Two phase flow – concepts and definitions

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Single phase flow

In general, the fluid can be in either of the following two states:

- **Liquid phase, such as water:**
  - Considered incompressible in most cases.
  - Pressure shocks and waves can occur.
  - Flow is classified as laminar or turbulent.

- **Gas phase, such as air or steam:**
  - Compressible in many applications.
  - Density very different from liquid state.
  - Properties vary considerably with temperature.
  - Flow is classified as laminar or turbulent.
Properties related to single phase flow

- The fluid is homogenous and in most cases be considered Newtonian, such that
  \[ \tau = \mu \frac{\partial u}{\partial y} \]

- Main properties are density, \( \rho \), and viscosity, \( \mu \), which are temperature dependent in general.

- Fluid classification is given by the Reynolds number
  \[ \text{Re} = \frac{\rho u d}{\mu} \]

- Pressure drop calculations are well defined for both laminar and turbulent flow.
Multi-phase flow

There are several combinations possible:

- Gas and liquid: Steam and water in geothermal pipes, oil and methane.
- Gas and solid particles: Sand transport, aluminium oxide transport. transport.
- Liquid and solids: Flowing mudslides, quicksands.
- Gas liquid and solids: Some cases in oil transport from reservoirs.

In most cases the flow is only in two case state.
Two phase calculations

All practical calculations are much more complex, when compared to single phase.

- Pressure drop calculations involve complex friction relations and momentum changes.
- Heat loss through pipe walls are complex, because of unstable boundary layers and phase mixing.
- Temperature variations in the fluid are difficult to predict, because of turbulence and phase mixing.
- Flow stability can vary with different two phase flow and violent flow changes can occur.

Conclusion: It is very difficult to derive theoretical formulas for the properties mentioned above.
Practical use of two phase flow calculations

In most cases it is necessary to use experiments and measurements to develop empirical relations. Examples of two phase flow situations in geothermal utilization involve:

- Heat exchangers, when working fluid is heated by external energy source.
- Vertical flow in geothermal wells.
- Flow in surface pipes in relation to geothermal power production.
- Performance of steam separators.
- Flow in low pressure parts of turbines.
Variations in phase ratios

Two different cases of flow can be considered:

1. Steady flow with respect to phase change, where no such change takes place. In most cases, pressure and temperature are constant.

2. Unsteady flow, where phase change takes place. This can happen when pressure drops at constant enthalpy and water boils to form steam.

An example of the second case is seen in a geothermal well, where water boils which in turn increases the amount of steam in the pipe.
Definitions – Mass flow ratio

This parameter is defined as $x$, and is the ratio between the mass flow of steam and the total mass flow, at a given time in a given cross section.

\[ x = \frac{\dot{m}_g}{\dot{m}_g + \dot{m}_l} = \frac{\dot{m}_g}{\dot{m}} \]

The ratio is sometimes denoted as *steam quality*, especially in relation with steam power cycles.

Note that the subscript $g$ denotes gas and $l$ denotes liquid.
Definitions – Static dryness

Here the definition $q$ is slightly different from $x$. $q$ is the ratio between the mass of steam and the total mass, at a given time in a given cross section.

$$q = \frac{m_g}{m_g + m_l} = \frac{m_g}{m}$$

Note that the unit for $m$ is $\text{kg/m}$. In general there is a difference between $x$ and $q$ if the flow velocity of the two phases are different.
Definitions – Slip ratio

This is the ratio between the flow velocities in gas and liquid, for a given cross section. Thus

\[ S = \frac{v_g}{v_l} \]

where \( v_g \) is gas phase velocity and \( v_l \) is liquid phase velocity. Note that for the special case \( S = 1 \) then \( x = q \).
Definitions – Void fraction

This parameter denotes the fraction between the area of steam and the total area of a given cross section of a pipe.

\[ \alpha = \frac{A_g}{A_g + A_l} = \frac{A_g}{A} \]

The following figure explains the relation in a pipe.
Volumetric dryness and mass velocity

1. The volumetric dryness is defined as the ratio between volumetric gas flow and total volumetric flow in a given cross section. The volumetric flow is usually given in $m^3/s$.

$$\beta = \frac{Q_g}{Q_g + Q_l} = \frac{Q_g}{Q}$$

2. The mass velocity is simply mass flow per unit area, or

$$G = \frac{\dot{m}}{A}$$

which has the unit $kg/(m^2s)$.
Relation between $q$ and $\alpha$

For each phase we have $m = \rho A$ and $A_l = (1 - \alpha)A$. Thus

$$q = \frac{\rho_g A_g}{\rho_g A_g + \rho_l A_l} = \frac{\rho_g \alpha A}{\rho_g \alpha A + \rho_l (1 - \alpha) A}$$

$$= \frac{\rho_g \alpha}{\rho_g \alpha + \rho_l (1 - \alpha)} = \frac{\alpha \rho_g}{\rho \alpha}$$

The value $\rho_\alpha$ is the mean density of the two phases in relation to the void fraction.
Relation between $\beta$ and $x$

Here we use that $\dot{m} = \rho Q$ for each of the phases. Also we have $\dot{m}_l = (1 - x)\dot{m}$. We get

$$
\beta = \frac{\dot{m}_g}{\rho_g} + \frac{\dot{m}_l}{\rho_l} = \frac{x\dot{m}}{\rho_g} + \frac{(1-x)\dot{m}}{\rho_l} = \frac{x}{\rho_g} + \frac{(1-x)}{\rho_l}
$$

A mean value of the density can be defined as

$$
\frac{1}{\rho_x} = \frac{x}{\rho_g} + \frac{(1-x)}{\rho_l}
$$

which results in the simple relation $\beta = x \frac{\rho_x}{\rho_g}$. 
Slip ratio relationships

For each phase we have \( \dot{m} = \rho A v \) which means that the slip ratio can be defined as

\[
S = \frac{\dot{m}_g \rho_l A_l}{\rho_g A_g \dot{m}_l} = \frac{x \rho_l}{(1 - x) \rho_g} \frac{1 - \alpha}{\alpha}
\]

From other relations we also have

\[
\frac{1 - x}{\rho_l} = \frac{1 - \beta}{\beta} \frac{x}{\rho_g}
\]

and therefore

\[
S = \left( \frac{\beta}{1 - \beta} \right) \left( \frac{1 - \alpha}{\alpha} \right)
\]