Finding the Pulsar Period and the Distance to the Cepheid Variable Bp Circini.

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In this paper, we use data from the LCOGT telescopes to calculate the period of the brightness pulsations of the Cepheid variable Bp Circini, and estimate the distance to it. We used measurements of the change in brightness of Bp Circini over time and showed that the fall and rise in brightness over time were statistically significant. Since Cepheids have a relationship between the pulsation period and the absolute magnitude, the absolute magnitude could be calculated from this data. The absolute magnitude, along with the apparent magnitude was then used to calculate the distance. The period of the brightness pulsations of Bp Circini was measured to be $P = 2.3 \pm 0.2$ days. The accepted value is $P = 2.4$ days. \cite{14} From the value of its apparent magnitude, the estimated distance to Bp Circini is $D_L = 1000 \pm 200$ pc.

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I. INTRODUCTION

Cepheid variables are stars that change its luminosity periodically over time. There is a relationship between the Cepheids’ pulsation period and their absolute magnitude, so they are used as standard candles for distance measurements. Cepheids are used to measure distances to galaxies in the Local group. \cite{3} \cite{4} In this paper, we will use measurements of the luminosity of the Cepheid Bp Circini to determine its period, using data from the LCOGT telescopes. We will also estimate the distance to Bp Circini from its period.

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II. THEORY OF CEPHEID STARS

A. Cosmic Distance Ladder

The methods used to measure distances in our universe are called the Cosmic Distance Ladder. These methods are shown schematically on figure 1. Since stars radiate light uniformly in all directions, the flux from a star is distributed over a spherical surface. As the star is farther away from earth, less portion of the flux is detected on earth. Therefore, the brightness of a star as seen from earth is generally no indicator of the distance to it. Therefore, other methods than direct flux measurements must be used for distance measurements. The name Cosmic Distance Ladder refers to the fact that different methods are used to measure different magnitudes of distances in the universe. For example, the second step in the ladder, the Stellar Parallax method, can only be used to measure the distance to nearby stars, or stars within a distance of approximately 1 kpc from earth. As a comparison, the distance from earth to the Galactic Center is around 8 kpc \cite{5} and the distance to Andromeda, the closest galaxy to the Milky Way, is about 780 kpc. \cite{6} Stellar Parallax is therefore only used to measure distances to stars within our galaxy. However, the last step in the ladder, to use Hubble’s Law to measure distances, can only be used to measure distances to galaxies that are moving away from the Milky Way. Hubble’s Law can be used to measure distances up to more than 1000 Mpc. Each step in the cosmic distance ladder relies on measurements from the previous steps, since the previous steps are used to scale the latter steps. Therefore, accurate measurements of the first steps gives more accurate measurements of all latter steps. \cite{7} \cite{4}

B. Cepheid Variables as Standard Candles

The fourth step in the ladder is a measurement of Cepheid variable stars. With Cepheid variables, one can measure distances up to about 100 Mpc. They generally have a period of days to months, and the relationship
between their period $P$ in days, and their absolute magnitude $M$ in visible filter is [8]

$$M = (-2.43 \pm 0.12)(\log_{10}(P) - 1) - (4.05 \pm 0.02) \quad (1)$$

This equation holds for classical Cepheids, as Bp Circini is.

If the apparent magnitude of two objects are $m_1$ and $m_{ref}$, and the objects have brightness measured from Earth $I_1$ and $I_{ref}$, then the following holds [9]

$$m_1 - m_{ref} = -2.5 \log_{10}\left(\frac{I_1}{I_{ref}}\right) \quad (2)$$

Therefore, if we choose a reference star, with a known apparent magnitude and brightness as measured from earth, then we can find the apparent magnitude of the variable star Bp Circini. The relationship between the absolute magnitude $M$ and the apparent magnitude $m$ of an object is

$$M = m - 5\log_{10}(D_L) - 1 \quad (3)$$

where $D_L$ is the distance to the object in parsecs. [10] Therefore, knowing the absolute and apparent magnitudes $M$ and $m$ of Bp Circini, from equations 1 and 2, we can find the distance $D_L$ to it by

$$D_L = 10^{(m-M)/5+1} \quad (4)$$

III. EXPERIMENTAL TECHNIQUE

The LCOGT telescopes were used to collect data of Bp Circini’s luminosity over time. The procedure of LCOGT measurements, and the pipeline for refining photos that the LCOGT telescopes take was discussed in the previous lab report on exoplanets. [11] As before, we eliminated photos manually that were not in focus or unusable. We could see that the picture was out of focus if the brightness of Bp Circini was obviously less than on the majority of the photos, and some photos had obvious defects such as a very large brightness gradient throughout the picture. All such photos were eliminated. We also eliminated photos by plotting the brightness of chosen stars and seeing on the plot which photos showed less brightness than others, as explained later.

