Exercise 21.56

The dipole moment of the water molecule \( \text{(H}_2\text{O)} \) is \( 6.17 \times 10^{-30} \text{ C} \cdot \text{m} \). Consider a water molecule located at the origin whose dipole moment \( \vec{p} \) points in the +x-direction. A chlorine ion (\( \text{Cl}^- \)), of charge \( -1.60 \times 10^{-19} \text{ C} \), is located at \( x = 3.00 \times 10^{-9} \text{ m} \). Assume that \( x \) is much larger than the separation \( d \) between the charges in the dipole, so that the approximate expression for the electric field along the dipole axis can be used.

Part A

Find the magnitude of the electric force that the water molecule exerts on the chlorine ion.

ANSWER:

\[
F = 6.58 \times 10^{-13} \text{ N}
\]

Correct

Part B

What is the direction of the electric force.

ANSWER:

- \( -x \)-direction
- \( +x \)-direction

Correct

Part C

Is this force attractive or repulsive?

ANSWER:

- attractive
- repulsive

Correct

Torque on a Dipole in a Uniform Field

Consider an electric dipole whose dipole moment (a vector pointing from the negative charge to the positive charge) is oriented at angle \( \theta \) with respect to the y axis. There is an external electric field of magnitude \( E \) (independent of the field produced by the dipole) pointing in the positive y direction. The positive and negative ends of the dipole have charges \( +q \) and \( -q \), respectively, and the two charges are a distance \( d \) apart. The dipole has a moment of inertia \( I \) about its center of mass. It will help you to imagine that the dipole is free to rotate about a pivot through its center.
Part A

What is the net force \( \vec{F}_{\text{net}} \) that the dipole experiences due to the electric field?

Express \( \vec{F}_{\text{net}} \) in terms of the given variables and the unit vectors \( \hat{i}, \hat{j} \).

**Hint 1.** What is the force on the positive charge?

What is the force \( \vec{F}_{\text{pos}} \) that the charge \( +q \) experiences due to the electric field in the \( y \) direction?

Express \( \vec{F}_{\text{pos}} \) in terms of the given variables and the unit vectors \( \hat{i}, \hat{j} \).

**ANSWER:**

\[
\vec{F}_{\text{pos}} = qE\hat{j}
\]

**ANSWER:**

\[
\vec{F}_{\text{net}} = 0\hat{i} + 0\hat{j}
\]

**Correct**

Part B

What is \( \tau_{\text{cm}}(\theta) \), the magnitude of the torque that the electric field exerts about the center of mass of the dipole?

Express the magnitude of the total torque in terms of the given quantities.

**Hint 1.** Find the torque on the positive charge

What is the magnitude of the torque \( \tau_{\text{pos}} \) about the center of mass of the dipole due to the force that the electric field exerts on the positive charge? Recall that the torque vector is given by \( \vec{\tau} = \vec{R} \times \vec{F} \), where \( \vec{R} \) is the position vector from the center of the dipole to the applied force \( \vec{F} \).

Express the magnitude of the torque in terms of the given quantities.
**Hint 1.** Find the force on the positive charge

What is the force \( \vec{F}_{\text{pos}} \) that the charge \( +q \) experiences due to the electric field in the \( y \) direction?

Express your answer in terms of the given variables and the unit vectors \( \hat{i}, \hat{j} \).

**Answer:**

\[ \vec{F}_{\text{pos}} = qE \hat{j} \]

**Hint 2.** What is the moment arm?

A simple way to compute the magnitude of the torque about a pivot point is to multiply the magnitude of the force by the moment arm, which is the distance from the line of force to the chosen pivot point (and to apply a negative sign if the torque is clockwise). What is the moment arm \( R_{\text{pos}} \) for the force on the positive charge in this problem?

Express your answer in terms of the given distances and angles.

