5,47.8,45.6,56.4,61.3,
     52.6,39.9,36.0,40.0,42.1,51.3,51.4,
     38.0,27.0,35.2,31.3,38.9)
```

a model can be fitted.

There a 4 parameters to be estimated, K, $B0$, r and q. It is advisable not to estimate all of these at once. e.g. you could estimate K, $B0$ and r with q fixed, then estimate r and q, then estimate all. Each time using the parameter estimates from the previous optimisation.

To estimate K, $B0$ and r, R code could look like this:

```
B0 <- 2*mean(Y)     # want starting biomass greater than catch
q <- mean(I)/B0     # as I = qB
K <- B0*1.3         # need some production ie K!=B0 and prefer K > B0
r <- 1.8
input <- c(K, B0, r)
B <- B0

ssefn <- function(input){
  K <- input[1]        # parameters
  B0 <- input[2]
  r <- input[3]

  B <- B0
  Yvec <- NULL
  Bvec <- NULL
  Ihat <- NULL
  yrs <- 1:length(Y)

  for(y in yrs){
    SY <- r*B*(1-B/K)     # production
    Bvec <- c(Bvec,B)
    Ihat <- c(Ihat,q*B)
    B <- B+SY-Y[y]
    B <- ifelse(B<0, 0, B)
  }
```

```
  SSE <- sum((I-Ihat)^2)
  return(SSE)
}

# to optimise
est <- nlm(ssefn, input, typsize=input, iterlim=1000)
```

The command `ifelse` combines the `if` and `else` statements. e.g.

```
x <- 1:10
ifelse(x<5, 0, x)
ifelse(x==5, 0, x)
ifelse(x%%2==0,  "even", "odd")
```

- Optimise the parameters using the data provided. You should do this in several steps using the values from the previous optimisation in the next optimisation.
- Plot the total biomass of the stock through time.
- Plot the equilibrium yield curve.
- Start with different parameter values e.g. $K = 10000$, $B0 = K/2$, $r = 1$ and $q = 0.01$. Are the resulting estimates different? Has your understanding of the stock changed?

Age–based dynamic production models

In your lecture notes there is R code to model age–based dynamic production models and optimise the parameters. Using the same data work though these examples, making sure you understand the aim of each step.