As explained in the previous report, [11] the total brightness of a single star was taken to be the total number of photons counted inside a circle of a specific radius $R$ around the maximum value of the star. The radius of the star was determined in the software SalsaJ [12] as before, and by inspecting where the brightness of the star had diminished significantly, we chose $R = 20$ pixels for Bp Circini. The center of the circle was again taken to be in a corner point of the pixel of maximum photon count, as in SalsaJ. We corrected for background noise by subtracting the average photon count in an annulus with inner radius $R_i = 30$ pixels and an outer radius of $R_o = 40$ pixels from the photon count of the star. Even though LCOGT’s pipeline corrects for background noise in various ways, the calibration frames that are used are not necessarily designed optimally for our measurements, so we correct the photos in this way in order to get more accurate measurements. [13]

The brightness of Bp circini increases by about 40% from its minimum value to its maximum value in the periodic cycle. [14]. However, our measurements are taken over 3 days in different locations which probably will have different weather, light pollution and other external conditions. Also, there is an error in the chosen exposure time due to the shutter mechanism. To account for this, we chose a constant magnitude reference star. The photon count of Bp Circini was divided by the photon count of the reference star to get a better estimate of the apparent brightness of the Cepheid.

A. Measurements

The measurements of the variable star Bp Circini were taken over a period of 3 days, from Tuesday 10th March 2015 at 0:00, to Friday 13th March 2015 at 0:00. The data was taken with a red filter and 0.01 s exposure time. The filter and exposure time were chosen from a test run of various filters and exposure times, and these parameters gave the best photos. The period of the brightness change in Bp Circini is 2.4 days, so by taking measurements over 3 days, we expect to see a full cycle of dimming and brightening of the star. We used LCOGT’s Visibility Tool to see that Bp Circini was observable with at least one of the LCOGT telescopes most of the time on these days, as can be seen on figure 2.
FIG. 2: The graph shows the visibility of Bp Circini in each telescope that belongs to the LCOGT. The areas that are shaded show times when it is visible, however, from around 10 to 14 and 19 to 22 in UT time, it will not be visible with any telescope.

IV. DATA SUMMARY

Figure 3, shows the flux of Bp Circini divided by the flux of the comparison star. Figures 4 and 5 show the total number of photons from Bp Circini and the comparison star respectively, which was used to see which photos were out of focus. The graph in figure 6 shows the same as in figure 3, with the outlier points discarded. Each point in the figures represent one picture from the telescope, that is a picture with an exposure time of 0.01 s.

FIG. 3: Flux of Bp Circini divided by the flux of a comparison star as a function of time. This data shows that some of the points are obviously outliers.

FIG. 4: Number of photons from Bp Circini as a function of time, after having corrected for background noise. All photos that had less than $1.7 \cdot 10^5$ photons were taken to be out of focus and discarded.

FIG. 5: Number of photons from the comparison star as a function of time, after having corrected for background noise. All photos that had less than $1.5 \cdot 10^4$ photons were taken to be out of focus and discarded.
By analyzing the data, we get that the null hypothesis is rejected with a confidence level of 99%. A plot of the best line fit is shown on figure 7.

\[ y = 7.01E-02x + 1.15E+01 \]

FIG. 6: Flux of Bp Circini divided by the flux of a comparison star as a function of time, with pictures not in focus discarded. We can see that there is a downward tendency of the data, until about 30 h after midnight, and after that the data seems to rise again, although we are missing measurements from about 35 h to 45 h after midnight.

FIG. 7: Best line of data in figure 6, from 0 h after midnight to 40 h after midnight, along with its equation. As can be seen from the equation, the slope of the best line is less than zero, as predicted by hypothesis testing.

\[ \epsilon_i = y_i - a - b \cdot x_i \]  

where \((x_i, y_i)\) are the points that the best line was fitted to. The method of hypothesis testing for a best line fit, assumes that the residues are normally distributed, with a mean 0 and some unknown standard deviation \(\sigma\). To test whether this is a reasonable assumption, we plot the residues \(y_i\) as a function of \(x_i\) and also make a normal probability plot of them, as can be seen on figures 8 and 9. The figures show that the residues do not seem to follow a normal curve very well, as expected from the fact that the brightness of a cepheid star is not piecewise linear in time. However, although there is a deviation from a normal distribution, we take the deviation to be small enough so that we can take our statistical analysis to be correct, that is, that the diminished brightness of Bp Circini over the time from 0 h to 40 h is not due to random fluctuations. [15]
FIG. 8: Plot of the residues of the data in figure 7 as a function of time. There seems to be a tendency for the residues to go down and up again. This could be due to the fact that we do not expect the brightness curve of Bp Circini to follow a linear fit. However, we would need more measurements to see whether the residues indeed have an upwards or downwards tendency, or if this is just because of random fluctuations in this case.

FIG. 9: Normal plot of the residues of the data in figure 7. If the residues follow a normal curve, they should lie close to the red line in the plot. The residues do not seem to fit very well to the line, they first are above it, then below it and then above it again. This is due to the fact that we do not expect a linear brightness curve for a cepheid, so we do not expect the residues to be perfectly normally distributed.