**Answer:**

\[ R_{\text{pos}} = \frac{d}{2} \sin(\theta) \]

**Hint 2.** Now consider the negative charge

Now find the magnitude torque on the negative charge. To find the magnitude of the net torque on the center of mass of the dipole, you need to consider whether the torques on the two charges act in the same or opposite directions. If both torques tend to rotate the dipole in the same direction, sum them to find the magnitude of the net torque, otherwise, subtract the smaller from the larger.

**Answer:**

\[ \tau_{\text{cm}}(\theta) = dqE\sin(\theta) \]

**Correct**

**Part C**

Using the above result, find the potential energy \( U(\theta) \) associated with the dipole's orientation in the field as a function of the angle \( \theta \) shown in the figure. Take the zero of the potential to occur when the dipole is at angle \( \pi/2 \); that is, \( U(\pi/2) = 0 \).

Express \( U(\theta) \) in terms of the given quantities.

**Hint 1.** The definition of potential energy

The potential energy associated with a conservative force \( F(x) \) is given by
This definition ensures that \( F(x) = -\nabla U(x) \). The potential energy function associated with a torque is found analogously, essentially by multiplying both sides of this equation by a distance (so force is converted to torque), and then integrating with respect to angle rather than distance (since the distance has already been absorbed into the functions). Thus if the torque is given by \( \tau(\theta) \), the associated potential energy is

\[
U(\theta) = -\int_{\theta_0}^{\theta} \tau(\theta') \, d\theta'.
\]

Note that the lower bound of this integral is arbitrary. You will have to determine an appropriate constant of integration.

**Hint 2. How to choose the constant of integration**

When you integrate to find \( U(\theta) \), choose the constant of integration to satisfy the condition \( U(\pi/2) = 0 \).

**ANSWER:**

\[
U(\theta) = -dq E \cos(\theta)
\]

Correct

The dipole moment \( \vec{P} \) of a neighboring pair of opposite charges of equal magnitude \( q \) is defined to be a vector from negative to positive charge of magnitude equal to the product of \( q \) and the distance between the charges.

### Part D

In terms of the dipole moment \( \vec{P} \), which of the following is an expression for the torque on a dipole in the field \( \vec{E} \)?

**Hint 1. How to approach this part**

The general relation between the torque, dipole moment and electric field may be hard to derive. However, you can easily compare the magnitude and/or direction of the torque derived for the particular case considered above with the magnitudes and/or directions of the expressions given below for the same physical situation.

**Hint 2. Formula for \( \vec{a} \times \vec{b} \)**

Let \( \vec{c} = \vec{a} \times \vec{b} \). Then

\[
|\vec{c}| = |\vec{a}||\vec{b}| \sin \theta,
\]

where \( \theta \) is the smaller angle between \( \vec{a} \) and \( \vec{b} \).
The direction of $\vec{c}$ is given by the right-hand rule: stretch out the thumb of your right hand and curl the remaining fingers from $\vec{a}$ to $\vec{b}$. Then your outstretched thumb gives the direction of $\vec{c}$.

**Hint 3. Formula for $\vec{a} \cdot \vec{b}$**

The formula for $\vec{a} \cdot \vec{b}$ is

$$\vec{a} \cdot \vec{b} = |\vec{a}| |\vec{b}| \cos \theta,$$

where $\theta$ is the smaller angle between $\vec{a}$ and $\vec{b}$.

**Hint 4. More on torques**

The direction of the torque vector $\vec{\tau}$ can be determined from the vector equation

$$\vec{\tau} = \vec{R} \times \vec{F},$$

where $\vec{R}$ is a vector that points from the pivot to the point of action of the force $\vec{F}$.

**ANSWER:**

- $\vec{P} \times \vec{E}$
- $\vec{E} \times \vec{P}$
- $\vec{P} \cdot \vec{E}$
- $-\vec{P} \cdot \vec{E}$

Correct

**Part E**

In terms of the dipole moment $\vec{P}$, which of the following is an expression for the potential energy of a dipole in the field $\vec{E}$?

**Hint 1. Formula for $\vec{a} \times \vec{b}$**

Let $\vec{c} = \vec{a} \times \vec{b}$. Then

$$|\vec{c}| = |\vec{a}| |\vec{b}| \sin \theta,$$

where $\theta$ is the smaller angle between $\vec{a}$ and $\vec{b}$.

The direction of $\vec{c}$ is given by the right-hand rule: stretch out the thumb of your right hand and curl the remaining fingers from $\vec{a}$ to $\vec{b}$. Then your outstretched thumb gives the direction of $\vec{c}$.

**Hint 2. Formula for $\vec{a} \cdot \vec{b}$**

The formula for $\vec{a} \cdot \vec{b}$ is

$$\vec{a} \cdot \vec{b} = |\vec{a}| |\vec{b}| \cos \theta,$$

where $\theta$ is the smaller angle between $\vec{a}$ and $\vec{b}$.

**ANSWER:**

- $\vec{P} \times \vec{E}$
- $\vec{E} \times \vec{P}$
- $\vec{P} \cdot \vec{E}$
- $-\vec{P} \cdot \vec{E}$
Alternative Exercise 22.68

An insulating sphere of radius 0.140 m has uniform charge density $6.70 \times 10^{-9} \text{ C/m}^3$. A small object that can be treated as a point charge is released from rest just outside the surface of the sphere. The small object has positive charge $4.80 \times 10^{-6} \text{ C}$.

**Part A**

How much work does the electric field of the sphere do on the object as the object moves to a point very far from the sphere?

Express your answer to three significant figures and include the appropriate units.

**ANSWER:**

$$W = 2.37 \times 10^{-5} \text{ J}$$

**Correct**

Alternative Exercise 22.71

In a certain region of space, the volume charge density $\rho$ has a uniform positive value.

**Part A**

Can $\vec{E}$ be uniform in this region?

**ANSWER:**

- yes
- no

**Correct**

**Part B**

Explain your answer on previous part.

**ANSWER:**
Part C

Suppose that in this region of uniform positive $\rho$ there is a "bubble" within which $\rho = 0$. Can $\vec{E}$ be uniform within this bubble?

ANSWER:

- yes
- no

Correct

Part D

Explain your answer on previous part.

ANSWER:

Graded, see 'My Answers' for details

Exercise 34.1

A candle 4.55 cm tall is 40.0 cm to the left of a plane mirror.

Part A

Where is the image formed by the mirror?

ANSWER:

- to the right of the mirror
- to the left of the mirror

Correct
Exercise 34.13

A dentist uses a curved mirror to view teeth on the upper side of the mouth. Suppose she wants an erect image with a magnification of 2.00 when the mirror is 1.25 cm from a tooth. (Treat this problem as though the object and image lie along a straight line.)

Part A

What kind of mirror is needed? Use a ray diagram to decide, without performing any calculations.

ANSWER:

The dentist needs a concave mirror.

Correct

Part B

What must be the focal length of this mirror?

ANSWER:

\[ f = 2.50 \text{ cm} \]

Correct

Part C

What must be the radius of curvature of this mirror?

ANSWER:

\[ R = 5.00 \text{ cm} \]

Correct
A candle is placed in front of a convex mirror.

Part A

Consider the following diagrams noting that the scale is different between diagrams. In these diagrams, C and F represent the center of curvature and the focal point of the convex mirror, respectively. The image formed by the mirror is obtained using the ray tracing technique. Which diagram(s) are correct?

Type A if you think that only diagram A is correct, type AB if you think that only diagrams A and B are correct, and so on.

**Hint 1. A ray parallel to the mirror axis**
A ray parallel to the axis of a convex mirror is reflected along a line that extends back through the focal point of the mirror.

**Hint 2. A ray that passes through the focal point**
A ray that moves along a line that extends through the focal point of a convex mirror is reflected parallel to the mirror axis.

**Hint 3. A ray that extends through the center of curvature**
A ray that moves along a line that extends through the center of curvature of a convex mirror is reflected back along the same line.

ANSWER:

Correct
A convex mirror always forms an image that is upright and reduced, independently of the distance of the object from the mirror.