2. Upward Tendency of Slope

For the time of 20 h to 60 h after midnight, we want to show that the data on figure 6 shows an upward tendency of slope. As for the downward tendency, we do not expect the slope to be linear, but we approximate the upward tendency with a line and see if it has a slope greater than zero, and the positive value of the slope is statistically significant. Again, we let $b$ be the slope of the best line fit of the data in figure 6, from 20 h to 60 h after midnight on 10th March, and $a$ be the intersection with $y$ axis. We use hypothesis testing with the null hypothesis

$$H_0 : a = 0$$

and the alternative hypothesis

$$H_0 : a > 0$$

By analyzing the data, we get that the null hypothesis is rejected with a confidence level of 99%. A plot of the best line fit is shown on figure 10.

FIG. 10: Best line and its equation of data from figure 6, from pictures taken 20 h to 60 h after midnight. As can be seen from the equation, the slope of the best line is less than 0, as predicted by hypothesis testing. There are only points on the graph from approximately 20 h to 30 h and again from 45 h to 55 h, so we are missing measurements in between those regimes. The points seem to have an upward tendency in time, but from this plot, we do not get any information of the curve between the regimes.

A plot of the residues of the data as a function of time and a normal probability plot of the residues can be seen on figures 11 and 12.
FIG. 11: Plot of the residues of the data in figure 10 as a function of time. There is no apparent tendency for the residues to follow a specific curve, and they do not seem to tend upwards or downwards as a function of time. This is the same behavior as a normal distribution of residues would have.

The data from 20 h to 60 h seems to have normally distributed residues so hypothesis testing on the linear fit is reasonable. This might be due to the fact that we do not have any measurements, and hence do not see any specific curve, from approximately 35 h to 45 h after midnight. We take our statistical hypothesis testing to be reasonable to estimate that the upward tendency of the points is not due to random fluctuations.

\[ P = 55 \pm 5 \text{ h} \] (10)

since the brightness of Bp Circini at 55 h after midnight is approximately the same as at midnight. This corresponds to

\[ P = 2.3 \text{ days} \pm 0.2 \text{ days} \] (11)

The accepted value is that the period of Bp Circini is \( P = 2.4 \text{ days} \), [14] so the accepted value is within our margin of error.

C. Determination of the distance to Bp Circini

The apparent magnitude of Bp Circini, \( M \), can be found from equation 1, by letting \( P = 2.3 \text{ days} \pm 0.2 \text{ days} \). Since the logarithmic function is increasing, and we know that \( \log(x) > 0 \) for \( x > 1 \), then we can use the error margin values of \( P = 2.5 \text{ days} \) and \( P = 2.1 \text{ days} \), along with the error margins in equation 1 to determine the error in \( M \). We get

\[ M = -2.5 \pm 0.2 \] (12)

The apparent magnitude of Bp Circini is \( m = 7.54 \pm 0.17 \). [14] The magnitude value of Bp Circini was looked up, instead of looking up the value for another star and calculating the apparent magnitude for Bp Circini with equation 2. We could have looked up the magnitude and flux value for a known star, for example Vega from which the scale is defined from; Vega is taken to have apparent magnitude of 0. [9] However, since Vega, and other recognized stars, are not on our photos along with Bp Circini, this would give a very crude estimate. We would have to look up the flux of Vega, and since the telescope we used is not able to capture all this flux, that would give a large amount of error. Therefore, we would not know how effectively the telescope would measure the flux of Vega, compared to the measurements we got from Bp Circini. The other possibility would have been to look up the apparent magnitude of a star in our photo that is not Bp Circini. However, since Bp Circini is the brightest star in our photos, the values of magnitude of the other stars in the photo are not a common knowledge of astrophysicists, like the value for Vega, so that would not add any sophistication to our calculations.

From the values \( M = -2.5 \pm 0.2 \) and \( m = 7.54 \pm 0.17 \), we calculate from equation 4, that the distance to Bp Circini is

\[ D_L = 1000 \pm 200 \text{ pc} \] (13)

Note that the error can be estimated from the extreme values of \( M \) and \( m \), since the power function \( f(x) = 10^x \)
is increasing. Therefore, the largest possible value of $D_L$ can be found from the largest possible value of $m$, $m = 7.71$, and the smallest possible value of $M$, $M = -2.7$ and analogous for the the smallest possible value of $D_L$.

VI. CONCLUSIONS

Data from the LCOGT telescopes, collected over 3 days, showed that the brightness of the Cepheid variable Bp Circini diminished and rose again during this timespan. Statistical hypothesis testing was used to show that the fall and rise in the brightness were statistically significant. The period of the brightness pulsations of Bp Circini was measured to be $P = 2.3$ days ±0.2 days. The accepted value is $P = 2.4$ days, [14] so the accepted value is within the margin of error of the measured value. From the value of the apparent magnitude of Bp Circini, which was looked up on Wikipedia [14], the estimated distance to the Cepheid variable is $D_L = 1000 \pm 200$ pc.

References