Part B

If the radius of curvature of the convex mirror is 50.0 cm, at what distance \( d_o \) from the mirror must the candle be placed so that its image is formed 20 cm from the mirror?

Express your answer in centimeters.

**Hint 1. How to approach the problem**
To determine at what distance the candle should be placed, you can use the mirror equation. To do that, you will need the focal length of the mirror and the image distance.
**Hint 2. The mirror equation**

The mirror equation for a mirror with a focal length $f$ is

$$\frac{1}{d_o} + \frac{1}{d_i} = \frac{1}{f},$$

where $d_o$ and $d_i$ are the object distance and the image distance, respectively.

**Hint 3. Find the focal length**

What is the focal length $f$ of a convex mirror of radius 50.0 cm?

Express your answer in centimeters.

**Hint 1. Focal length of spherical mirrors**

The focal length of a spherical mirror is always half the radius of the mirror. Moreover, conventionally it has a positive sign when the focal point is in front of the mirror, and a negative sign when the focal point is behind the mirror.

ANSWER:

$$f = -25.0 \text{ cm}$$

**Hint 4. Find the image distance**

Considering the sign convention used with mirrors, what is the image distance $d_i$?

Express your answer in centimeters.

**Hint 1. Sign convention for mirrors**

Recall that image distance is positive for a real image, that is, an image that forms in front of a mirror, and negative for a virtual image, that is, an image that forms behind a mirror.

ANSWER:

$$d_i = -20 \text{ cm}$$

ANSWER:

$$d_o = 100 \text{ cm}$$

**Correct**

The convex mirror is now replaced by a concave mirror with the same radius of curvature.

**Part C**

Consider the following diagrams, where now C and F represent, respectively, the center of curvature and the focal point of the concave mirror. Again, the image formed by the mirror is found by ray tracing. Choose the correct diagram(s).

Type A if you think that only diagram A is correct, type AB if you think that only diagrams A and B are correct, and so on.
**Hint 1.** A ray parallel to the mirror axis
A ray parallel to the axis of a concave mirror is reflected through the focal point of the mirror.

**Hint 2.** A ray that passes through the focal point
A ray that passes through the focal point of a concave mirror is reflected parallel to the mirror axis.

**Hint 3.** A ray that passes through the center of curvature
A ray that moves along a line that passes through the center of curvature of a concave mirror is reflected back along the same line.

**ANSWER:**

Correct

**Part D**

Assuming that the candle remains at a distance $d_o$ from the mirror (the same distance found in Part B), at what distance $d_i$ from the mirror will its image form now?

Express your answer in centimeters.

**Hint 1.** How to approach the problem
As before, you can use the mirror equation to find the correct answer, but be careful to use the correct focal length for a concave mirror.

**Hint 2.** Find the focal length
What is the focal length $f$ of a concave mirror of radius 50.0 cm?

Express your answer in centimeters.

**Hint 1.** Focal length of spherical mirrors
The focal length of a spherical mirror is always half the radius of the mirror. Moreover, conventionally it has a positive sign when the focal point is located in front of the mirror, and a negative sign when the focal point is behind the mirror.
Problem 34.58

Part A

A magnifying glass uses a converging lens with a refractive power of 20 diopters. What is the magnification if the image is to be viewed by a relaxed eye (image at the far point) having a near point of 25 cm and a far point at infinity?

ANSWER:

- 4.0 x
- 2.0 x
- 3.0 x
- 1.0 x
- 5.0 x

Correct

Problem 34.48

Part A

A person can read the newspaper when it is held at 60 cm from his eyes. What should the focal length of his contact lenses be to allow him to read the newspaper comfortably at a distance of 30 cm?

ANSWER:
Exercise 35.8

Coherent light with wavelength 440 nm falls on a pair of slits. On a screen 1.90 m away, the distance between dark fringes is 3.90 mm.

Part A

What is the slit separation?

Express your answer to three significant figures and include the appropriate units.

ANSWER:

\[ d = 0.214 \text{ mm} \]

Score Summary:

Your score on this assignment is 93.8%.
You received 9.38 out of a possible total of 10 points.